

Unit A: Problem Analysis

Half Course III

HALF COURSE III

Unit A: Problem Analysis

Hours: 9

General Learning Outcome:

Develop and use mathematical strategies to solve problems in different situations.

The intent of this unit is to provide a range of interesting problems of a primarily non-algebraic nature. These problems augment the work in other units.

Specific Outcome

A-1 Solve problems using a variety of non-algebraic approaches.

PROBLEM ANALYSIS

Instructional Materials

- *Essentials of Mathematics 11*
- See Appendix I for possible activities.
- See Appendix II for additional resources.

**PRESCRIBED LEARNING
OUTCOMES**

General Outcome

Develop and use mathematical strategies to solve problems in different situations.

Specific Outcome(s)

A-1 solve problems using a variety of non-algebraic approaches

SUGGESTIONS FOR INSTRUCTION

Examples of non-algebraic approaches include geometry, networks, flow charts, organizational charts, simulations, etc.

Remember that for activities in this unit, the journey is more important than the destination. It is beneficial to discuss multiple approaches to solving these problems, particularly when the approaches have been developed by students. Are some approaches “better” than others? Why? On what grounds?

The problems contained in Appendix I are intended to provide material which is interesting in its own right and which complements the other units of the program. It is illustrative rather than exhaustive. Some activities have been chosen to illustrate a wide variety of job and consumer applications of mathematics that are largely non-algebraic. Others have been chosen because they are intrinsically interesting or because they challenge students to find and to use new ways of analyzing and thinking mathematically. All students do not need to engage in the same activities.

The activities in Appendix I are presented in **no** particular sequence. Teachers are encouraged to supplement this set of activities with material from other sources, such as the Internet. A preliminary list of possible resources is included in Appendix II.

It is suggested that these problems and activities be interspersed throughout the course as either extensions, enrichment, or a change of pace in the day-to-day work of the classroom. Some of them will link directly to particular units, but most are independent and **may** be used at any time. One approach would be to introduce problem analysis with a few days, possibly up to a week, of work on these activities. Intersperse the remainder throughout the course.

-
- ✓ Communications
 - ✓ Connections
 - ✓ Organization and Structure
 - ✓ Patterns
 - ✓ Problem Solving
 - ✓ Reasoning
 - Technology
 - ✓ Visualization
-

SUGGESTIONS FOR ASSESSMENT

Students' progress should be assessed over long periods of time. Look, for example, for an increasing use of a variety of problem-solving strategies and increasingly sophisticated explanations. Anecdotal records of how students work in pairs or groups on these activities is appropriate. Well-developed solutions and examples of reasoning could become part of a student's portfolio.

Problem-solving activities are generally not appropriate on pencil-and-paper timed tests.

**SUGGESTED LEARNING
RESOURCES**

Print

- Austin, J.D. *Applications of Secondary School Mathematics*. Reston, VA: NCTM, 1991.
- Giblin, P., and I. Porteous. *Challenging Mathematics*. Toronto/New York: Oxford University Press, 1990.
- Hirsch, C.R., and R.A. Laing. *Activities of Learning and Teaching*. Reston, VA: NCTM, 1993.
- Mathematical Association of America and National Council of Teachers of Mathematics. *A Sourcebook of Applications of School Mathematics*. Reston, VA: NCTM, 1980.
- National Council of Teachers of Mathematics. *NCTM Student Math Notes*. Reston, VA: NCTM, n.d.
- Senior 3 Consumer Mathematics (35S) Part III: A Course for Distance Learning*. Winnipeg, MB: Manitoba Education, Training and Youth, 2001.
— Cover Assignments
- Swetz, F., and J.S. Hartzler. *Mathematical Modeling in the Secondary School Curriculum*. Reston, VA: NCTM, 1991.
- Two journals which contain useful teaching ideas are:
- The Mathematics Teacher*. National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA: 22091-1593.
- Mathematics in School*. The Mathematical Association, 259 London Road, Leicester, UK: LE2 3BE.
- See Appendix II for a list of additional resources.

