

*Unit B: Analysis of Games and Numbers*

*Half Course IV*

## ***HALF COURSE IV***

### **Unit B: Analysis of Games and Numbers**

**Hours: 5**

#### **General Learning Outcome:**

**Develop, use, and justify mathematical strategies by analyzing a variety of puzzles and games; develop an awareness of how numbers are used in society.**

*The material provided for this unit should be used throughout the course to provide a change of pace in a context which is enjoyable, yet calls for mathematical and logical thinking.*

#### **Specific Outcomes**

- B-1 Demonstrate the use of an appropriate strategy in solving puzzles and playing games involving patterns.
- B-2 Be aware of how numbers are used throughout society.

# ***ANALYSIS OF GAMES AND NUMBERS***

## **Instructional Materials**

- *Essentials of Mathematics 11*
- See Appendix I for possible activities.
- See Appendix II for additional resources.

**PRESCRIBED LEARNING  
OUTCOMES**

**General Outcome**

Develop, use, and justify mathematical strategies by analyzing a variety of puzzles and games; develop an awareness of how numbers are used in society.

**Specific Outcome(s)**

B-1 demonstrate the use of an appropriate strategy in solving puzzles and playing games involving patterns

**SUGGESTIONS FOR INSTRUCTION**

The objective of this unit is to have students play the games and find winning strategies. Students are expected to find the strategies and be able to explain them by demonstration, orally, or in writing.

Sufficient time should be devoted to playing and enjoying a game before analysis begins. Then, allow students to discuss the game and articulate their “winning” strategies.

Finding the strategy is the first step. The subsequent steps are of equal importance. Students will be examining their own thinking processes. Such examinations are not easy and may be an unusual experience for them. The actual results may be trivial, but finding the result is the real objective. Another key objective is the communication of strategies and thought processes.

Teachers should try these activities before giving them to students.

The activities are independent of grade and could be repeated. A variation on a Senior 2 activity may be appropriate in Senior 3.

It is not intended that the activities be taught in a block of time, but, rather, dealt with periodically during the year.

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- |                                 |                           |
|---------------------------------|---------------------------|
| ✓ Communications<br>Connections | ✓ Patterns                |
| ✓ Number Sense                  | ✓ Problem Solving         |
| ✓ Organization and<br>Structure | ✓ Reasoning<br>Technology |
|                                 | ✓ Visualization           |
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## SUGGESTIONS FOR ASSESSMENT

Participation and the willingness to accept the challenge of the activities is important. Keep a daily record.

You may wish to keep anecdotal notes on how students develop their strategies.

Recreational activities are an appropriate context for journal writing on both content and attitudinal factors.

SUGGESTED LEARNING  
RESOURCES**Print**

*Senior 3 Consumer Mathematics (35S) Part IV: A Course for Distance Learning.* Winnipeg, MB: Manitoba Education, Training and Youth, 2001.

— Cover Assignments

Baron, C., et al. *Essentials of Mathematics 11.* Victoria, BC: British Columbia Ministry of Education, 2002.

Blocksma, Mary. *Reading the Numbers: A Survival Guide to the Measurements, Numbers, and Sizes Encountered in Everyday Life.* New York, NY: Penguin Books, 1989.  
[ISBN 0-14-01.0654-5]

Hopkins, N.J., J.W. Mayne, and J.R. Hudson. *The Numbers You Need.* Detroit, MI: Gale Research, Inc., 1992. [ISBN 0-8103-8373-X]

Muschla, G.R., and J.A. Muschla. *Hands-On Math Projects with Real-Life Applications.* West Nyack, NY: The Center for Applied Research in Education, 1996.  
[ISBN 0-87628-384-9]

National Council of Teachers of Mathematics. *Historical Topics for the Mathematics Classroom.* Reston, VA: Author, 1969, 1989. [ISBN 0-87353-281-3]

Posamentier, A.S., and J. Stepelman. *Teaching Secondary School Mathematics: Techniques and Enrichment Units.* Toronto, ON: Merrill, 1990, 1986. [ISBN 0-675-21209-X]

See Appendix II for more resources.

