

Unit A: Problem Analysis

Half Course IV

HALF COURSE IV

Unit A: Problem Analysis

Hours: 9

General Learning Outcome:

Develop and use mathematical strategies to solve problems in different situations.

The intent of this unit is to provide a range of interesting problems of a primarily non-algebraic nature. These problems augment the work in other units.

Specific Outcome

A-1 Solve problems using a variety of non-algebraic approaches.

PROBLEM ANALYSIS

Instructional Materials

- *Essentials of Mathematics 11*
- See Appendix I for possible activities.
- See Appendix II for additional resources.

**PRESCRIBED LEARNING
OUTCOMES**

General Outcome

Develop and use mathematical strategies to solve problems in different situations.

Specific Outcome(s)

A-1 solve problems using a variety of non-algebraic approaches

SUGGESTIONS FOR INSTRUCTION

Examples of non-algebraic approaches include geometry, networks, flow charts, organizational charts, simulations, etc.

Remember that for activities in this unit, the journey is more important than the destination. It is beneficial to discuss multiple approaches to solving these problems, particularly when the approaches have been developed by students. Are some approaches “better” than others? Why? On what grounds?

The problems contained in Appendix I are intended to provide material which is interesting in its own right and which complements the other units of the program. It is illustrative rather than exhaustive. Some activities have been chosen to illustrate a wide variety of job and consumer applications of mathematics that are largely non-algebraic. Others have been chosen because they are intrinsically interesting or because they challenge students to find and to use new ways of analyzing and thinking mathematically. All students do not need to engage in the same activities.

The activities in Appendix I are presented in **no** particular sequence. Teachers are encouraged to supplement this set of activities with material from other sources, such as the Internet. A preliminary list of possible resources is included in Appendix II.

It is suggested that these problems and activities be interspersed throughout the course as either extensions, enrichment, or a change of pace in the day-to-day work of the classroom. Some of them will link directly to particular units, but most are independent and **may** be used at any time. One approach would be to introduce problem analysis with a few days, possibly up to a week, of work on these activities. Intersperse the remainder throughout the course.

-
- ✓ Communications
 - ✓ Connections
 - ✓ Organization and Structure
 - ✓ Patterns
 - ✓ Problem Solving
 - ✓ Reasoning
 - Technology
 - ✓ Visualization
-

SUGGESTIONS FOR ASSESSMENT

Students' progress should be assessed over long periods of time. Look, for example, for an increasing use of a variety of problem-solving strategies and increasingly sophisticated explanations. Anecdotal records of how students work in pairs or groups on these activities is appropriate. Well-developed solutions and examples of reasoning could become part of a student's portfolio.

Problem-solving activities are generally not appropriate on pencil-and-paper timed tests.

SUGGESTED LEARNING RESOURCES

Print

- Austin, J.D. *Applications of Secondary School Mathematics*. Reston, VA: NCTM, 1991.
- Giblin, P., and I. Porteous. *Challenging Mathematics*. Toronto/New York: Oxford University Press, 1990.
- Hirsch, C.R., and R.A. Laing. *Activities of Learning and Teaching*. Reston, VA: NCTM, 1993.
- Mathematical Association of America and National Council of Teachers of Mathematics. *A Sourcebook of Applications of School Mathematics*. Reston, VA: NCTM, 1980.
- National Council of Teachers of Mathematics. *NCTM Student Math Notes*. Reston, VA: NCTM, n.d.
- Senior 3 Consumer Mathematics (35S) Part IV: A Course for Distance Learning*. Winnipeg, MB: Manitoba Education, Training and Youth, 2001.
— Cover Assignments
- Swetz, F., and J.S. Hartzler. *Mathematical Modeling in the Secondary School Curriculum*. Reston, VA: NCTM, 1991.
- Two journals which contain useful teaching ideas are:
- The Mathematics Teacher*. National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA: 22091-1593.
- Mathematics in School*. The Mathematical Association, 259 London Road, Leicester, UK: LE2 3BE.
- See Appendix II for a list of additional resources.

Appendix I

Teacher Information: Sunset Hockey League

Skills Required

- pattern recognition
- basic arithmetic

When to Do

This activity may be done at any time.

Teaching Information

- Students may need some direction to get started on a basic time card
- Extensions could include finding the rink's profit given staff wages, supplies, etc.

