

Unit B: Analysis of Games and Numbers

Half Course III

HALF COURSE III

Unit B: Analysis of Games and Numbers

Hours: 5

General Learning Outcome:

Develop, use, and justify mathematical strategies by analyzing a variety of puzzles and games; develop an awareness of how numbers are used in society.

The material provided for this unit should be used throughout the course to provide a change of pace in a context which is enjoyable, yet calls for mathematical and logical thinking.

Specific Outcomes

B-1 Demonstrate the use of an appropriate strategy in solving puzzles and playing games involving patterns.

B-2 Be aware of how numbers are used throughout society.

ANALYSIS OF GAMES AND NUMBERS

Instructional Materials

- *Essentials of Mathematics 11*
- See Appendix I for possible activities
- See Appendix II for additional resources

**PRESCRIBED LEARNING
OUTCOMES**

General Outcome

Develop, use, and justify mathematical strategies by analyzing a variety of puzzles and games; develop an awareness of how numbers are used in society.

Specific Outcome(s)

B-1 demonstrate the use of an appropriate strategy in solving puzzles and playing games involving patterns

SUGGESTIONS FOR INSTRUCTION

The objective of this type of activity is to have students play the game and find winning strategies. Students are expected to find the strategies and be able to explain them by demonstration, oral, and written communication.

Devote sufficient time to playing and enjoying a game before analysis begins. Then allow students to discuss the game and articulate their “winning” strategies.

Finding the strategy is the first step, although the subsequent steps are of equal importance. The actual results may be of less consequence, but a discussion of the results is important. Throughout the activity, students should be encouraged to examine their own thinking processes, something that is not easy and perhaps not a common experience for many. The examination should lead to communication and exploration of the strategies.

As this applies to students, it applies equally to teachers. Teachers need to be prepared for the activities. They should try them before the activities are given to students. Teachers will find a wide variation among their students and in the activities that students find easy or difficult.

These activities could be thought of as “problem solving.” It is not intended that they be taught in a block of time, but, rather, dealt with periodically during the year.

-
- ✓ Communications
 - Connections
 - ✓ Number Sense
 - ✓ Organization and Structure
 - ✓ Patterns
 - ✓ Problem Solving
 - ✓ Reasoning
 - Technology
 - ✓ Visualization
-

SUGGESTIONS FOR ASSESSMENT

Participation and the willingness to accept the challenge of the activities is important. Keep a daily record.

You may wish to keep anecdotal notes on how students develop their strategies.

Recreational activities are an appropriate context for journal writing on both content and attitudinal factors.

**SUGGESTED LEARNING
RESOURCES**

Print

Senior 3 Consumer Mathematics (35S) Part III: A Course for Distance Learning.

Winnipeg, MB: Manitoba Education, Training and Youth, 2001.

— Cover Assignments

Baron, C., et al. *Essentials of Mathematics 11.* Victoria, BC: British Columbia Ministry of Education, 2002.

Blocksma, Mary. *Reading the Numbers: A Survival Guide to the Measurements, Numbers, and Sizes Encountered in Everyday Life.* New York, NY: Penguin Books, 1989. [ISBN 0-14-01.0654-5]

Hopkins, N.J., J.W. Mayne, and J.R. Hudson. *The Numbers You Need.* Detroit, MI: Gale Research, Inc., 1992. [ISBN 0-8103-8373-X]

Muschla, G.R., and J.A. Muschla. *Hands-On Math Projects with Real-Life Applications.* West Nyack, NY: The Center for Applied Research in Education, 1996. [ISBN 0-87628-384-9]

National Council of Teachers of Mathematics. *Historical Topics for the Mathematics Classroom.* Reston, VA: NCTM, 1969, 1989. [ISBN 0-87353-281-3]

Posamentier, A.S., and J. Stepelman. *Teaching Secondary School Mathematics: Techniques and Enrichment Units.* Toronto, ON: Merrill, 1990, 1986. [ISBN 0-675-21209-X]

**PRESCRIBED LEARNING
OUTCOMES**

B-2 be aware of how numbers are used throughout society

SUGGESTIONS FOR INSTRUCTION

Teachers should present a topic showing how numbers are used in society and have students discuss the use of numbers.

An example: Read the following information on gold and answer the questions that follow.

Gold

The karat system is confusing because “karat” is spelled with a “c” when referring to precious stones, but with a “k” when applied to gold. The two systems are not the same—although a one-carat diamond is fairly respectable, one-karat gold is not. When applied to gold, a karat represents a ratio of gold to alloy (other metals)—100 percent gold is 24k, 14k is 14 parts gold, and 10 parts alloy and so on.

Why 24 parts? Why not 16 or 100? Centuries ago, gold was weighed using the troy weight system in which a pound equalled 12 ounces. A karat was used to describe 1/2 ounce of pure gold, or 1/24 of a troy pound.

A 24k (100 percent) gold is too soft to be made into jewellery. In the United States most gold jewellery is 14k gold—14 parts gold to 10 parts other metals, although 18k jewellery is also popular. By law, gold jewellery must contain at least 10 karats of gold to be labeled as karat gold. “Gold-filled” jewellery is jewellery made from a base metal with a thin layer of gold bonded to it. (It makes more sense to call this “metal-filled gold.”) If jewellery is gold-filled with better than 10k gold (the minimum required for gold-filled jewellery), it is marked something like 14KGF—meaning 14 Karat-Gold-Filled.