**PRESCRIBED LEARNING  
OUTCOMES**

B-2 be aware of how numbers are used throughout society

**SUGGESTIONS FOR INSTRUCTION**

Teachers should present a topic showing how numbers are used in society and have students discuss the use of numbers.

An example: Read the following information on paper and answer the questions that follow.

**Paper**

What does “20-pound” mean when applied to paper? And a ream is 500 sheets of 8½ x 11-inch stock, isn't it? There are many types of commercial paper: When a ream (usually a stack of 500 sheets of paper measuring 2 feet by 3 feet) weighs approximately 20 pounds at a temperature of 70 degrees and a humidity of 50 percent, it's called 20-pound paper. This is how some commercial paper is sized. The heavier the weight, of course, the thicker the paper usually is, but the pound designation always applies to that oversized stack, not the 500 8½ x 11-inch sheets you buy at the stationers.

A ream is not always 500 — sometimes it's 480 sheets. And the size of the sheet is not always 2 feet by 3 feet — sometimes it's 17 inches by 22 inches. Usually you can get 2000 letterhead sheets out of a ream, but not always.

It's usually at the printer's that you have to choose the weight of your paper. Despite the confusion of size and the number of sheets in a ream, here are a few general guidelines:

**Some Common Commercial Paper Weights**

- 9 lbs                      Onionskin
- 16 lbs                     Mimeographing paper
- 20 lbs                     Standard typing paper
- 24 lbs                     Standard letterhead paper
- 60 lbs                     Good for printing on both sides
- 65 lbs                     Business cards, postcards
- 100 lbs                    Magazine paper (coated stock)
- 120 lbs                    Poster board

In addition to commercial paper, art paper is also weighted, but is sized by a different standard. The size of the sheets in the stack to be weighed (again 480 to 500 sheets) depends on what the paper is for. The standard size for watercolour paper is 22 inches by 30 inches, but other art papers may not be the same.

- ✓ Communications    ✓ Patterns
- Connections        ✓ Problem Solving
- ✓ Number Sense        ✓ Reasoning
- ✓ Organization and    ✓ Technology
- Structure            ✓ Visualization

**Paper:** Reprinted by permission of Mary Blocksma, from *Reading the Numbers* by Mary Blocksma, © 1989 by Mary Blocksma.

*(continued)*

SUGGESTIONS FOR ASSESSMENT

SUGGESTED LEARNING  
RESOURCES

Print (continued)

Sutcliffe, Andrea. *Numbers: How Many, How Long, How Far, How Much . . . All the Numbers You'll Ever Need*. New York, NY: Harper Perennial, 1996.  
[ISBN 0-06-273362-1]

Reimer, W., and L. Reimer. *Historical Connections in Mathematics*. Fresno, CA: AIMS Educational Foundation, 1992.  
[ISBN 1-881431-35-5]

*Scientific American*. The paradox box. San Francisco, CA: W.H. Freeman, 1975.

Verhille, C., and R. Blake. "The Peg Game." *Mathematics Teacher* (January, 1982): 39-43.

**Note 1:** Many bookstores carry problem and puzzle books.

**Note 2:** *The Mathematics Teacher* and *Mathematics Teaching in the Middle School* from the National Council of Teachers of Mathematics have useful articles and activities.

**PRESCRIBED LEARNING  
OUTCOMES**

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**SUGGESTIONS FOR INSTRUCTION**

**PRESCRIBED LEARNING  
OUTCOMES**

B-2 be aware of how numbers are used throughout society  
– *continued*

**SUGGESTIONS FOR INSTRUCTION**

**Some Common Art Paper Weights**

- 8 to 48 lbs tracing papers
- 13 to 20 lbs bond papers
- 28 to 32 lbs ledger papers
- 65 to 80 lbs cover papers (commercial papers)
- 90 to 400 lbs watercolour papers

**Questions**

1. What is a ream of paper?
2. What does “24 lb” mean when applied to paper?
3. What do the different weights of paper describe?
4. Have you worked with any of the different kinds of paper listed above? Would you agree with the statement that the greater the weight, the thicker the paper?
5. Why is the grading system for art paper different from the one used for grading commercial paper? Could you compare the papers for art and commercial uses?
6. Gather samples of different types of papers. Arrange them from thinnest to thickest. Do they match the weights listed above? (Do not weigh the paper; simply compare them to the one given previously.)
7. Compare the area of commercial paper to the area of watercolour paper.
8. How many sheets of 8½ x 11-inch paper do you get from a ream of commercial paper? (Do the calculations for both 2 feet by 3 feet and 17 inches by 22 inches.)