Solution

1. 34 teams
2. \$41.91 per team
3. \$38.24 per hour

Blackline Master: Sunset Hockey League

The Sunset Hockey League has rented a rink from 8:00 a.m. to 5:00 p.m. on Saturday, and from 6:00 a.m. to noon on Sunday. Each team will share the ice with another team for an hour-long practice or game. The ice must be cleaned at the beginning of each day, and every two hours thereafter. The cleaning of the ice requires 15 minutes.

1. How many teams can be booked to play?
2. If the rental costs, including staff and cleaning, total \$1425, find the cost for each team.
3. If each cleaning costs \$12.50, find the hourly rate for the staff.



Teacher Information: Understanding Your Vehicle

Skills Required

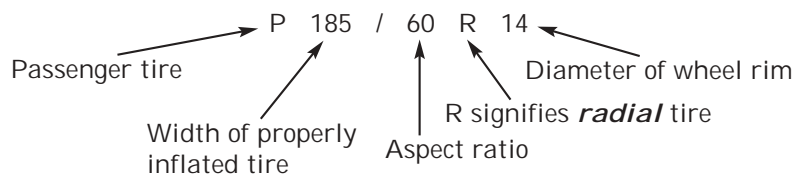
- careful reading
- organization of information

When to Do

While this activity may be done at any time, the content relates to Unit D: Operating a Vehicle.

Teaching Information

This activity is primarily informational and will be of particular interest to those students who get excited about cars. It looks at how numbers are used: (1) to designate tire sizes and uses; and (2) to indicate the viscosity of motor oil. An example of the five pieces of information in a tire designation:



Solutions

1. Tire Codes

- P195/60R15 87S: Passenger car tire; inflated width 195 cm; tire section height is 60% of width; radial tire with 15-inch rim.
- P205/75R14 XL: Passenger car tire; inflated width 205 cm; tire section is 75% of the width; radial tire with 14-inch rim.
- LT235/85R16 10E: Light truck tire; inflated width 235 cm; tire section is 85% of the width; radial tire with 16-inch rim.
- 33 × 12.50R15LT 6C: Light truck radial tire; tire diameter is 33 inches; tire width is 12.5 inches; 15-inch rim.

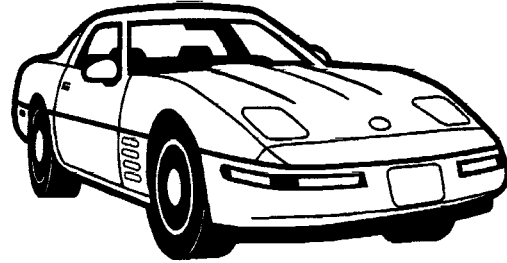
Note: The intent is to have students be able to give the five pieces of information in each of the above items. The remaining information would be enrichment. In (a) 87 is a load index; the load index range is 75-100 with 75 representing 851 pounds and 100 representing 1760 pounds. The "S" is a speed rating of up to 112 mph. Numbers in truck tires may refer to the ply of the tire. The final numbers and letters may also be a reference to traction, treadwear, or heat resistance.

Understanding Your Vehicle, Tires: Reprinted by permission of Mary Blocksma from *Reading the Numbers* by Mary Blocksma, © 1989 by Mary Blocksma.

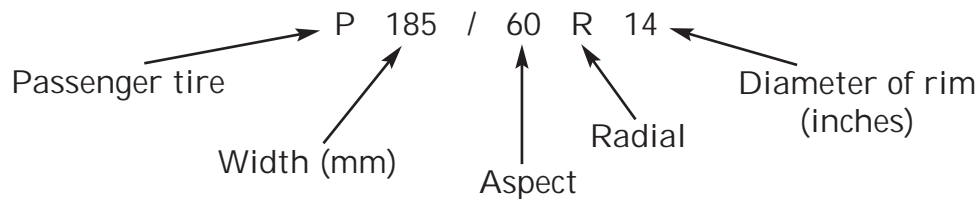
Blackline Master: Understanding Your Vehicle

Tires

What do the numbers and letters that are on the tires of your vehicle really mean? While the tire sizing system may change and sizing in some companies may vary, the following information should give you a better understanding.



A typical tire size: P185/60R14. What these letters and numbers mean is explained below:



"P" means the tire is intended for a **passenger vehicle**. "LT" would refer to a tire for a **light truck**.

"185" refers to the width of a properly inflated tire from sidewall to sidewall in millimetres. If this number contains a decimal, it means the tire width is given in inches.