Gold can also be electroplated to metal and used in costume jewellery. The layer on electroplated jewellery must be at least seven millionths of an inch thick and at least 10k gold. (Blocksma, pp. 88-89.)

Questions

1. Why is 14k gold 14 parts pure gold and 10 parts alloy?
2. a) For 18k gold how many parts are pure gold and how many parts are alloy?
 b) Express this as a ratio of parts gold to parts alloy.
 c) What other ratios can be formed from the information provided? What do they represent?
 d) What percent of 18k gold is pure gold?

Gold: Reprinted by permission of Mary Blocksma, from *Reading the Numbers* by Mary Blocksma, © 1989 by Mary Blocksma.

- | | |
|---------------------------------|---------------------------------|
| ✓ Communications
Connections | ✓ Patterns
✓ Problem Solving |
| ✓ Number Sense | ✓ Reasoning
Technology |
| ✓ Organization and
Structure | ✓ Visualization |

(continued)

SUGGESTIONS FOR ASSESSMENT

SUGGESTED LEARNING
RESOURCES

Print (continued)

Sutcliffe, Andrea. *Numbers: How Many, How Long, How Far, How Much . . . All the Numbers You'll Ever Need*. New York, NY: Harper Perennial, 1996. [ISBN 0-06-273362-1]

Reimer, W., and L. Reimer. *Historical Connections in Mathematics*. Fresno, CA: AIMS Educational Foundation, 1992. [ISBN 1-881431-35-5]

Scientific American. The paradox box. San Francisco, CA: W.H. Freeman, 1975.

Verhille, C., and R. Blake. "The Peg Game." *Mathematics Teacher* (January, 1982): 39-43.

Note 1: Many bookstores carry problem and puzzle books.

Note 2: *The Mathematics Teacher* and *Mathematics Teaching in the Middle School* from the National Council of Teachers of Mathematics have useful articles and activities.

Note 3: See Appendix II for some additional resources.

**PRESCRIBED LEARNING
OUTCOMES**

B-2 be aware of how numbers are used throughout society
– *continued*

SUGGESTIONS FOR INSTRUCTION

3. a) For 10k gold, how many parts are pure gold and how many parts are alloy?
b) Express this as a ratio of parts gold to parts alloy.
c) What percent of 10k gold is pure gold?
4. Investigate the price of gold. (The price of one troy ounce of gold is listed in the financial section of many daily newspapers.) Compare this to the cost of gold jewellery of various karats at local jewellery stores. What conclusions can you draw?

Solutions

1. Gold is made up of 24 parts. So if there are 14 parts gold, there are 10 parts left over for other metals.
2. a) 18 parts gold and 6 parts alloy
b) 18 : 6 or 3 : 1
c) Answers will vary. For example:
18 : 24 — parts gold to total parts
6 : 24 — parts alloy to total parts
d) $\frac{18}{24} \times 100 = 75\%$
3. a) 10 parts gold and 14 parts alloy
b) 10 : 14 or 5 : 7
c) $\frac{10}{24} \times 100 \approx 42\%$

- | | |
|---------------------------------|---------------------------------|
| ✓ Communications
Connections | ✓ Patterns
✓ Problem Solving |
| ✓ Number Sense | ✓ Reasoning
Technology |
| ✓ Organization and
Structure | ✓ Visualization |

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Appendix I

Teacher Information: Number Tricks

Skills Required

- arithmetic

Materials

- calculator

When To Use

This activity may be done at any time.

Teaching Information

These should be fun. With some students you may wish to examine the algebra behind the tricks. Here is the algebra for Tricks 1, 2, and 3.

Trick 1:

$$\frac{\frac{3(2x-6)}{2} + 9}{3} = x$$

The result each time will be 3.

Trick 2:

$$\frac{\frac{3(2x-6)}{2} + 9}{3} = x$$

The result each time is the number chosen.

Trick 3:

The number before the decimal will be your house numbers and the number after the decimal is your age.

Blackline Master: Number Tricks

Trick 1

1. Choose any number
2. Add 5
3. Double the result
4. Subtract 4
5. Divide by 2
6. Subtract the original number

Try this several times with different numbers. What do you get?

Trick 2

1. Choose any number
2. Double it
3. Subtract 6
4. Multiply by 3
5. Divide by 2
6. Add 9
7. Divide by 3

Try this several times with different numbers. What do you get?

Now try making up some number tricks of your own!

Trick 3

1. Start with your house number (you may wish to use a calculator for this one!)
2. Double it, and then add 5
3. Multiply the sum by 50
4. Now add your age to the product
5. Add 365 days for the number of days in a year
6. Subtract 615
7. Place a decimal point where you would if the answer were dollars and cents.



If you did these calculations carefully and correctly, you will notice something special about the result. What is it?

Teacher Information: Mission Impossible

Skills Required

- logical reasoning

Materials

- graph paper to make a chart to help organize the information

When To Use

This activity may be done at any time.

Teaching Information

Students may want to make a chart that allows for entering information. The chart will be quite big, but students may find it useful. For example, if you just had the colour, nationality, and soft drink factors, the chart would look like this.

