

Unit C: Relations and Formulas

Half Course III

HALF COURSE III

Unit C: Relations and Formulas

Hours: 13

General Learning Outcome:

Represent and interpret relations in a variety of contexts.

The intent of this unit is for students to develop an understanding of linear relations in a real-life context.

Specific Outcomes

C-1 Express a linear relation of the form $y = mx$

- in words
- as a formula
- with a table of values
- as a graph

C-2 Interpolate and extrapolate values from the graph of a linear relation

C-3 Determine the slope of a linear relation and describe it in words

C-4 Express a linear relation of the form $y = mx + b$

- in words
- as a formula
- with a table of values
- as a graph

C-5 Interpret the graph of a relation and describe it in words

C-6 Construct a graph of a relation from its description in words

C-7 Evaluate formulas

RELATIONS AND FORMULAS

Instructional Materials

- *Essentials of Mathematics 11*
- Graph paper
- Ruler
- Calculator

Connections to Problem Analysis and Analysis of Games and Numbers

Any of the Problem Analysis and Analysis of Games and Numbers activities may be interspersed with problems from the Relations and Formulas unit.

PRESCRIBED LEARNING OUTCOMES

General Outcome

Represent and interpret relations in a variety of contexts.

Specific Outcome(s)

C-1 express a linear relation of the form $y = mx$

- in words
- as a formula
- with a table of values
- as a graph

SUGGESTIONS FOR INSTRUCTION

The intention of this unit is to develop the concepts of a linear relation in context. A pure, algebraic approach is not recommended.

Example 1

Timmy works part-time, earning \$7 per hour. His earnings depend on how many hours he works. The time, or hours worked, is the independent variable. Express this relationship in words, in a formula, as a table of values, and as a graph.

Solution

Word: As the number of hours worked increases, the pay increases.

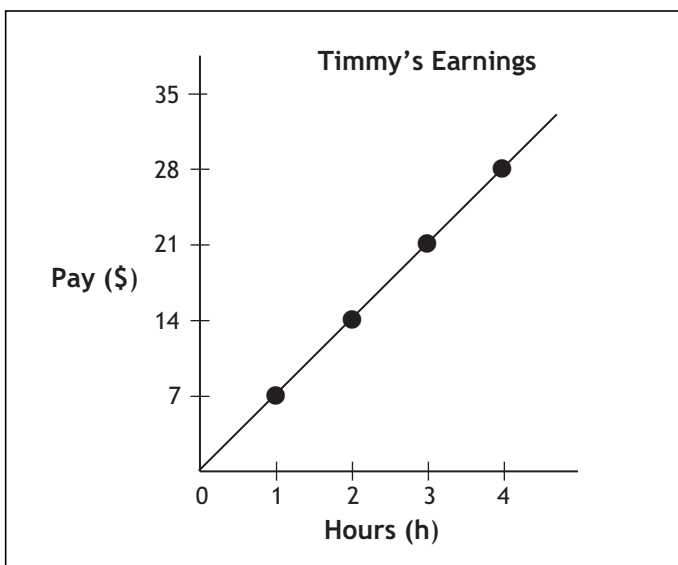
Formula: (Dependent = constant x independent)

$$\text{Pay} = \$7 \times \text{hours worked}$$

Table of Values:

<i>h</i>	Pay
0	0
1	7
2	14
3	21
4	28

Graph:



- ✓ Communications
- ✓ Connections
- ✓ Number Sense
- ✓ Organization and Structure
- ✓ Patterns
- ✓ Problem Solving
- ✓ Reasoning
- Technology
- ✓ Visualization

(continued)

SUGGESTIONS FOR ASSESSMENT

1. Identify the dependent variable
 - a) Distance walked vs calories burned
 - b) Gas consumed vs distance driven
 - c) Test marks vs hours of studying
 - d) Driving speed vs value of ticket received
2. Write each of the above as a formula, placing the dependent variable first, dependent = $m \times$ independent. Graph each of the above.
3. Ensure that all graphs have the independent variable along the x -axis (horizontal).

 Students can generate their own examples, expressing each in words, formulas, tables of values, and graphs. These would be excellent portfolio items.

SUGGESTED LEARNING RESOURCES

Print

- Senior 3 Consumer Mathematics (35S) Part III: A Course for Distance Learning.* Winnipeg, MB: Manitoba Education, Training and Youth, 2001. — Module 4, Lesson 1
- Baron, C., et al. *Essentials of Mathematics 11.* Victoria, BC: British Columbia Ministry of Education, 2002.
- Montesanto, R., and D. Zimmer. *Models and Patterns: Experimenting with Linear Equations.* Toronto, ON: D.C. Heath Canada, 1994. [ISBN 0-669-95482-9]