"60" is the **aspect ratio** of the tire. This ratio is a percentage between the tire section height and the section width. In our example, the height is 60 percent of the width. A low aspect ratio should give the car better control since the tire rides low and wide. The advantage of a high aspect ratio is more give on a bumpy road.

"R" refers to a **radial** tire. A "B" would mean a **belted bias** tire and a "D" signifies a **diagonal bias** tire. All refer to the way or direction that layers of rubber are added to the tire.

"14" refers to the diameter of the rim of the wheel in inches. 13, 14, 15, or 16 are common rim sizes that have been standardized throughout the automobile industry. While it is possible to select a different width, aspect ratio, or type of tire, the rim diameter cannot be changed.

Understanding Your Vehicle, Tires: Adapted by permission of Mary Blocksma, from *Reading the Numbers* by Mary Blocksma, © 1989 by Mary Blocksma.

What about $31 \times 10.5R15LT$? The "31" refers to the diameter of the tire in inches. The width of the tire, "10.5", is also in inches. The rest is the same as P tires.

In light truck tires, these numbers may precede a number and letter; e.g., LT220/80R16 8C. The number (8) refers to the ply rating (layers of rubber added to the tire) and the letter (C) refers to the load range. The lower the letter, the smaller the load-carrying capacity of the tire.

In passenger car tires, if the number is followed by a two-digit number and letter (i.e., P155/80R13 79S), the "79S" represents a speed rating. There are a number of speed-rating systems and knowledge of each goes beyond the scope of this course.

1. Explain what is meant by each of the following tire codes:

a) P195/60R15 87S

b) P205/75R14 XL

c) LT235/85R16 10E

d) $33 \times 12.50R15LT$ 6C

2. With your teacher's permission, make a trip to your school parking lot. Find and record at least two different tire codes, then complete the following table:

Tire Code	Meaning	Type of Vehicle

Oil

When the owner’s manual of your vehicle should recommend a specific grade of oil, such as SAE 10W/30, do you understand what the references mean?

SAE means that numbers rating the oil have been assigned by the Society of Automotive Engineers.

The “W” stands for winter. Oils with two viscosity ratings (two numbers) are oils that have been tested at both cold and hot temperatures. **Viscosity** refers to the thickness of the oil, or its ability to flow. The colder it becomes, the thicker the oil gets. An oil needs to be thin enough to flow in winter so that an engine can start but thick enough to perform its lubricating function at high temperatures.

The “10” has no units; it arbitrarily stands for oil of a certain thinness tested at 0°F. This first number may be rated 5, 10, 15, or 20.

- 10W oil flows half as fast as 5W oil
- 15W oil flows half as fast as 10W oil
- 20W oil flows half as fast as 15W oil

Generally, the colder the winter temperature, the lower the “W” rating needs to be.

The second number, a regular rating, shows the viscosity of the oil at 210°F. The second number may be rated at 20, 30, 40, or 50. Again, if the number is higher, the oil is thicker and runs slower. The higher the number, the better the protection against hot temperatures.

Understanding Your Vehicle, Oil: Adapted by permission of Mary Blocksma, from *Reading the Numbers* by Mary Blocksma, © 1989 by Mary Blocksma.

Teacher Information: The Horse Ranch

Skills Required

- simple arithmetic
- spatial arrangement
- logical reasoning

When to Do

This activity may be done at any time.

Teaching Information

Students should be provided with one or more copies of the worksheet containing diagrams of the farmer's pastures. Teachers may want to have students working in pairs. For Question #1 in the activity, challenge students to see how many different solutions they can come up with.

Solutions

1. Many solutions are possible. One solution is shown at the right:

5	7	6
7	Home	7
6	7	5

2. The maximum number of horses is 72:

0	18	0
18	Home	18
0	18	0

3. The minimum number of horses is 36 :

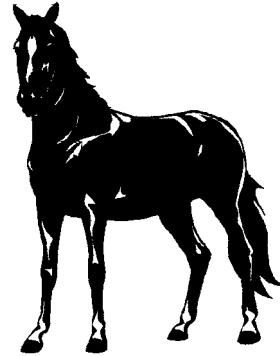
9	0	9
0	Home	0
9	0	9

The Horse Ranch: From Reich, Otto, "Horsing Around with Numbers," *Grainews* 23 (6): 3, April, 1997.
Sauder, David, "Neighbors Rotate Horses," *Grainews* 23 (10): 6, June, 1997.