**Note:** If micrometers are used in the class, students could do an experiment dealing with measuring the thickness of paper and deciding how many sheets would fit into various spaces or estimating the number of sheets that would fit into a certain size binder. Are there other measuring instruments you can use? For example, 500 sheets could be measured using a ruler, and then the measurement could be divided by 500 to find the thickness of one sheet. Other ideas for experiments are possible.

- ✓ Communications
- ✓ Connections
- ✓ Number Sense
- ✓ Organization and Structure
- ✓ Patterns
- ✓ Problem Solving
- ✓ Reasoning
- Technology
- ✓ Visualization

**SUGGESTIONS FOR ASSESSMENT**

**SUGGESTED LEARNING  
RESOURCES**

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# Appendix I

## Teacher Information: Probability Stones Game

### Skills Required

- knowledge of simple probabilities

### Materials

- gameboard
- coloured markers (coins, bingo chips, etc.)
- two six-sided dice per group

### When To Use

This game may be played at any time or used as an introduction to the Applications of Probability unit.

### Teaching Information

- The game can be played by small groups of 2 or 3. Each group should have a pair of six-sided dice and a copy of the gameboard.
- Go over the rules with the class before beginning. A sample game played on the overhead might be useful.
- An enlarged gameboard that is laminated may be useful.