PRESCRIBED LEARNING OUTCOMES

C-1 express a linear relation of the form $y = mx$

- in words
- as a formula
- with a table of values
- as a graph

– *continued*

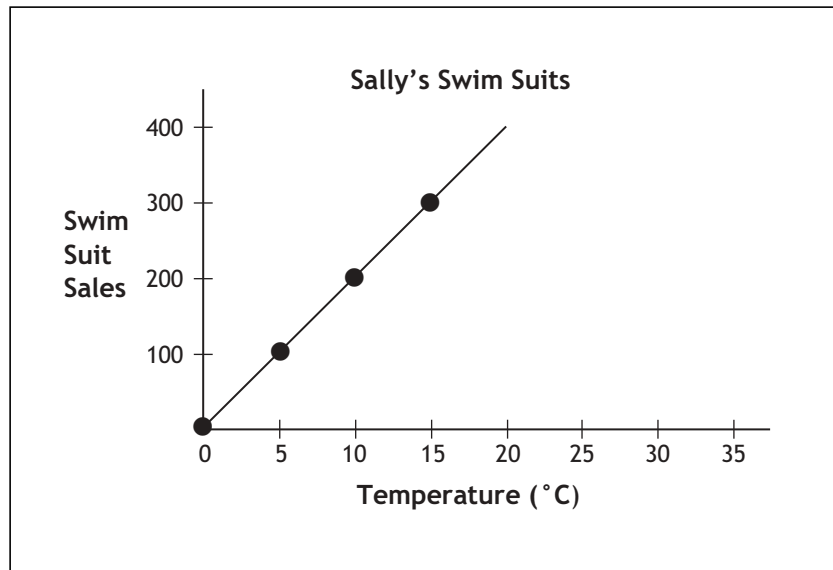
- | | |
|------------------------------|-------------------|
| ✓ Communications | ✓ Patterns |
| ✓ Connections | ✓ Problem Solving |
| ✓ Number Sense | ✓ Reasoning |
| ✓ Organization and Structure | Technology |
| | ✓ Visualization |

SUGGESTIONS FOR INSTRUCTION

Students will need to practice reading basic graphs and understanding the linear relationship depicted.

Example 2

Identify the dependent and independent variables and describe the relationship in words.



Solution

independent variable: temperature (°C)

dependent variable: swim suit sales

As the temperature increases, swim suit sales increase.

C-2 interpolate within and extrapolate values from the graph of a linear equation

- | | |
|------------------------------|-------------------|
| ✓ Communications | ✓ Patterns |
| ✓ Connections | ✓ Problem Solving |
| ✓ Number Sense | ✓ Reasoning |
| ✓ Organization and Structure | Technology |
| | ✓ Visualization |

Distinguish between **interpolation** and **extrapolation**.

Interpolation: the process of determining values between known points on the graph

- a) From Example 1, if Timmy worked 2½ hours, what would his earnings be? (\$17.50)
- b) From Example 2, if Sally sold 250 swim suits, what would the temperature be? (12° C)

Extrapolation: the process of determining values beyond the know data points

- a) From Example 1, if Timmy worked 5 hours, what might his earnings be? (\$35)
- b) From Example 2, if the temperature reaches 30°, how many swim suits might be sold? (500)

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Students should refer to their own graphs, and develop questions using interpolation and extrapolation. These questions could be shared with the class.

When students' work is presented to their peers, more meaning is attached to the activity, and increased interest results.

Print

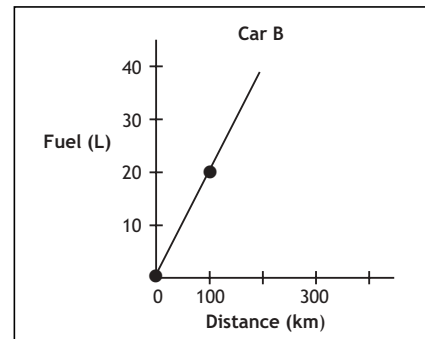
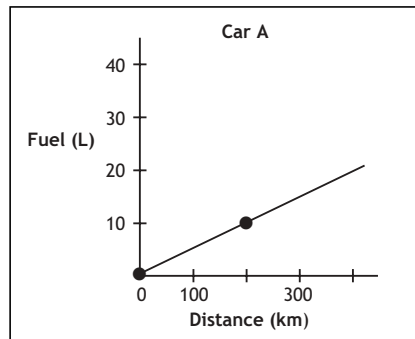
*Senior 3 Consumer
Mathematics (35S) Part
III: A Course for Distance
Learning.* Winnipeg, MB:
Manitoba Education,
Training and Youth,
2001.
— Module 4, Lesson 1

**PRESCRIBED LEARNING
OUTCOMES**

C-3 determine the slope of a linear relation and describe it in words

SUGGESTIONS FOR INSTRUCTION

Compare the following graphs:



Car B uses more fuel than Car A. The line on the graph is much steeper. Slope can be determined by the amount of the rise (vertical) divided by the amount of the run (horizontal).

$$\text{slope} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

For Car A, the rise (vertical change) from the origin (0,0) to the first data point is 10, and the run (horizontal change) is 200.

The slope is $\frac{10}{200}$ or $\frac{1}{20}$.