Blackline Master: The Horse Ranch

A farmer had a quarter-section of land (160 acres) and fenced it into nine equal pastures. In the middle, he built a big house with a large picture window facing in each of the four directions. He then placed six horses in each of the eight pastures for a total of 48 horses.

6	6	6
6	Home	6
6	6	6



A neighbour who had two horses came to him and asked him if he could look after her two horses while she went away for a holiday. The farmer replied, "No, I cannot because I have my pastures and house arranged so that when I look out any window: north, south, east or west, I see 18 horses and your two additional horses would change that."

The neighbour with the two horses responded, "If I can put in my two horses and not change your count of 18 in each direction, will you let me put them in?" The farmer then replied, "Yes." In answering the following questions, you may wish to use the pastures provided in the worksheet on page IV-A-32.

The Horse Ranch: From Reich, Otto, "Horsing Around with Numbers," *Grainews* 23 (6): 3, April, 1997.
 Sauder, David, "Neighbors Rotate Horses," *Grainews* 23 (10): 6, June, 1997.

1. How was the neighbour able to add two horses to the farmer's pastures without disturbing the required balance of 18 horses being visible in each direction?

	Home	

2. After this incident received local publicity, local layabouts realized that their orderly neighbour only noticed the number of horses to be seen in each direction and not the total number of horses. Any dishonest party could get free grazing for his/her own horses by slipping them into the eight pastures without disturbing the required balance of 18 in each direction. What is the maximum number of horses that could be grazing in the farmer's pastures without his noticing that there were more than 48 in total?

	Home	

3. Predictably, what one dishonest neighbour could figure out, so could another (even more) dishonest neighbour. This party was a horse thief, and set out to remove as many horses as possible without disrupting the rule of 18 horses per side. What is the least number of horses the thief would need to leave in the pastures so that the farmer wouldn't notice the missing horses?

	Home	

Worksheet

	Home	

	Home	

	Home	

	Home	

	Home	

	Home	

	Home	

	Home	

	Home	

Teacher Information: Miscellaneous Problems

Skills Required

- basic arithmetic
- logical reasoning
- geometric formula evaluation

When to Do

These problems may be distributed at any time.

Teaching Information

- Students could be given all nine questions at once and asked to hand in solutions to any five questions. This would allow students to choose questions they are interested in or are able to do.
- Students could be given one page at a time and asked for solutions to the problems.

Solutions

1. 32 students
2. 72 years old
3. pumpkins = 8 lbs.; melons = 4 lbs.
2 pumpkins and 1 melon = 20 lbs.
4. children's ages—1, 8, 8
apartment number 17
5. 27 bananas
6. 0.82 cm^2
7. no
8. 4 cubes
9. 760 men

3. Three pumpkins and two melons weigh 32 pounds. Four pumpkins and three melons weigh 44 pounds. All pumpkins weigh the same and all melons weigh the same. What is the weight of two pumpkins and one melon?

4. Frank is from the local school board and greets Rhoda at the door. They have the following conversation:

Frank: I need to know how old your three kids are.

Rhoda: The product of their ages is 64.

Frank: I still don't know their ages.

Rhoda: The sum of their ages is the same as my apartment number.

Frank: I still don't know their ages.

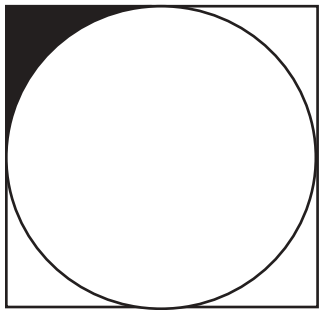
Rhoda: The older two are twins.

Frank: Now I know their ages! Thanks.

How old are Rhoda's kids and what is Rhoda's house number?

5. Four men were stranded in the jungle. Since they had eaten all of their food, they went to work gathering bananas. Once they finished gathering bananas, they were tired and all four fell asleep. An hour later, one of the men awoke and was feeling quite hungry so he ate one-third of the bananas—this was more than his proper share. Since he felt full he quickly fell back to sleep. The second man woke an hour later and, being hungry, ate one-third of the remaining bananas and quickly fell back to sleep. The third man did the same. When the fourth man awoke, he took only his rightful share of the remaining bananas. Then there were six bananas left. How many bananas did the men gather?

6. A circle of radius 4 cm is inscribed in a square. Find the area of the shaded part.



7. Phil and Mary both work evenings. Phil is free every ninth evening; Mary every sixth evening. Phil is free this coming Sunday; Mary is free the next Monday. When, if ever, will they have a free evening together? Explain your reasoning.
8. The surface of a 4 cm x 2 cm rectangular block is painted red. The block is then cut into 24 cubes and each cube has a 1-cm edge. How many of these cubes have exactly one face painted red?