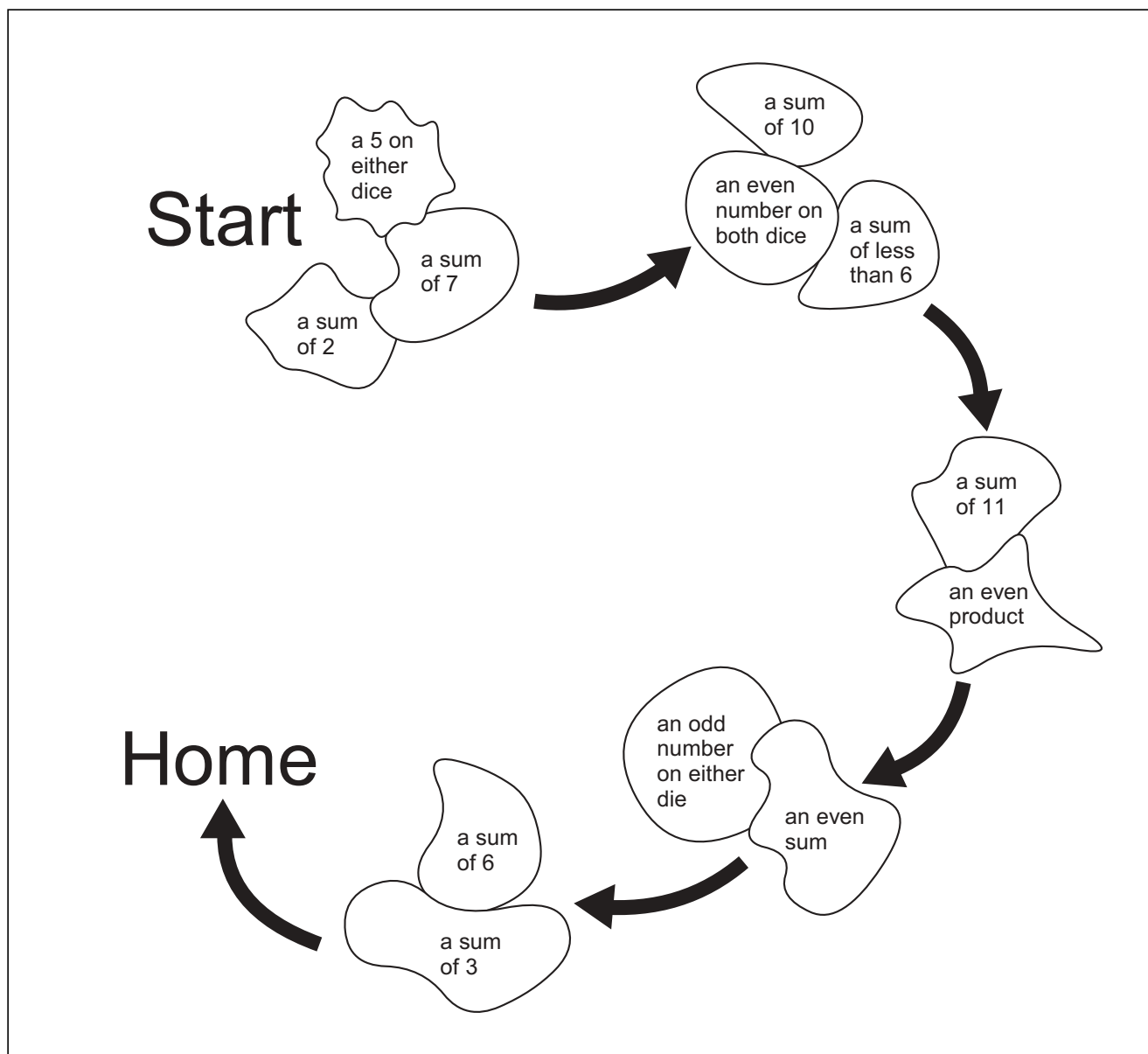
### Extensions

- Have students use different kinds of dice (4-sided, 8-sided, 12-sided, etc.) with the given board.
- Have students create a different gameboard for the game.
- Have students write about the game and the strategies used in their journal or notebook.

## Blackline Master: Probability Stones Game

### Rules

- The object of the game is to reach home first.
- Choose who will go first.
- Each player places his or her counter in any one of the three shapes at the Start.
- The first player rolls two six-sided dice. If the result of the roll matches the stone the player's counter is on, the counter gets moved to the next group of stones (follow the arrows). Play passes to the next person.
- If the result of the roll does not match the stone the player is on, the counter remains where it is and play passes to the next person.



## Teacher Information: Stamp Collector

### Skills Required

- pattern recognition
- visualization

### Materials

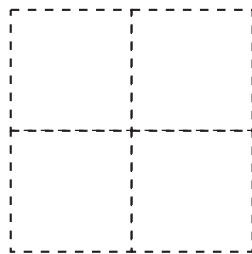
- blackline masters

### When To Use

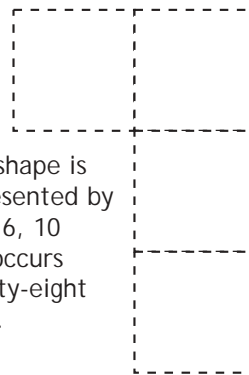
This activity may be done at any time.

### Solution

There are five basic ways to have four stamps together.



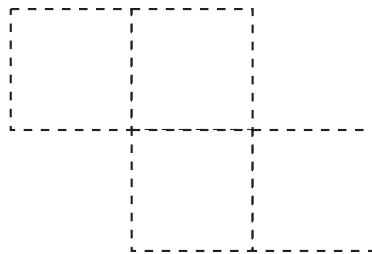
This shape is represented by 1, 2, 5, 6 and occurs six ways.



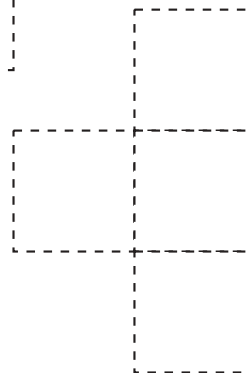
This shape is represented by 1, 2, 6, 10 and occurs twenty-eight ways.



This shape is represented by 1, 2, 3, 4 and occurs three ways.



This shape is represented by 1, 2, 6, 7 and occurs fourteen ways.



This shape is represented by 2, 5, 6, 10 and occurs fourteen ways.

In total, the stamp collector can tear four stamps off in 65 ( $6 + 28 + 3 + 14 + 14$ ) different ways.