For Car B, the rise from the origin to the first data point is 20, and the run is 100. The slope (rise/run) is $\frac{20}{100}$ or $\frac{1}{5}$.

Formula:

$$\text{Car A — } F = \frac{1}{20}d$$

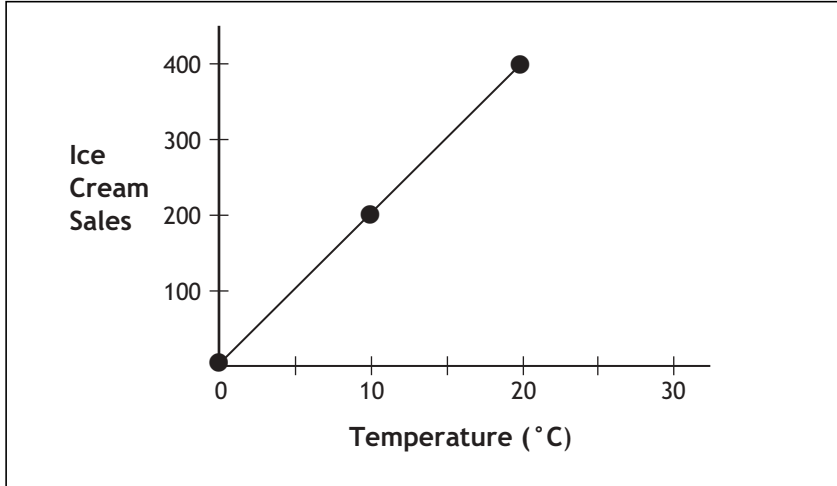
$$\text{Car B — } F = \frac{1}{5}d$$

where F = fuel used in L and d = distance in km

- ✓ Communications
- ✓ Connections
- ✓ Number Sense
- ✓ Organization and Structure
- ✓ Patterns
- ✓ Problem Solving
- ✓ Reasoning
- Technology
- ✓ Visualization

SUGGESTIONS FOR ASSESSMENT

Teachers should display graphs, asking students to determine the slope, and describe the relationship of the two variables.



$$\text{slope} = \frac{200 \text{ (rise)}}{10 \text{ (run)}}$$

“For every 10° increase in temperature, the sales of ice cream increase by 200.”

Interpolation: How many sales would you expect at 15°?

Extrapolation: How many sales would you expect at 30°?

Formula: $S = 20t$

where S = amount of sales and t = temperature in °C

SUGGESTED LEARNING RESOURCES

Print

Senior 3 Consumer Mathematics (35S) Part III: A Course for Distance Learning. Winnipeg, MB: Manitoba Education, Training and Youth, 2001.

— Module 4, Lesson 2

**PRESCRIBED LEARNING
OUTCOMES**

- C-4 express a linear relation of the form $y = mx + b$
- in words
 - as a formula
 - with a table of values
 - as a graph

SUGGESTIONS FOR INSTRUCTION

Many situations in life involve “fixed” costs, plus a constant amount per item (hours, people, etc.). Renting a car usually costs a fixed amount per day, plus a constant amount per kilometre driven.

This graph shows the cost of renting a hall. The fixed amount is \$500, plus \$2.50 per person.

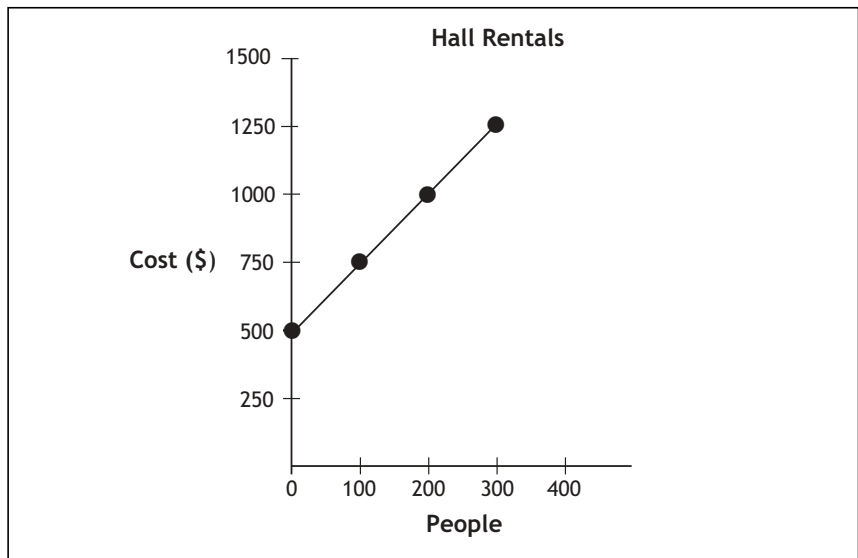


Table of Values:

People	0	100	200	300
Cost (\$)	500	750	1000	1250

$$\text{slope} = \frac{(\text{rise})}{(\text{run})} = \frac{500}{200} = 2.5$$

Formula: $C = 2.5n + 500$

where C = cost and n = number of people

Students should interpolate within and extrapolate beyond the graphs of linear relations.