	Coke	Dr. Pepper	Orange Crush	Pepsi	7-Up	American	Canadian	English	Irish	Norwegian
Blue										
Brown										
Green										
Red										
Yellow										
American										
Canadian										
English										
Irish										
Norwegian										

Solution

Yellow	Blue	Red	Brown	Green
Norwegian	American	English	Canadian	Irish
Orange Crush	Coke	Dr. Pepper	7-Up	Pepsi
Remington	Federal	Winchester	Western	Peters
Redhead	Canvasback	Mallard	Bluebill	Teal

Blackline Master: Mission Impossible

The following puzzle is not an easy one; it could swallow up a considerable amount of time. However, if you organize the information, the mission is certainly possible. Good luck!

The following 15 facts are all you need to solve it:

1. There are five hunting cabins on a lake. Each cabin is a different colour, and is inhabited by a man of a different nationality, each drinking a different kind of soft drink, firing a different brand of shotgun shell, and shooting a different duck.
2. The Englishman lives in the red cabin.
3. The Canadian shoots only bluebills.
4. Pepsi is drunk in the green cabin.
5. The American drinks Coke.
6. The green cabin is immediately to the right (your right) of the brown cabin.
7. The hunter who uses Winchester shells shoots mallards.
8. Remington shells are shot by the man in the yellow cabin.
9. Dr. Pepper is drunk in the middle cabin.
10. The Norwegian lives in the first cabin on the left.
11. The man who buys Federal shells lives in the cabin next to the cabin of the man who shoots redheads.
12. Remington shells are used in the cabin next to the cabin where the man lives who shoots canvasbacks.
13. The hunter who fires Western shells drinks 7-Up.
14. The Irishman loads up with Peters shells.
15. The Norwegian lives next to the blue cabin.

Who drinks the Orange Crush and who shoots the teal?

Teacher Information: The Peg Farm

Skills Required

- organizing information

Materials

- can be constructed from a small piece of wood with 11 holes drilled partway through the board
- golf pegs or bingo chips may be used for pegs

When To Use

This activity may be done at any time.

Teaching Information

There is a pattern to the moves. It may be best for students to start with only a single peg of each colour and three holes, then, use two pegs of each colour and so on to help develop the pattern. Predicting the number of moves relative to the number of pegs can be treated as an extension.

Blackline Master: The Peg Farm

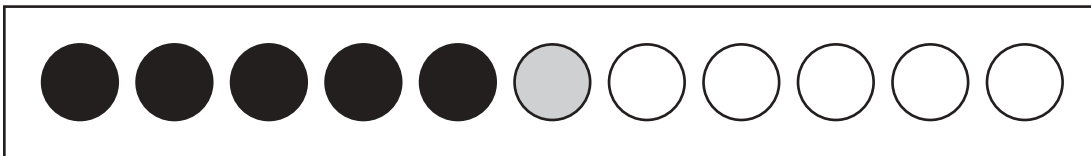
The board below shows two sets of pegs in slots. "Black" pegs are on the left and "white" pegs are on the right. There is an empty centre slot.

The object of the game is to exchange the positions of the black and white pegs.

Rules

- You can only move one peg at a time.
- You can only move "forward" (i.e., black to right, white to left).
- You may move a peg into an adjacent empty slot, or
- You may jump exactly one peg (of either colour) into an empty slot.

Can you find a way to complete the task? (There is a pattern of moves.) Describe and explain the pattern.



Teacher Information: Farm Folly

Skills Required

- arithmetic

Materials

- a spreadsheet would be useful so students can use guess and check

When To Use

This activity may be done at any time.

Teaching Information

Encourage students to use a spreadsheet. It may also be done algebraically (although it is not recommended for all students).

Let x = the number hens

Let y = the number of sheep

Then x = the number of heads of the hens, and

$2x$ = the number of legs of the hens

y = the number of heads of the sheep, and

$4y$ = the number of legs of the sheep

$$\therefore x + y = 29$$

$$2x + 4y = 100$$

From the first equation, $y = -x + 29$. Substitute this expression for y into the second equation. This yields a solution of $x = 8$.

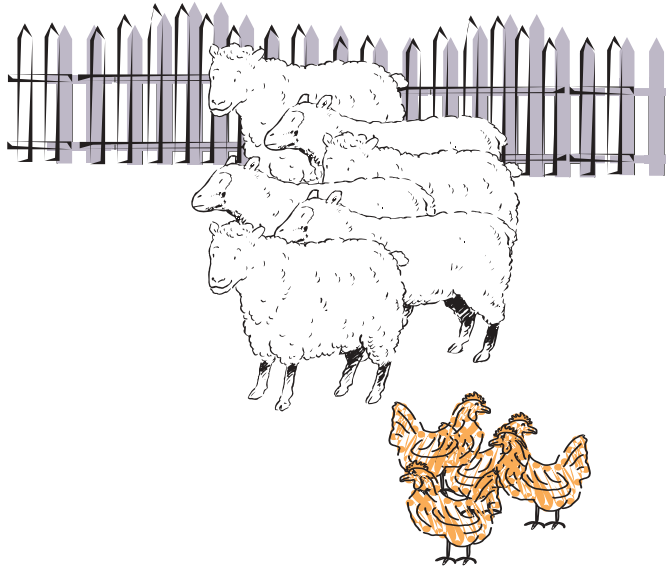
So there are 8 hens and $29 - 8 = 21$ sheep.