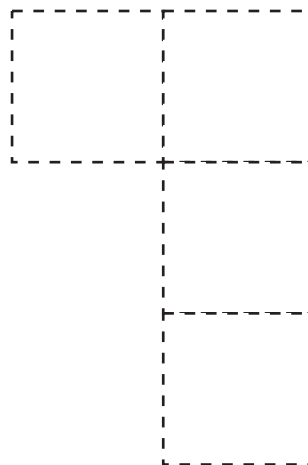
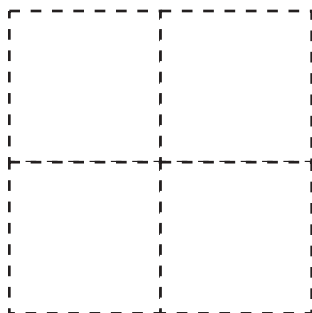
## Blackline Master: Stamp Collector

A stamp collector has a sheet of 12 stamps, as pictured below.

1	2	3	4
5	6	7	8
9	10	11	12

When a fellow collector asked to buy four stamps, she specified that all four stamps must be joined together along their sides. How many different ways could the collector tear them loose from the sheet above? Explain your reasoning.

Hint: Two possibilities are shown below: 1–2–5–6 and 1–2–6–10.



## Teacher Information: Multiplying by Numbers Ending in 5

### Skills Required

- number sense
- pattern recognition
- visualization

### Materials

- calculator

### When To Use

This activity may be done at any time.

### Teacher Information

Here is a quick way to square numbers ending in 5. Have students discover the pattern by using a calculator to complete the following squares:

$$15^2 = 225$$

$$25^2 = 625$$

$$35^2 = 1225$$

$$45^2 = ???$$

The pattern: Each result ends in 25. The digits preceding the 25 are determined by multiplying the original preceding digit by itself increased by one. For example, to square 45, the last two digits are 25 and the preceding digits are  $4 \times (4 + 1) = 20$ . Therefore,  $45^2 = 2025$ .

In algebraic form:  $n5^2 = n(n + 1)25$  where  $n(n + 1)$  represents the preceding digit and one more than this digit.

1. Have students use the pattern to determine  $55^2$ ,  $65^2$ ,  $75^2$ ,  $85^2$ , and  $95^2$ .
2. Will this method work if there is more than one digit preceding the 5 (for example,  $105^2$ )? Determine a few others (for example,  $115^2$ ,  $205^2$ ).

**Note:** This type of pattern recognition can be reinforced through the use of mental math.

## Teacher Information: Dinner with the Wellingtons

### Skills Required

- logical reasoning

### Materials

- blackline master

### When To Use

This activity may be done at any time.

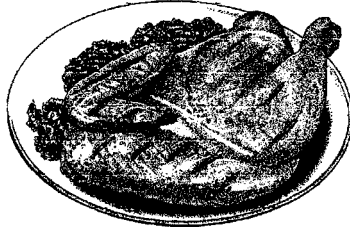
### Solution

Students will find it helpful to use the grid provided.

	Ham	Chicken	Pork	Salmon	Steak	Carrots	Corn	Peas	Broccoli	Beans (green)
Tim	O	X	O	O	O	X	O	O	O	O
Marilyn	O	O	O	X	O	O	O	O	X	O
Heather	O	O	O	O	X	O	X	O	O	O
Britany	X	O	O	O	O	O	O	X	O	O
Stefany	O	O	X	O	O	O	O	O	O	X
Carrots	O	X	O	O	O					
Corn	O	O	O	O	X					
peas	X	O	O	O	O					
broccoli	O	O	O	X	O					
beans (green)	O	O	X	O	O					

## Blackline Master: Dinner with the Wellingtons

The Wellington family went out to dinner last evening. Tim, Marilyn, Heather, Britany, and Stefany each ordered a different main course. Ham, chicken, pork, salmon, and steak were the only main courses on the menu. Each family member also ordered a baked potato, salad, and a different vegetable. The five vegetables on the menu were carrots, corn, peas, broccoli, and beans (green). Use the information below to determine what main course and vegetable each family member had.



1. The person who had steak and corn sat on the left of Tim and on the right of Stefany.
2. The person who had chicken doesn't play tennis; but the person who had ham, the person who had broccoli, and Heather played doubles tennis yesterday with Stefany.
3. The person who had broccoli did not have it with pork; Britany did not order pork either.
4. The person who had ham recently saw Tim; the person who had broccoli helped Tim wrap a present for Tim's father.
5. Marilyn's and Stefany's vegetables begin with the same letter as Tim's and Heather's vegetables.