9. A colonel attempts to form a square by arranging the men in his regiment in rows. He finds that he has 31 soldiers left over. He needs 24 more soldiers in order to increase the side of the square by one person. How large is his regiment?

Appendix II

Additional Resources

Print

The Association of Teachers of Mathematics. *Eight Days a Week: Puzzles, Problems and Questions to Activate the Mind*. The Association of Teachers of Mathematics.

ISBN 1-898-611-09-2.

Baron, Celia, Don Bradford, Angela Kaiser, David Sufrin, Dave Tambellini, and Rick Wunderlich. *Essentials of Mathematics 11*. Victoria, BC: British Columbia Ministry of Education, 2002.

Brecker, Erwin. *Lateral Logic Puzzles*. Sterling Publishing Company, Inc.

ISBN 0-8069-0618-9.

Bremner, John. *Mensa Maths Wizards for Kids*. Carleton Books Limited.

ISBN 1-85868-555-9.

Carter, Philip, Ken Russell, and John Bremner. *The Ultimate Puzzle Challenge*. Carlton Books Ltd. ISBN 1-85868-716-0.

DeSpezio, Michael A. *Giant Book of Challenging Thinking Puzzles*. Sterling Publishing Company, Inc. ISBN 0-8069-2087-4.

Forte, Imogene, and Sandra Schur. *180 Icebreakers to Strengthen Critical Thinking and Problem-Solving Skills*. Incentive Publications, Inc. ISBN 0-86530-345-2.

Graham, Evelyne M. *Think-A-Grams*. Critical Thinking Press and Software.

ISBN Numbers: Book A1: 0-89455-329-1

Book A2: 0-89455-430-1

Book B1: 0-89455-330-5

Book B2: 0-89455-431-X

Book C1: 0-89455-331-3

Book C2: 0-89455-432-8

Hunter, J.A.H. *Entertaining Mathematical Teasers and How to Solve Them*. Dover Publications, Inc. ISBN 0-486-24500-4.

Matt-Smith, Geoffrey. *Mathematical Puzzles for Beginners and Enthusiasts*. Dover Publications, Inc. ISBN 0-486-20198-8.

Nash, Helen, and Dorothy Masterson. *Humorous Cryptograms*. Sterling Publishing Company, Inc. ISBN 0-8069-3982-6.

National Council of Teachers of Mathematics. *How to Evaluate Progress in Problem Solving*. National Council of Teachers of Mathematics. ISBN 0-87353-241-4.

Sloane, Paul, and Des MacHale. *Improve Your Lateral Thinking*. Sterling Publishing Company, Inc. ISBN 0-8069-1374-6.

Weber, Ken. *Five Minute Mysteries for the Armchair Detective*. Stoddart Publishing Co., Ltd. ISBN 0-7737-5210-2.

Internet

There are many sites on the Internet with problems and puzzles. If you are using a search engine to find these sites, search using the words "Mathematics Puzzles Problems."

As of February 2003, the following sites were available:

AAA Math

<<http://www.aaamath.com>>

This site has games and practice sheets for various grade levels and topics. There are links to other sites on the web with games and puzzles.

Algebra Story and Word Problems

<http://www2.hawaii.edu/suremath/intro_algebra.html>

There are word problems for various subjects on this site. As well, there are helpful hints to assist in problem solving. Some of the problems may be too algebraic for Senior 3 Consumer Mathematics students.

Breaking Away from the Mathbook

<<http://www.math.nmsu.edu/breakingaway/main.html>>

Although the site is subtitled **Creative Projects for K-8**, some of the projects may be suitable for Senior 3 Consumer Mathematics students. One that might be interesting for students is *Creasing Paper Along Curves*.

Math Forum

<<http://mathforum.org>>

This is a good site to begin searching for problems and puzzles. One feature is **Problems of the Week**. New problems are available as well as a library of previous problems. Students can submit their answers and get some feedback. There are links to other math sites and several departments that are useful.

Word Problems for Kids

<<http://www.stfx.ca/special/mathproblems/welcome.html>>

This is a Canadian site with word problems, hints, and solutions from previous mathematics competitions. The problems are sorted by grade level. Choosing problems from Grades 5 through 9 will lead to a wealth of non-algebraic problems.