Blackline Master: Farm Folly

A farmer looks into a field and she sees a mixture of hens and sheep. She counts the number of the legs of all the animals. There are 100. She counts the heads of the animals; there are 29.

How many sheep and how many hens are in the field?

Note: There are a variety of ways to solve this problem. A spreadsheet might prove useful.



Teacher Information: Happy Numbers and Their Friends

Skills Required

- arithmetic

Materials

- paper and pencil

When To Use

This activity may be done at any time.

Teaching Information

There are three separate activities and information about a fourth type of number. Teachers may want to split the activity up so it does not overwhelm the students.

Solutions

1. 11 is not a happy number
2. 1, 7, 10, 13, 19, 23, 28, 31, 32, 44, 49, 68, 70, 79, 82, 86, 91, 94, 97, 100
3. $4 - 16 - 37 - 58 - 89 - 145 - 42 - 20 - 4$
4. No
5. No
6. No; No
7. The first four perfect numbers are: 6, 28, 496, 8128
8. Deficient numbers from 1 to 20: 2, 3, 4, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 19
Abundant numbers from 1 to 20: 12, 18, 20
Note that all prime numbers are deficient. Why?

Extension

Find another pair of amicable numbers.

Blackline Master: Happy Numbers and Their Friends

Number theory is a fascinating branch of mathematics which studies the characteristics of the counting numbers. Here are several simple examples.

Happy Numbers

A happy number is a counting number for which the sum of the squares of the digits eventually results in 1.

Is 19 a happy number?

Compute the sum of the squares of the digits: $1^2 + 9^2 = 1 + 81 = 82$

Stop if the result is 1; otherwise repeat the process.

Find the sum of the squares of the digits of 82: $8^2 + 2^2 = 64 + 4 = 68$

$6^2 + 8^2 = 36 + 64 = 100$

$1^2 + 0^2 + 0^2 = 1$

Stop! The result is 1.

Therefore, 19 is a happy number!

1. Determine whether 11 is a happy number.
2. Twenty of the first 100 counting numbers are happy. Find them. Use any short cuts you discover.
3. It turns out that every number that is not happy enters a repeating cycle of eight numbers. You may have discovered this in working on the previous activity. The cycle never ends because it repeats itself and never becomes 1. Determine this cycle of eight numbers.
4. Is the sum of two happy numbers always a happy number?
5. Is the product of two happy numbers always a happy number?
6. Is 1998 a happy number year? What about 2000?

Happy Numbers and Their Friends: From Reimer, W. and L. Reimer, "Happy Numbers and Their Friends," *Historical Connections in Mathematics*. Copyright © 1992 by AIMS Education Foundation

Perfect Numbers

A perfect number is a counting number that is equal to the sum of its proper factors, i.e., those factors other than itself.

6 is a perfect number
because the sum of the proper factors of 6 is equal to 6: $1 + 2 + 3 = 6$

Perfect numbers do not occur too frequently, but there is another one less than 30.

7. Can you find it? Can you find any others?

Abundant and Deficient Numbers

As we discovered above, many numbers are not perfect. Such numbers are either abundant or deficient. An abundant number is any counting number where the sum of its proper factors is greater than the number itself. For example, 12 is an abundant number because

$$1 + 2 + 3 + 4 + 6 = 16 > 12.$$

On the other hand, 8 is a deficient number because the sum of its proper factors is less than 8:

$$1 + 2 + 4 = 7 < 8.$$

8. For the numbers from 2 to 20, determine which are perfect, abundant, and deficient. Do they form any sort of pattern? Can you tell more about such numbers if you also examine those from 12 to 50? or to 100?

Amicable Numbers

Amicable means friendly. Amicable numbers are pairs of numbers such that the sum of the proper factors of each is equal to the other number. These are difficult to find. The smallest such pair is 220 and 284:

The sum of the proper factors of 220 is:

$$1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284$$

The sum of the proper factors of 284 is:

$$1 + 2 + 4 + 71 + 142 = 220$$

Teacher Information: Roman Numerals

Skills Required

- reading comprehension

Materials

- paper and pencil

When To Use

This activity may be done at any time.

Teaching Information

Have students work in pairs or small groups, and have someone in each group read the information to the others in the group.

Solutions

1.

Roman Numerals	Arabic Numerals
XXIX	29
XXXV	35
MCD	1400
CLIX	159
MMXIV	2014
MDCCXCIV	1794
MCMXCVIII	1998

2. LXXXXVIII

3. $XLIX = [50 - 10] + [10 - 1] = 49$

4. There is long history here regarding the development of computation. In essence, Arabic numerals are easier for computation. Arabic numbers have a zero.

5. $494 = CDXCIV = CCCCLXXXIIII$

Blackline Master: Roman Numerals

Read the information in the box below. Then, answer the questions that follow. Be sure to show all necessary work in reaching your answer.