In order to solve this logic problem, it is helpful to organize the information in a grid such as on the following page.

To use this grid, place an "X" wherever you can say Yes and "O" where it is clearly No. In this case, statement 3 tells us that Britany did not eat pork; therefore, we can place an O at the intersection of Britany and pork. Use the clues provided by the statements to fill in the grid with Xs and Os.

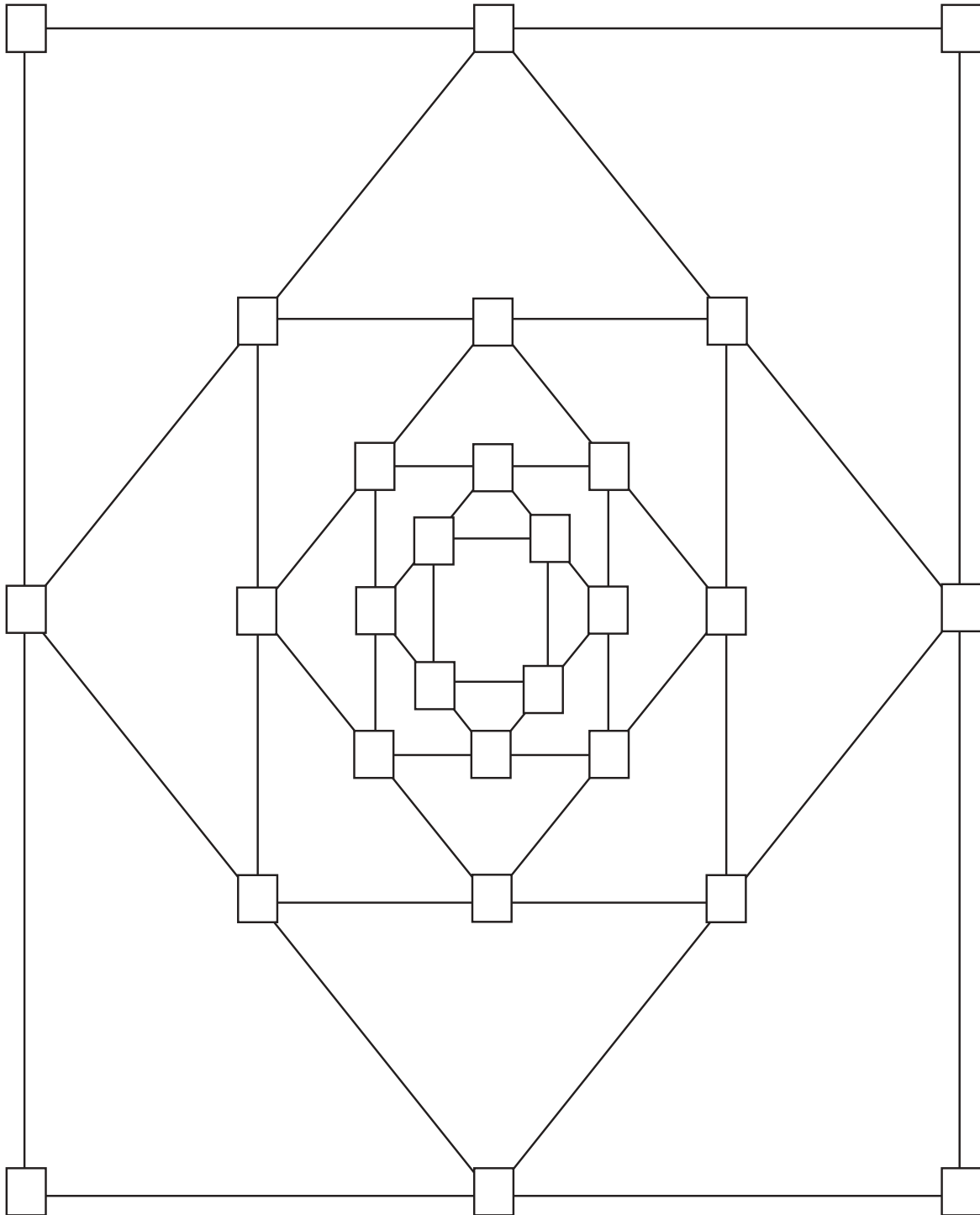
	Ham	Chicken	Pork	Salmon	Steak	Carrots	Corn	Peas	Broccoli	Beans (green)
Tim										
Marilyn										
Heather										
Britany			O							
Stefany										
Carrots										
Corn										
peas										
broccoli										
beans (green)										