Appendix I

Teacher Information: Designing a Work Schedule

Skills Required

- basic arithmetic
- organization of data

When to Use

This activity may be done with the Income and Debt unit or at any time.

Teaching Information

- Go over the scenario with students.
- Ask students if they feel there are many possibilities for schedules. Further probe as to why they believe there would be many possibilities.
- The work schedules created will depend on the assumptions students make:
 - How many managers are there?
 - When there are fewer than ten people in the restaurant, does the manager do more than one job?
 - What is the minimum number of employees allowed in the restaurant? (Can a manager work alone on the night shift?)
 - Are there any holidays that will affect the work schedule?
- Tell students that the goal is to provide a schedule that is fair.
 - Ask students to write out their assumptions before beginning.
- After completing their schedule, students may discuss the fairness of the work schedule. Journal writing on this may be appropriate.
- Peer evaluations may be appropriate.

Possible Solution

There are many possible solutions. To check if the schedule works, the following jobs must be covered:

Hours	Manager	Cook	Waiter
12:00–02:00	1	1	1
02:00–04:00	1	1	1
04:00–06:00	1	1	1
06:00–08:00	1	3	3
08:00–10:00	1	2	2
10:00–12:00	1	2	3
12:00–14:00	1	3	3
14:00–16:00	1	1	1
16:00–18:00	1	2	2
18:00–20:00	1	3	3
20:00–22:00	1	2	2
22:00–24:00	1	1	1

Note: A manager can do all jobs. The numbers listed are the least number for each job.

Blackline Master: Designing a Work Schedule

A new restaurant is opening. The restaurant will be open 24 hours a day. After doing some market research, it is determined that the following employees are needed to staff the restaurant:

- A manager has to be on duty at all times. Managers are able to do all jobs in the restaurant.
- One cook is needed for every 20 customers in the restaurant.
- One waiter or waitress is needed for every five occupied tables.

The average number of customers and occupied tables for every two-hour block are shown in the table below:

Time	12:00 AM – 2:00 AM	2:00 AM – 4:00 AM	4:00 AM – 6:00 AM	6:00 AM – 8:00 AM	8:00 AM – 10:00 AM	10:00 AM – 12:00 PM
# of customers	8	5	10	45	20	40
# of tables	4	3	4	15	7	12

Time	12:00 PM – 2:00 PM	2:00 PM – 4:00 PM	4:00 PM – 6:00 PM	6:00 PM – 8:00 PM	8:00 PM – 10:00 PM	10:00 PM – 12:00 AM
# of customers	55	20	35	45	30	10
# of tables	15	5	10	14	8	3

Work Shifts

- Daily minimum is 3 hours and daily maximum is 8 hours.
- Weekly minimum is 40 hours.

Create a seven-day schedule for the restaurant. State any assumptions you have made. Try to use the fewest number of people at all times.

Extension

Write a report on how a business schedules its employees.

Teacher Information: Traffic Jam

Skills Required

- arithmetic
- diagramming

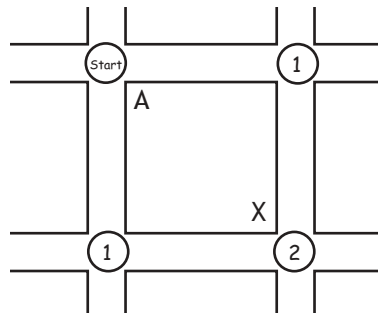
When to Do

This problem may be introduced at any time. It contains elements of geometry, spatial arrangement, and mathematical modelling.