Like Dracula, Roman numerals rise from their coffin in the dead of night and stamp themselves into buildings, slip into books, number chapters, and force themselves into student outlines. Decoding Roman numerals is not really difficult, though, and might even be an amusing pastime.

Here are the letters used to symbolize numbers in the Roman numeral system:

1	=	1	C	=	100
V	=	5	D	=	500
X	=	10	M	=	1000
L	=	50	There is no zero.		

To read a number, simply add up the numbers from left to right (large numbers to small ones).

Roman Numerals	Arabic Numerals
II	$1+1 = 2$
III	$1+1+1 = 3$
VIII = V+III	$5+3 = 8$
XXVII = XX+V+II	$10+10+5+2 = 27$
LXXXVI = L+XXX+V+I	$50+30+5+1 = 86$
CCLI = CC+L+I	$200+50+1 = 251$
MMMCCCVI = MMM+CCC+V+I	$3000+300+6 = 3306$

The Romans, much like the rest of us, were always looking for an easier way to do things, so after a while, subtraction was introduced to shorten the bulkier numbers. The subtracting involves only the numbers 4 and 9 wherever they occur, including 14, 19, 24, 29, etc., as well as 40 and 90, 400 and 900, and 4000 (technically Roman numerals can count only to 4999). So you subtract smaller numerals that precede larger ones: instead of writing MMMMCCCCXXXIIII (4444), you can shorten it to MMMMCDXLIV ($4000+[500-100]+[50-10]+[5-1]$). Here are some easier examples:

IV	=	V-I	=	4
IX	=	X-I	=	9
XIV	=	X+(V-I)	=	14
XL	=	L-X	=	40
XCIX	=	(C-X)+(X-I)	=	99

Adding and subtracting Roman numerals, much less multiplying and dividing them, is literally unthinkable. Hence their demise.

1. Fill in the following chart:

Roman Numerals	Arabic Numerals
XXIX	
	35
MCD	
	159
MMXIV	
	1794
MCMXCVIII	

- If the idea of subtracting to get 4s and 9s had not been thought of, how would you write the number 99 in Roman numerals?
- Show why the Roman numeral XLIX is equivalent to the Arabic numeral 49.
- Why were Roman numerals replaced by Arabic numerals in common usage?
- Give two different ways to write the number 494 in Roman numerals.

Teacher Information: Balances

Skills Required

- arithmetic
- logical reasoning

Materials

- paper and pencil

When To Use

This activity may be done at any time.

Teaching Information

The idea of a balance may be difficult for students. Work through the examples with the students using the overhead. See Appendix II for additional balance resources.

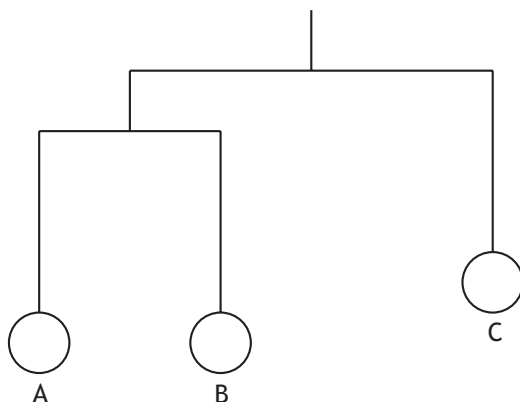
Solutions

1. $A = 5, B = 5, D = 2.5, E = 2.5$ (kg)
2. $B = 7, C = 7, D = 14, E = 7, F = 3.5, G = 3.5$ (kg)
3. $A = 64, B = C = 32, D = E = F = 8, G = H = 4, I = 32, J = 64, K = 64, L = 128, M = 32, N = 16, O = 8, P = 4, R = 2$ (g)

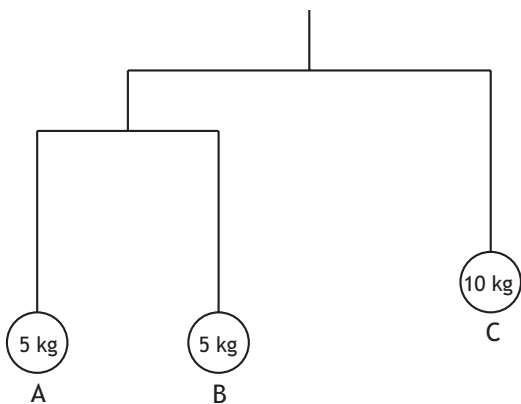
Blackline Master: Balances

In each of the following mobiles, the weights attached must balance the connecting rods.

Example: The total weight is 20 kg.

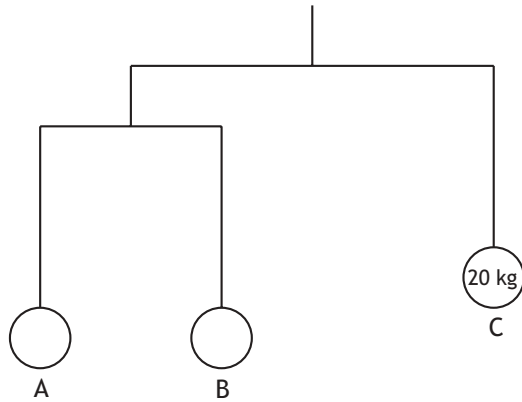


In this mobile, weight A must equal weight B so that this part of the mobile balances. To balance the entire mobile, $A + B = C$, which implies that weight C must be 10 kg. The completed balance is shown below.