- | | |
|------------------------------|-------------------|
| ✓ Communications | ✓ Patterns |
| ✓ Connections | ✓ Problem Solving |
| ✓ Number Sense | ✓ Reasoning |
| ✓ Organization and Structure | Technology |
| | ✓ Visualization |

(continued)

SUGGESTIONS FOR ASSESSMENT

Students could develop an investigation to show linear relations in the forms $y = mx$ and $y = mx + b$.

Have students gather graphs of linear relations from various sources. Sources can include newspapers, magazines, Internet, brochures, etc.

Students should analyze a graph and express the relation it represents in the following forms:

- words
- table of values
- formula

Students should interpolate and extrapolate values from the graph.

**SUGGESTED LEARNING
RESOURCES****Print**

Senior 3 Consumer Mathematics (35S) Part III: A Course for Distance Learning. Winnipeg, MB: Manitoba Education, Training and Youth, 2001.
— Module 4, Lesson 4

PRESCRIBED LEARNING OUTCOMES

- C-4 express a linear relation of the form $y = mx + b$
- in words
 - as a formula
 - with a table of values
 - as a graph
- *continued*

SUGGESTIONS FOR INSTRUCTION

Complete this activity with students.

- Distinguish between relations in the form $y = mx$ and $y = mx + b$. Describe differences in the word description, the table of values, the formula, and the graph.

Example

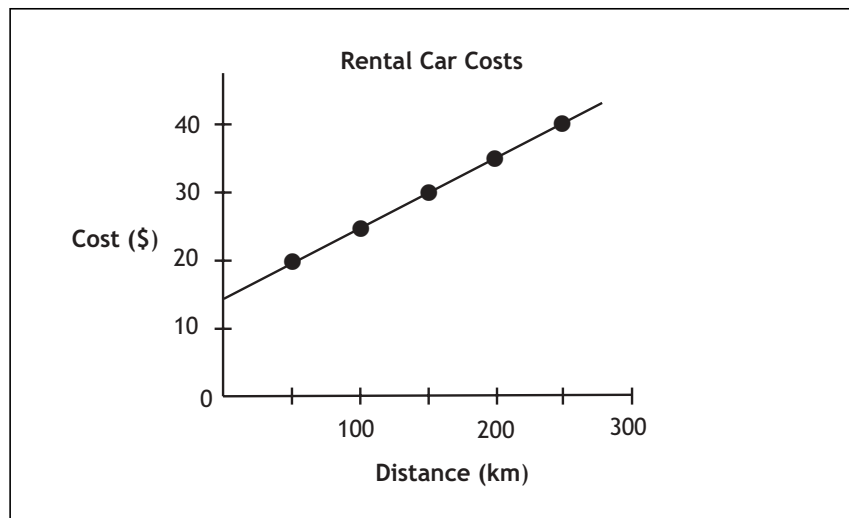
The cost to rent a car is calculated at \$0.10 per kilometre plus a fixed charge of \$15.00 per day. Create a table of values and a graph for renting a car. Express the relationship as a formula.

Solution

Table of values:

Distance (km)	50	100	150	200	250
Cost (\$)	20	25	30	35	40

Graph:



Formula:

Dependent variable = slope x independent + fixed value

$$\begin{array}{ccccccc}
 C & = & 0.10 & \times & D & + & 15.00 \\
 \uparrow & & \uparrow & & \uparrow & & \uparrow \\
 \text{Cost (\$)} & & = \text{slope} & \times & \text{distance (km)} & + & \text{fixed value (\$)} \\
 & & & & \text{or} & & \\
 & & & & \text{rate} & &
 \end{array}$$

Students should interpolate within and extrapolate beyond the graph of a linear equation.

- | | |
|------------------------------|-------------------|
| ✓ Communications | ✓ Patterns |
| ✓ Connections | ✓ Problem Solving |
| ✓ Number Sense | ✓ Reasoning |
| ✓ Organization and Structure | Technology |
| | ✓ Visualization |

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

PRESCRIBED LEARNING OUTCOMES

C-5 interpret the graph of a relation and describe it in words

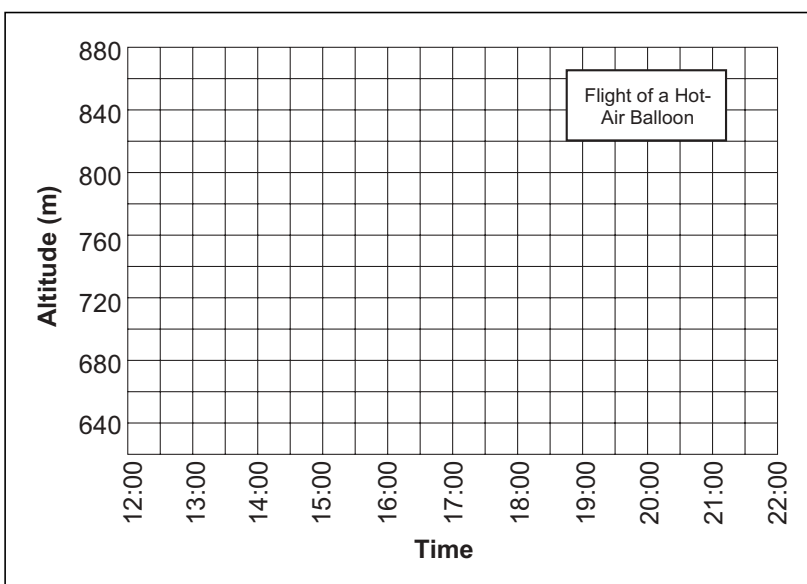
SUGGESTIONS FOR INSTRUCTION

Example:

The altimeter on a hot-air balloon recorded the following altitudes over a period of time.

Time	12:00	13:00	14:00	15:00	16:00	17:00	18:00	19:00	20:00
Altitude (m)	800	820	840	860	860	860	800	740	680

- a) State the dependent and independent variables in this problem.
- b) Plot these points on the following grid.



- c) Calculate the slopes of the line segments from (use a negative sign if necessary):
 - i) 12:00 – 15:00 ii) 15:00 – 17:00 iii) 17:00 – 20:00
- d) In words, describe the flight of this hot-air balloon from 12:00 to 20:00.
- e) Determine the formula that describes each of the following portions of the balloon’s flight:
 - i) 12:00 – 15:00 ii) 15:00 – 17:00 iii) 17:00 – 20:00
- f) Use the graph to determine the missing information:

Time	13:30		16:30	
Altitude (m)		720		850

- ✓ Communications
- ✓ Connections
- ✓ Number Sense
- ✓ Organization and Structure
- ✓ Patterns
- ✓ Problem Solving
- ✓ Reasoning
- Technology
- ✓ Visualization

(continued)

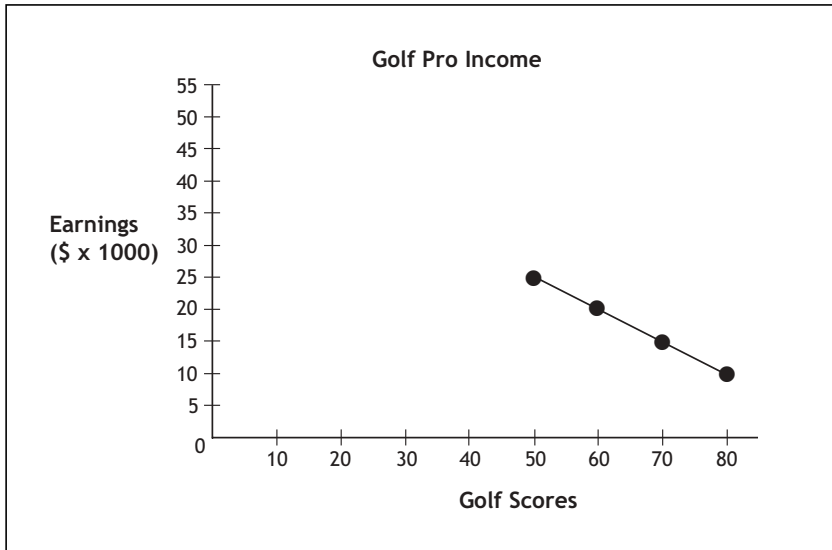
SUGGESTIONS FOR ASSESSMENT

Ask students to find graphs in the media and provide

- calculations of the slope
- written descriptions of the relation
- formulas for the relation

Extension

Describe the linear relation depicted by the following graph.



- a) At what point does the graph become unreasonable?
- b) If a golfer had 0 shots, would he earn money?

Solution

- a) The graph is unreasonable for shots less than 50 and greater than 80.
- b) The golfer would not earn money for 0 shots.

SUGGESTED LEARNING RESOURCES

Print

Senior 3 Consumer Mathematics (35S) Part III: A Course for Distance Learning. Winnipeg, MB: Manitoba Education, Training and Youth, 2001.
— Module 4, Lesson 6