Teaching Information

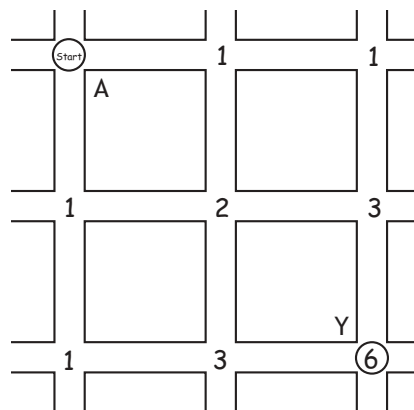
Before having students work on this problem, use the problem-solving strategy, "looking at a simpler case," and examine the pattern.

How many ways are there to get from A to X?



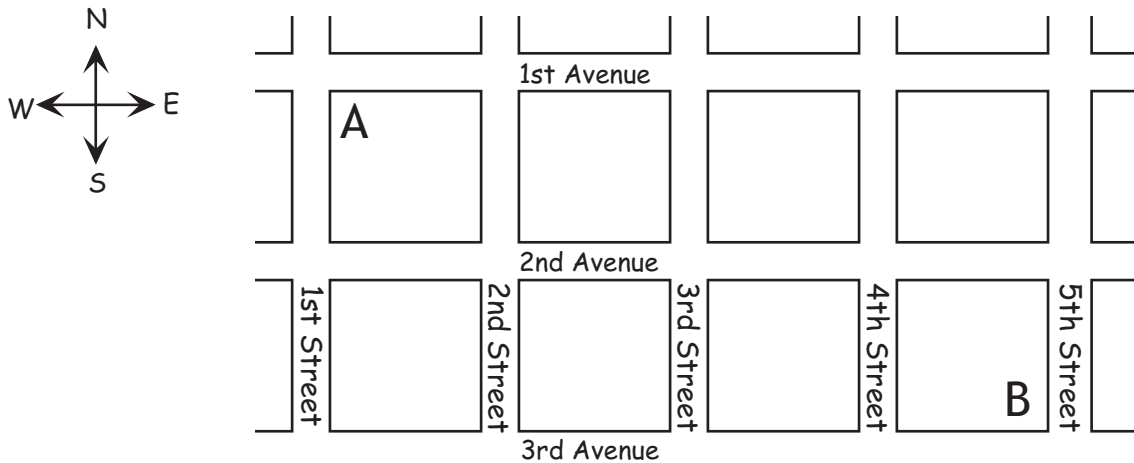
To keep track of the possible paths, put a number in each intersection to represent the number of ways traffic can get to that intersection travelling only south or east.

How many ways are there to get from A to Y?



Blackline Master: Traffic Jam

Construction on a new building has resulted in an increased traffic flow from Pylon Construction headquarters to the construction site. In the following diagram, **A** represents Pylon Construction headquarters and **B** represents the construction site. How many different ways are there for Pylon Construction workers to get to the construction site if all traffic travels south or east on the streets shown?



Extension

Although traffic in an area is heavy, suppose that street repairs must be made. Which intersection should be closed to have the least effect on traffic flowing from Pylon Construction to the construction site? Explain.

Teacher Information: Baseball Salaries

Skills Required

- basic arithmetic
- pattern recognition
- critical thinking

When to Do

This may be done at any time.