Balances: From Reimer, W. and L. Reimer. "Balances," *Historical Connections in Mathematics*. Copyright © 1992 by AIMS Educational Foundation.

Now find the missing weights in the mobile shown immediately below:

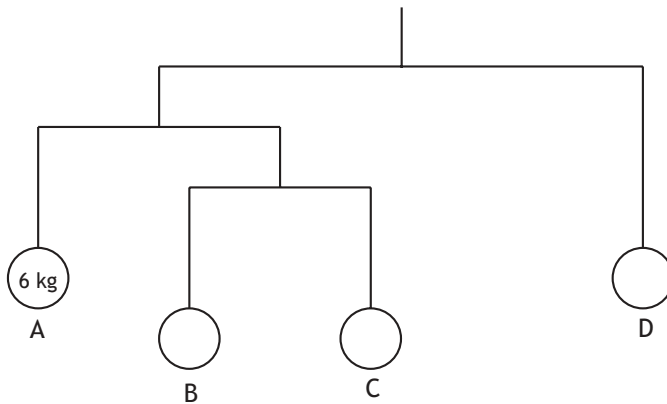


Here is another one to try:

Notice that $B + C = A$, and $B = C$.

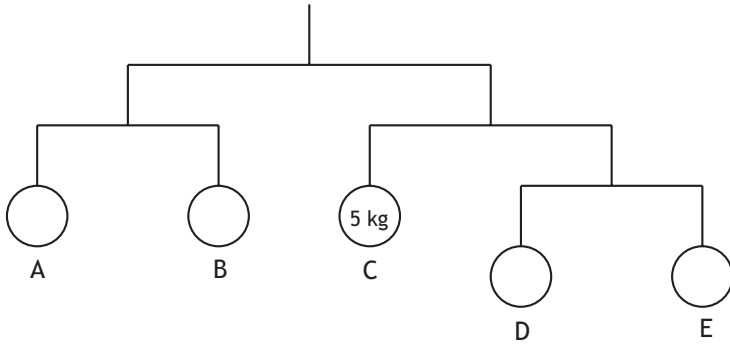
To find D, notice that $A + B + C = D$.

D = _____

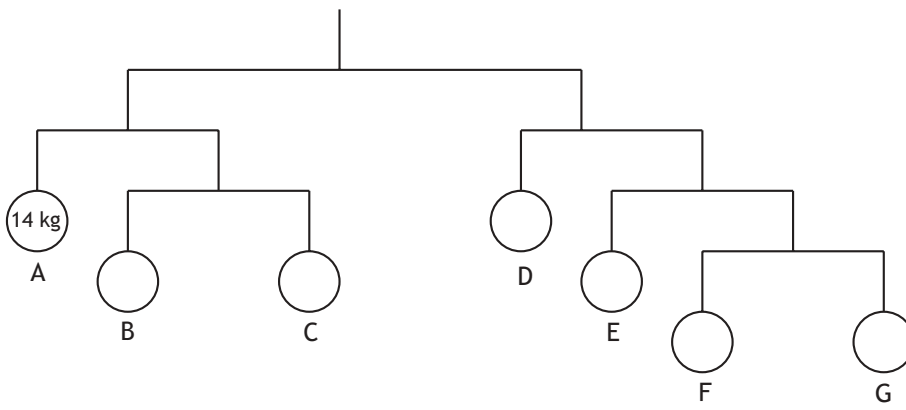


Now try these:

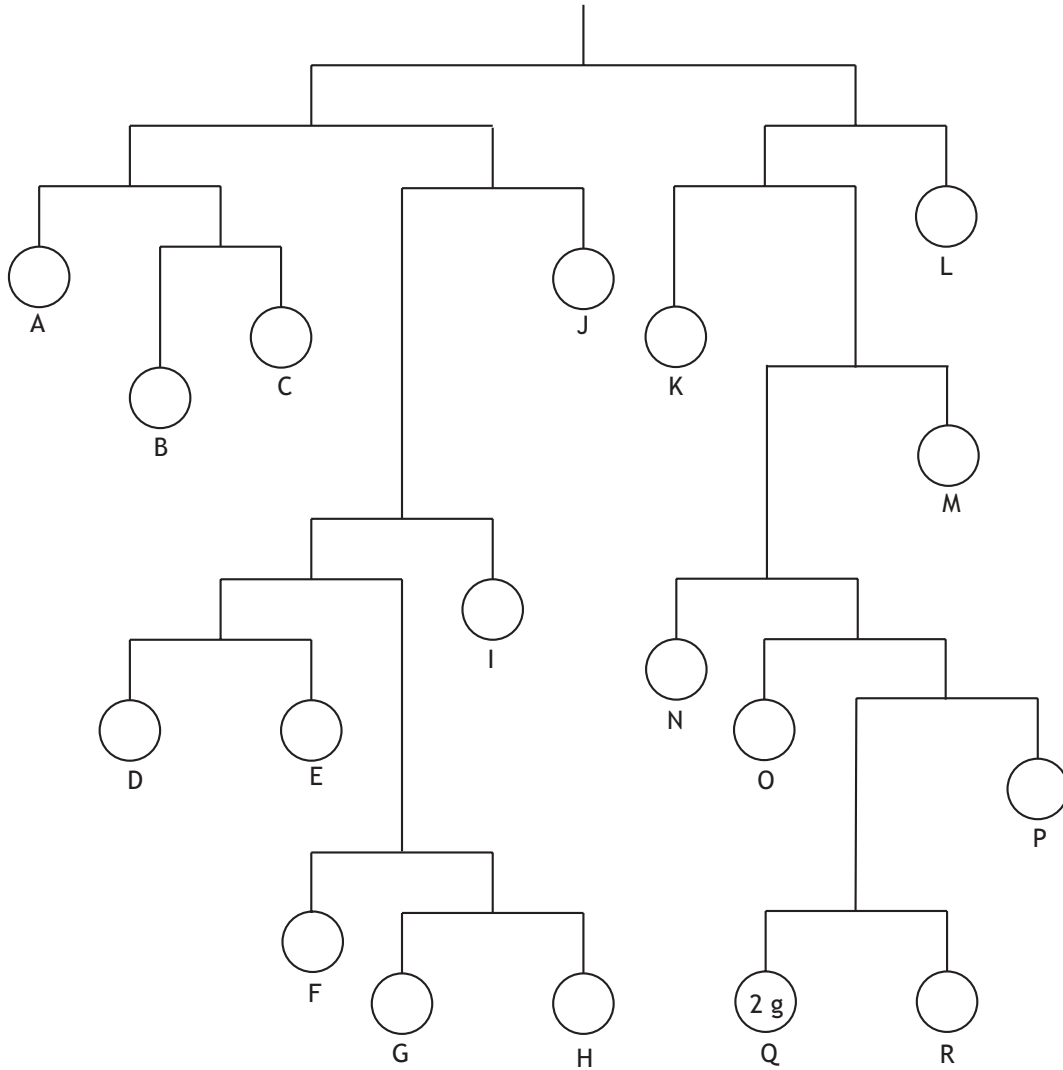
1.



2.



3.



4. Make your own mobile balances as above. Be sure to provide an answer. Trade questions with a partner.

Teacher Information: Surface Additions I

Skills Required

- basic arithmetic
- spatial visualization

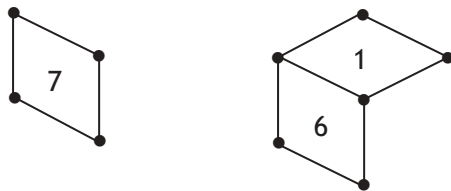
When To Use

This activity may be done at any time.

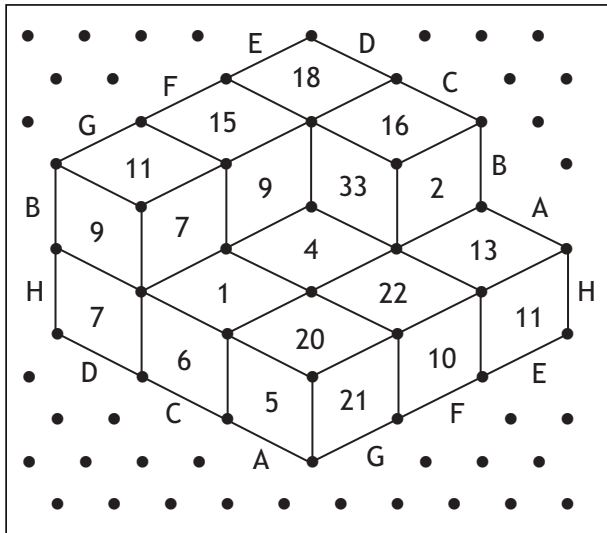
Teaching Information

Start the puzzle with the students. Have the puzzle on the overhead and go over where the pieces might go. Put in the two pieces that can only go in one place.

For example:



Solution



Notes: There are two pieces that can go in right away: the single "7" piece over D and the "1/6" piece over C. Continuing on the C line, you need either 18 or 4 and two sides of the cubes to equal 53. The two sides for the cube on line C can have a possible total of 25, 20, or 49. Only 49 will combine with 4 to get 53. So C is filled in and the "18" piece will need to go in the corner (under D, E). Following along line E, we have $18 + 16 + 2 = 36$. We need 24 and we have two possibilities with the "15/9" and "10/11" pieces. Checking along the D line, we already have $18 + 7 = 25$. We need 35 more