Internet

<www.statcan.ca/english/kits/glanc1.htm>

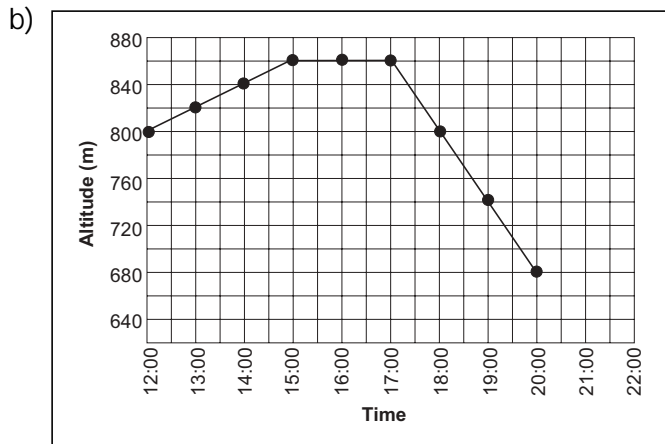
**PRESCRIBED LEARNING
OUTCOMES**

C-5 interpret the graph of a relation and describe it in words
– *continued*

SUGGESTIONS FOR INSTRUCTION

Solution

a) dependent: altitude (*m*)
independent: time



c) i) 12:00–15:00 rise: $860 - 800 = 60$ m
run: $15:00 - 12:00 = 3$ hours

$$\text{slope: } \frac{60}{3} = 20 \text{ m/hr}$$

ii) 15:00–17:00 rise: $860 - 860 = 0$ m
run: $17:00 - 15:00 = 2$ hours

$$\text{slope: } \frac{0}{2} = 0 \text{ m/hr}$$

iii) 17:00–20:00 rise: $680 - 860 = -180$ m
run: $20:00 - 17:00 = 3$ hours

$$\text{slope: } \frac{-180}{3} = -60 \text{ m/hr}$$

d) From 12:00 to 15:00, the balloon rose at a rate of 20 m/hr. From 15:00 to 17:00, the balloon didn't change height. From 17:00 to 20:00, the balloon started to descend at a rate of 60 km/h.

e) Let *A* = altitude and *t* = time (as hours after 12:00).

i) $A = 20t + 800$

ii) $A = 860$

iii) $A = -60t + 1160$

f)

Time	13:30	19:20	16:30	17:30
Altitude (m)	850	720	860	850

- | | |
|------------------------------|-------------------|
| ✓ Communications | ✓ Patterns |
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| | ✓ Visualization |

(continued)

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

PRESCRIBED LEARNING OUTCOMES

C-6 construct a graph of a relation from its description in words

SUGGESTIONS FOR INSTRUCTION

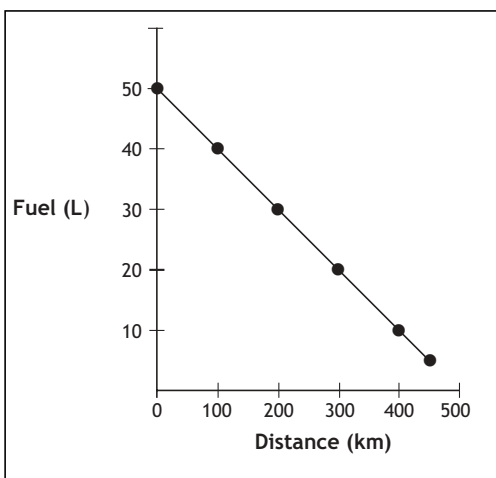
Example 1

Construct a graph for each of the following relations:

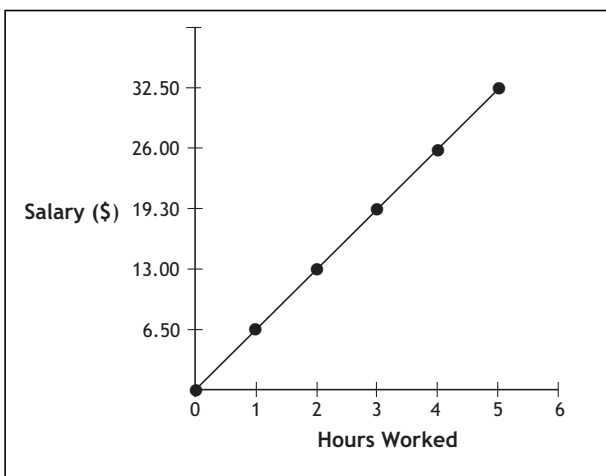
- a) A vehicle has 50 L of fuel in its tank. For every 100 km driven, 10 L of fuel are consumed. The trip is 450 km.
- b) Brianne works part-time in a store, and earns \$6.50 per hour.
- c) Sarah is sewing purses. Each purse requires a half square metre of fabric.
- d) Lynne is buying shrubs. Each shrub costs \$15.

Solution

a)



b)



- ✓ Communications
- ✓ Connections
- ✓ Number Sense
- ✓ Organization and Structure
- ✓ Patterns
- ✓ Problem Solving
- ✓ Reasoning
- Technology
- ✓ Visualization

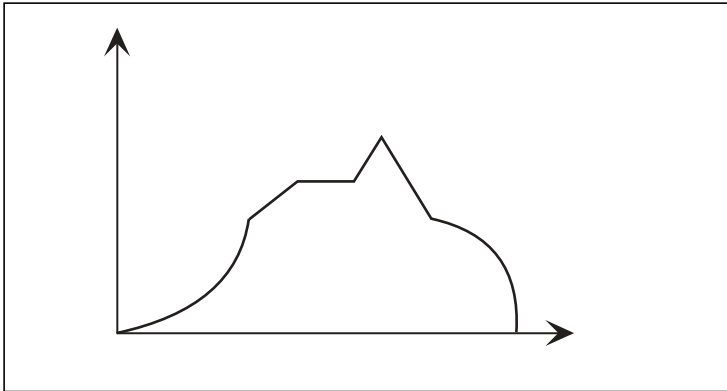
(continued)

SUGGESTIONS FOR ASSESSMENT

Graphs should include labels, appropriate scales, and data points.