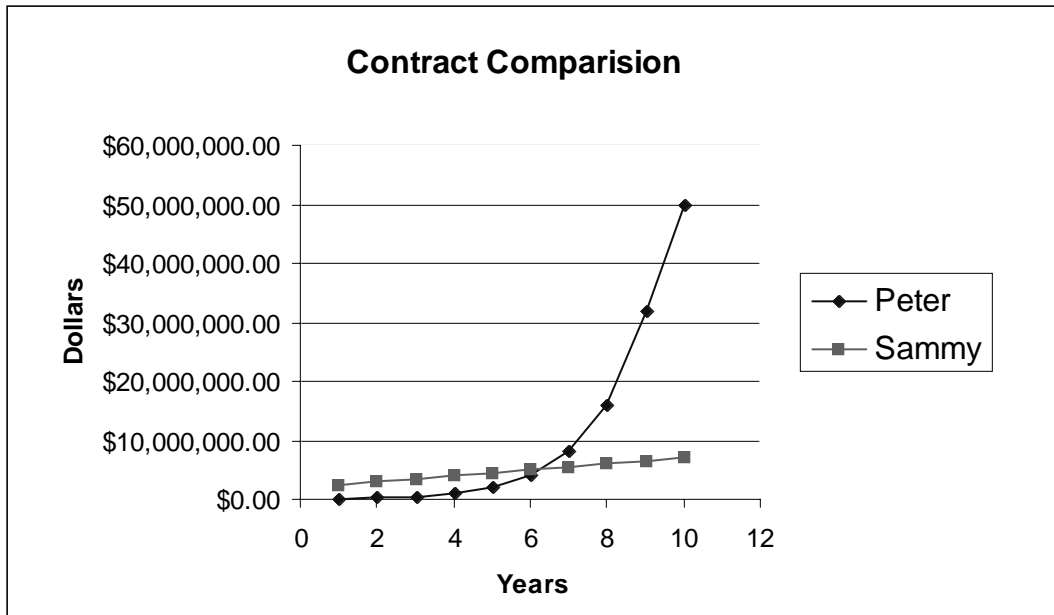
Teaching Information

- Discussion of injuries, league expansion or contraction, career longevity.
- Extensions could include Peter on a Canadian team (40% tax) and Sammy on an American team (20%). Now compare the contracts.
- Students could create their own questions and present them to the class, or use as portfolio items.

Solution

1.

Number of Years	Peter	Sammy
1	\$125,000	\$2,500,000
2	\$250,000	\$3,000,000
3	\$500,000	\$3,500,000
4	\$1,000,000	\$4,000,000
5	\$2,000,000	\$4,500,000
6	\$4,000,000	\$5,000,000
7	\$8,000,000	\$5,500,000
8	\$16,000,000	\$6,000,000
9	\$32,000,000	\$6,500,000
10	\$50,000,000	\$7,000,000



2. Peter—longer career, more income
Sammy—shorter career, more income
3. Year 7
4. Visit <www.mlb.com>
5. Possible answers include: possibility of injury, possibility of being cut, bonuses
6. Answers will vary. The rationale behind the decision is more important than the decision itself.

Blackline Master: Baseball Salaries

Two MLB rookies choose different types of contracts. Peter Pitcher negotiates a salary which pays \$125,000 in his first year and doubles for each season that he plays with a ceiling of \$50,000,000. Sammy Slugger's contract pays \$2,500,000 in his first year and gives him a \$500,000 increment in each subsequent year that he plays.

1. Graph these data either by hand or using a spreadsheet.
2. Who has negotiated the better contract? Explain.
3. In which year would Peter earn more than Sammy?
4. The careers of many MLB players last only a few years. Find out the average number of playing years.
5. What are the assumptions or factors to consider when looking at a contract?
6. If you were a player's agent, what would you recommend?



Teacher Information: Magic Squares

Skills Required

- basic arithmetic
- pattern recognition

When to Do

This activity may be done at any time.

Teaching Information

Definition: A magic square is a square array of numbers arranged so that the sum of numbers in each row, column, and diagonal is the same.

For other magic square activities, see *Senior 2 Consumer Mathematics: A Foundation for Implementation* (2002).