## Blackline Master: Numsquare

**Instructions:** Place any four numbers in the four corner squares. Place the difference of the two numbers in the square between them. Continue.

**Objective:** Not to end with four 0s in the middle squares.



## Teacher Information: Manitoba Statistics in Your Life

### Skills Required

- reading presented data from a list and a graph
- percentage skills

### Materials

- blackline masters
- calculator

### When To Use

Although this activity may be done at any time, it could be used with the Owning and Operating a Vehicle unit or the Data Analysis unit.

### Teaching Information

These are real data based on MPI's Annual Report. Some basic calculations are required to answer the questions. Students will need to know (or look up) the population figures for Winnipeg and for Manitoba.

### Solutions

$$1. \frac{176\,555}{93\,473 + 93\,421} \times 100 = 94.5\%$$

$$2. \frac{10\,339}{93\,473 + 93\,421} \times 100 = 5.5\%$$

$$3. \frac{358\,129\,000}{93\,473 + 93\,421} = \$1916.21$$

$$4. 186\,894 : 658\,602 \text{ or } 10\,383 : 36\,589$$

5. Some drivers have more than one vehicle registered in their name. Some drivers have no vehicles registered in their name.

6. 1 146 331 (June 1, 1997 according to Manitoba Regional Health Authority.)  
<[www.gov.mb.ca/health/annstats/19971998/p1t1.html](http://www.gov.mb.ca/health/annstats/19971998/p1t1.html)>

$$7. \frac{658\,602}{1\,146\,331} \times 100 = 57.5\%$$

**Solutions (continued)**

$$8. \frac{93\,421}{93\,473 + 93\,421} \times 100 = 49.99\%$$

$$9. \frac{93\,473}{93\,473 + 93\,421} \times 100 = 50.01\%$$

10. 648 764 (June 1, 1997 according to Manitoba Regional Health Authority.)  
<[www.gov.mb.ca/health/annstats/19971998/p1t1.html](http://www.gov.mb.ca/health/annstats/19971998/p1t1.html)>

11. Percent of Manitoba population that lived in Winnipeg:

$$\frac{648\,764}{1\,146\,331} \times 100 = 56.6\%$$

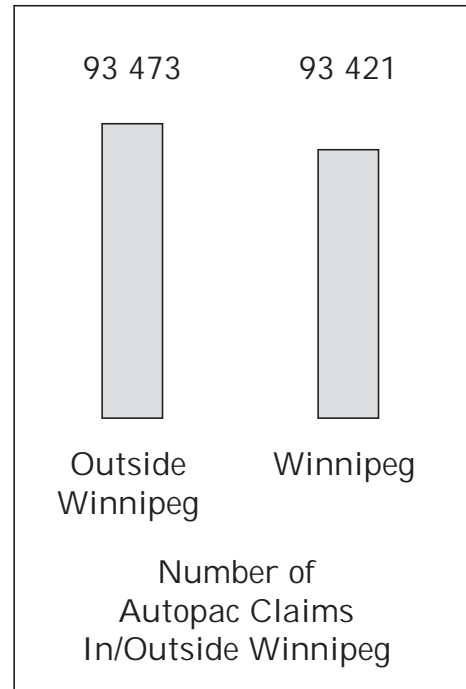
Although approximately 57% of the Manitoba population lived in Winnipeg, only 50% of the Autopac claims came from Winnipeg.

## Blackline Master: Manitoba Statistics in Your Life

The statistics in this activity come from the Manitoba Public Insurance Corporation. This Corporation provides the automobile insurance known as Autopac. Think about the information provided below and respond to the questions that follow.