Problem

Write a scenario for the following graph. Be sure to identify the independent and dependent variables.



SUGGESTED LEARNING RESOURCES

Print

Senior 3 Consumer Mathematics (35S) Part III: A Course for Distance Learning. Winnipeg, MB: Manitoba Education, Training and Youth, 2001.
— Module 4, Lesson 6

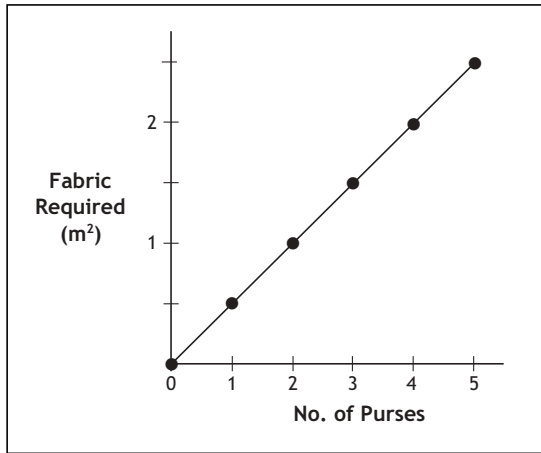
**PRESCRIBED LEARNING
OUTCOMES**

C-6 construct a graph of a relation from its description in words
– *continued*

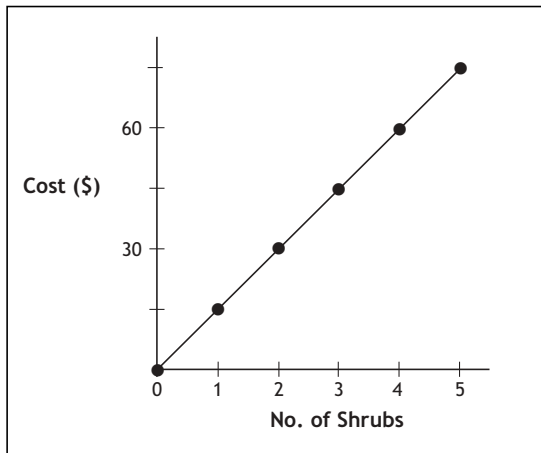
SUGGESTIONS FOR INSTRUCTION

Solution (continued)

c)



d)



- ✓ Communications
- ✓ Connections
- ✓ Number Sense
- ✓ Organization and Structure
- ✓ Patterns
- ✓ Problem Solving
- ✓ Reasoning
- Technology
- ✓ Visualization

(continued)

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

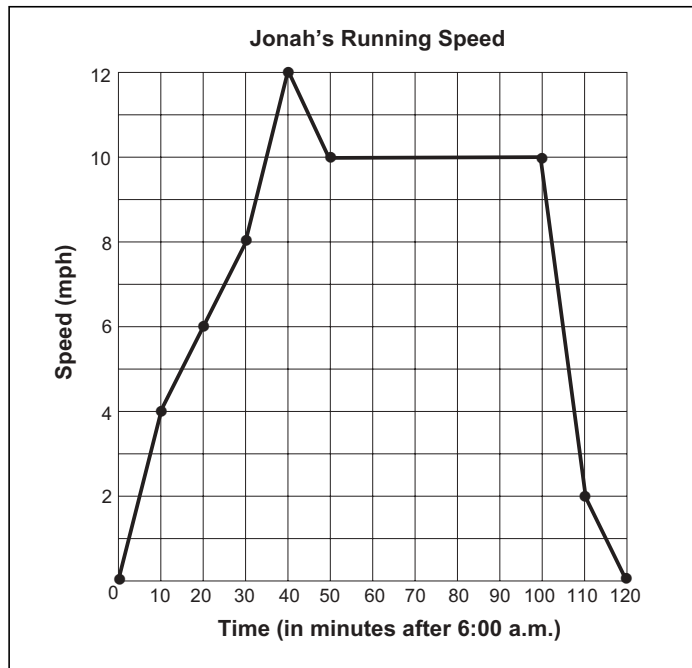
**PRESCRIBED LEARNING
OUTCOMES**

C-6 construct a graph of a relation from its description in words
– *continued*

SUGGESTIONS FOR INSTRUCTION

Example 2

Jonah is a marathon runner. On Saturday, he ran a part of his training program. The following graph shows his speed as he ran. He started running at 6:00 a.m.



- What is his speed at 6:30 a.m.? (*Answer: 8 mph*)
- At what time is he going the fastest and what is his speed at that time? (*Answer: At 6:40 a.m. he is running 12 mph.*)
- What happens to his speed between 6:55 a.m. and 7:15 a.m.? (*Answer: He is running at a constant speed of 10 mph.*)
- Describe his run.