Part A: Solutions

magic sum = 15

2	9	4
7	5	3
6	1	8

magic sum = 15

6	7	2
1	5	9
8	3	4

magic sum = 15

8	3	4
1	5	9
6	7	2

magic sum = 15

8	1	6
3	5	7
4	9	2

magic sum = 15

4	3	8
9	5	1
2	7	6

magic sum = 15

8	1	6
3	5	7
4	9	2

Part B: Solutions

magic sum = 48

A	12	8	28
	32	16	0
	4	24	20

magic sum = 72

B	20	16	36
	40	24	8
	12	32	28

magic sum = 36

C	3	18	15
	24	12	0
	9	6	21

magic sum = 60

D	16	12	32
	36	20	4
	8	28	24

magic sum = 84

E	40	20	24
	12	28	44
	32	36	16

magic sum = 54

F	15	12	27
	3	18	7
	9	24	21

magic sum = 45

F	24	3	18
	9	15	21
	12	27	6

magic sum = 63

H	24	27	12
	9	21	33
	30	15	18

magic sum = 24

I	2	16	6
	12	8	4
	10	0	14

Extension

- Have students generate magic squares of their own.
- Have students generate magic squares using integers, fractions, or decimals.

Blackline Master: Magic Squares

A magic square is an arrangement of the numbers in a square matrix with each number occurring exactly once, and such that the sum of the numbers in any row, any column, or any main diagonal is the same. This sum is often called the "magic sum." The following magic square uses the digits 1 to 9 and has a magic sum of 15.

magic sum = 15

2	7	6
9	5	1
4	3	8

Part A

For each of the following magic squares, determine the magic sum and then complete the square by filling in the missing numbers using the digits 1 to 9.

magic sum =

		4
	5	
6		

magic sum =

6		
	5	
		4

magic sum =

	3	
	5	
	7	

magic sum =

8		
	5	
		2

magic sum =

4	3	8

magic sum =

4	9	2

Part B

For each of the following magic squares, determine the magic sum and then complete the square by filling in the missing numbers.

magic sum =

A

12		28
	16	
		20

magic sum =

B

20		36
		8
		28

magic sum =

C

	18	15
	12	
9		

magic sum =

D

	12	32
	20	
8		

magic sum =

E

40		
	28	
	36	16

magic sum =

F

		27
	18	7
		21

magic sum =

F

24		
9	15	
12		

magic sum =

H

		12
	21	
30	15	

magic sum =

I

		6
12	8	
10		

Extension

magic sum =

magic sum =

magic sum =

magic sum =

magic sum =

magic sum =

magic sum =

magic sum =

Teacher Information: Miscellaneous Problems

Skills Required

- pattern recognition
- Venn diagrams
- proportional reasoning
- number theory (factors, multiples)
- spatial reasoning

When to Do

This may be done at any time.

Teaching Information

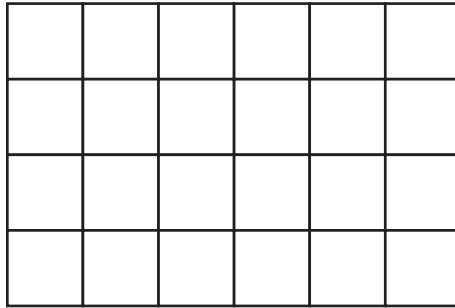
- These problems could be handed out to students in question pairs or as an entire set.
- If handed out in pairs, students could hand in the solution to one out of the two problems.
- If handed out as an entire set, students could be asked to hand in solutions to four of the seven questions and have number eight as a bonus question. This would allow students an opportunity to choose problems that interest them or that they feel are easier.

Solutions

1. 50
2. 42
3. Leah
4. 8 hours
5. 100 minutes
6. 61 mangoes
7. 27 cm^2
8. $(8 - 2\pi) \text{ cm}^2$ or $\cong 1.72 \text{ cm}^2$