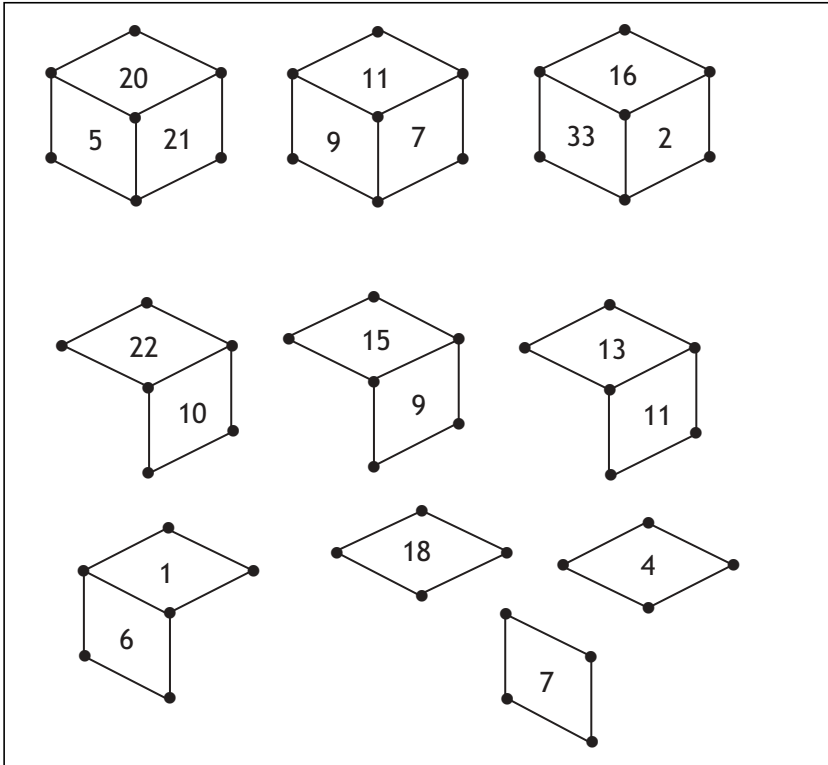
from two sides of a cube and either 15 or 13. The remaining cubes have a sum of 25 or 20 on the two needed sides. Only $20 + 15 = 35$. Therefore, "15/9" goes on the F and "13/11" goes on line E. The last two pieces fit in as above.

Extensions

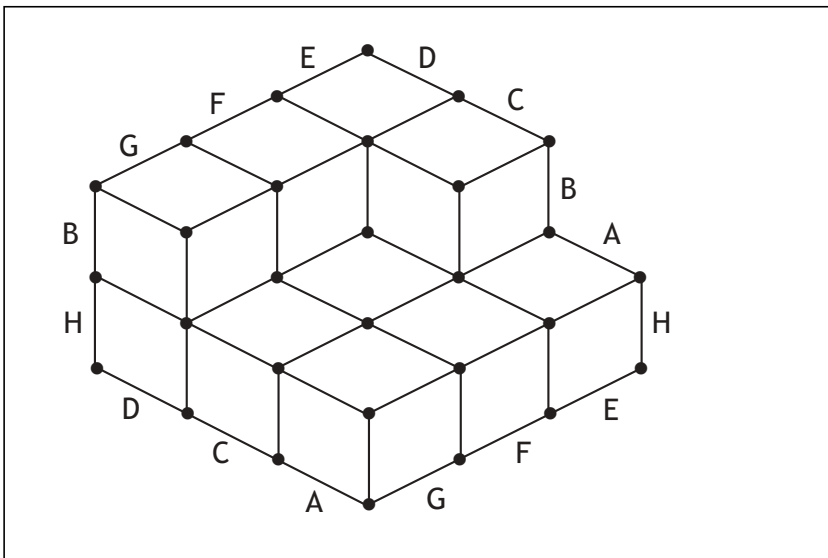
- Students can change the target number and make a new puzzle using the same structure.
- Students can draw a different structure using isometric paper and create their own puzzle.

Blackline Master: Surface Additions I

Fit the pieces below into the diagram so that the numbers on the surfaces of each lettered row (from A to A, B to B, etc.) add up to the target number. Do not rotate pieces or attempt to fit a piece into a section of the diagram that does not match the piece exactly.



Target Number: 60



Extension

Target Number:

Appendix II

Additional Resources

Print

The Diagram Group. *The Little Giant Encyclopedia of Games for 1 or 2*. Sterling Publishing Company Inc. ISBN 0-8069-0981-1.

Fleisher, Paul. *Brain Food: Games That Make Kids Think*. Zephyr Press. ISBN 1-56976-072-1.

Kroner, Lou. *In the Balance: Algebra Logic Puzzles*. Creative Publications. Grades 4–6: ISBN 0-7622-0551-2
Grades 7–9: ISBN 0-7622-0552-0

Note: The Grades 4–6 book has some tough puzzles and is not necessarily just for Grades 4–6.

Nasht, Helen, and Dorothy Masterson. *Humorous Cryptograms*. Sterling Publishing Company Inc. ISBN 0-8069-3982-6.

Tuller, Dave, and Michael Rios. *Mensa Math and Logic Puzzles*. Sterling Publishing Company Inc. ISBN 0-8069-4199-5.

Internet

There are many sites on the Internet with problems and puzzles. When searching for problems and puzzles, use the words “mathematical games.”

As of February 2003, the following sites were available:

Fun Brain

<<http://www.funbrain.com>>

This site offers some interactive games. Some of the games could be adapted to pencil-and-paper games.

Puzzlemaker

<<http://www.puzzlemaker.com>>

This site allows teachers to create their own puzzles. One of the puzzle types is cryptograms. When using the site, be sure to print the answer with the puzzle as all puzzles are created new with each visit.

This Is Mega Mathematics

<<http://www.c3.lanl.gov/mega-math/>>

There are a variety of activities here. One of the activities involves map colouring. Another activity involves games with graphs.