(*Answer: Jonah starts running at 6:00 a.m. and gradually increases his speed until 6:40 a.m. For the next 10 minutes he decreases his speed slightly. Then, for the next 50 minutes, he runs at a constant speed. In the next 10 minutes he slows down considerably. In the last 10 minutes he slows down less quickly before stopping altogether.*)

- | | |
|------------------------------|-------------------|
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(*continued*)

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

**PRESCRIBED LEARNING
OUTCOMES**

C-6 construct a graph of a relation from its description in words
– *continued*

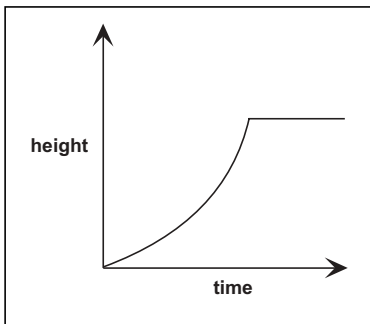
SUGGESTIONS FOR INSTRUCTION

Example 3

Draw a graph of each of the following situations. Be sure to identify the dependent and independent variables and label the axes appropriately.

a) The height of a rose bush over time.

Solution

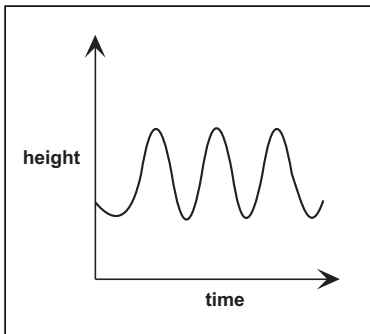


Independent: time

Dependent: height

b) The height of a particular seat on a ferris wheel for three revolutions.

Solution

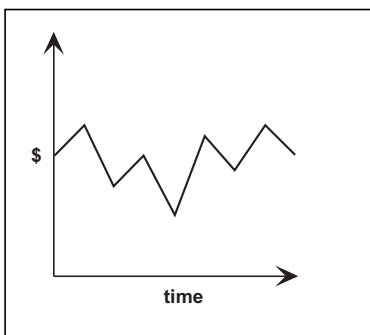


Independent: time

Dependent height

c) The value of a mutual fund during a month.

Solution



Independent: time

Dependent: value (\$)

- ✓ Communications
- ✓ Connections
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- ✓ Reasoning
- Technology
- ✓ Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

PRESCRIBED LEARNING
OUTCOMES

C-7 evaluate formulas

SUGGESTIONS FOR INSTRUCTION

 Students will revisit this topic in *Senior 4 Consumer Mathematics* (40S).

Evaluate formulas by substituting known values into the right-hand side of the formula, and determine the value for the unknown value on the left-hand side.

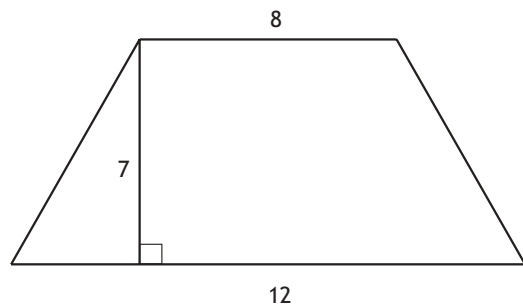
Example 1

Find the area of a trapezoid with bases of 8 cm and 12 cm, and a height of 7 cm.

Solution

$$a = 8, b = 12, h = 7$$

$$\begin{aligned} A &= \left(\frac{a+b}{2} \right) h \\ &= \left(\frac{8+12}{2} \right) (7) \\ &= \left(\frac{20}{2} \right) (7) \\ &= 70 \text{ cm}^2 \end{aligned}$$


Example 2

The amount of energy required to separate charges depends on the voltage developed and the amount of charge moved. If W is the energy in joules (J), Q is the charge in coulombs (C), and V is the resulting voltage in volts (V), then

$$V = \frac{W}{Q}$$

If it takes 35 J of energy to move a charge of 5 C from one point to another, what is the voltage between the two points?

Solution

$$\begin{aligned} V &= \frac{W}{Q} \\ V &= \frac{35 \text{ J}}{5 \text{ C}} \\ V &= 7 \text{ V} \end{aligned}$$

Students could be provided with a formula sheet from which to choose formulas, or the formulas could be presented in the problem. A variety of formulas should be used in this section.

- | | |
|------------------------------|-------------------|
| ✓ Communications | ✓ Patterns |
| ✓ Connections | ✓ Problem Solving |
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| ✓ Organization and Structure | Technology |
| | ✓ Visualization |

SUGGESTIONS FOR ASSESSMENT

Have students collect various formulas and describe where they are used. They could also show the sample table of values and the graph to illustrate the formula.

**SUGGESTED LEARNING
RESOURCES**

Print

Senior 3 Consumer Mathematics (35S) Part III: A Course for Distance Learning. Winnipeg, MB: Manitoba Education, Training and Youth, 2001.
— Module 4, Lesson 7

Internet

American Technical Publishers, Inc.(a list of formulas used in electrical work)
<www.americantech.org/eresources/resources.cfm>