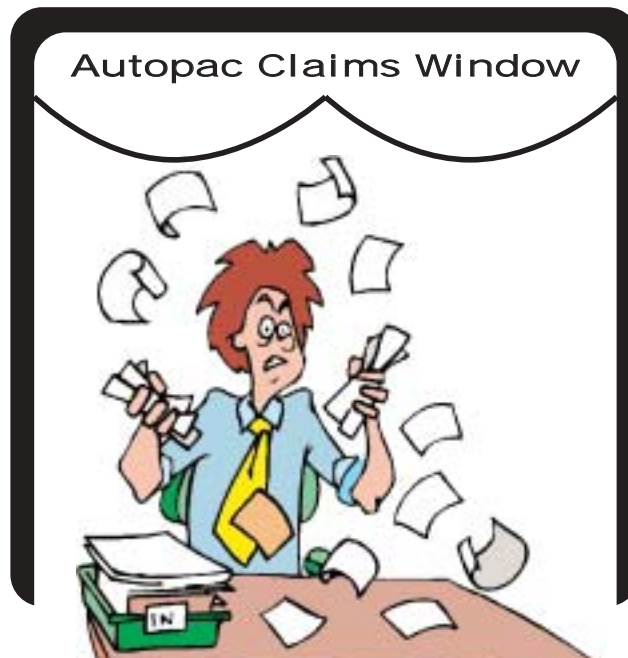
**For the period March 1, 1997 – February 28, 1998**

- Average number of Autopac claims reported per working day: **769**
- Total Autopac claims reported: **186 894**
- Bodily injury claims reported: **10 339**
- Property damage claims reported: **176 555**
- Total theft claims reported in Winnipeg: **8309**
- Total theft claims reported elsewhere in the province: **1550**
- Percentage increase in total theft claims over last year: **0 %**
- Autopac agency outlets as of 28 February 1998: **352**
- Calls taken by the Autopac line: **773 303**
- Autopac policies in force – 1997 average: **755 761**
- Licensed drivers in Manitoba in 1997: **658 602**
- Claims incurred (\$000): **358 129**



### Questions

1. What percent of the number of Autopac claims were claims for property damage?
2. What percent of the number of Autopac claims were for bodily damage?
3. What was the average cost per claim?
4. What is the ratio of total Autopac claims to licensed drivers in Manitoba?
5. Why are the number of Autopac policies in force not the same as the number of licensed drivers in the province?
6. What was the population of Manitoba in 1997?
7. What percent of the population of Manitoba were licensed drivers?
8. What percent of the Autopac claims were inside Winnipeg?
9. What percent of the Autopac claims were outside Winnipeg?
10. What was the population of Winnipeg in 1997?
11. How does the percent of Autopac claims in 1997 compare with the percent of the Manitoba population in Winnipeg in that year?



## Appendix II

## Additional Resources

### Print

The Diagram Group. *The Little Giant Encyclopedia of Games for 1 or 2*. Sterling Publishing Company Inc. ISBN 0-8069-0981-1.

Fleisher, Paul. *Brain Food: Games That Make Kids Think*. Zephyr Press. ISBN 1-56976-072-1.

Nasht, Helen, and Dorothy Masterson. *Humorous Cryptograms*. Sterling Publishing Company Inc. ISBN 0-8069-3982-6.

Tuller, Dave, and Michael Rios. *Mensa Math and Logic Puzzles*. Sterling Publishing Company Inc. ISBN 0-8069-4199-5.

### Internet

There are many sites on the Internet with problems and puzzles. When searching for problems and puzzles, use the words "mathematical games."

As of February 2003, the following sites were available:

#### *Fun Brain*

<<http://www.funbrain.com>>

This site offers some interactive games. Some of the games could be adapted to pencil-and-paper games.

#### *Puzzlemaker*

<<http://www.puzzlemaker.com>>

This site allows teachers to create their own puzzles. One of the puzzle types is cryptograms. When using the site, be sure to print the answer with the puzzle as all puzzles are created new with each visit.

#### *This Is Mega Mathematics*

<<http://www.c3.lanl.gov/mega-math/>>

There are a variety of activities here. One of the activities involves map colouring. Another activity involves games with graphs.