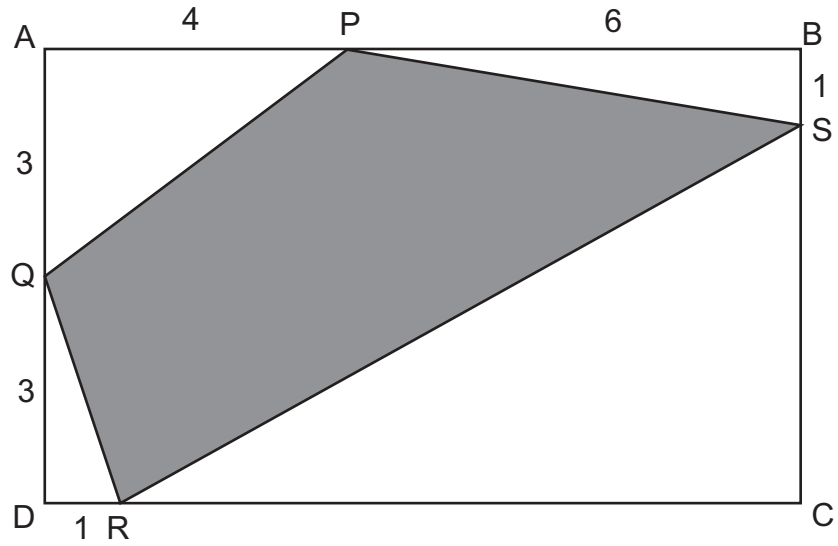
Blackline Master: Miscellaneous Problems

1. A 4 x 6 rectangle is divided into a number of smaller squares as shown. Find the number of squares of *all sizes* in this diagram.



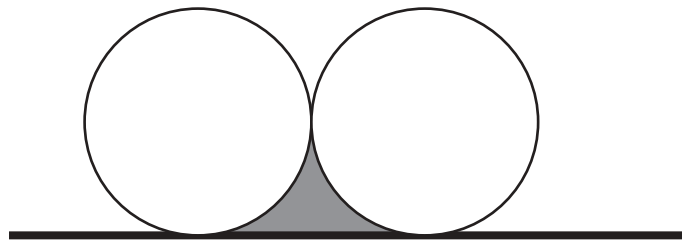
2. Two hundred delegates attended an international gathering. It was discovered that 84 spoke English, 70 spoke French, and 40 spoke Spanish. Fourteen spoke French and Spanish, 12 spoke English and Spanish, and 15 spoke French and English. These numbers include the five people who spoke all three languages. How many delegates spoke none of these languages?

7. In rectangle ABCD below, all the given dimensions on the diagram are in centimetres. Find the area of the shaded region.



Extension

8. Two circles touch one another and also touch the line. The radius of both circles is 2 cm. Find the area of the shaded region.



Appendix II

Additional Resources

Print

The Association of Teachers of Mathematics. *Eight Days a Week: Puzzles, Problems and Questions to Activate the Mind*. The Association of Teachers of Mathematics.

ISBN 1-898-611-09-2.

Baron, Celia, Don Bradford, Angela Kaiser, David Sufrin, Dave Tambellini, and Rick Wunderlich. *Essentials of Mathematics 11*. Victoria, BC: British Columbia Ministry of Education, 2002.

Brecker, Erwin. *Lateral Logic Puzzles*. Sterling Publishing Company, Inc.

ISBN 0-8069-0618-9.

Bremner, John. *Mensa Maths Wizards for Kids*. Carleton Books Limited.

ISBN 1-85868-555-9.

Carter, Philip, Ken Russell, and John Bremner. *The Ultimate Puzzle Challenge*. Carlton Books Ltd. ISBN 1-85868-716-0.

DeSpezio, Michael A. *Giant Book of Challenging Thinking Puzzles*. Sterling Publishing Company, Inc. ISBN 0-8069-2087-4.

Forte, Imogene, and Sandra Schur. *180 Icebreakers to Strengthen Critical Thinking and Problem-Solving Skills*. Incentive Publications, Inc. ISBN 0-86530-345-2.

Graham, Evelyne M. *Think-A-Grams*. Critical Thinking Press and Software.

ISBN Numbers: Book A1: 0-89455-329-1

Book A2: 0-89455-430-1

Book B1: 0-89455-330-5

Book B2: 0-89455-431-X

Book C1: 0-89455-331-3

Book C2: 0-89455-432-8

Hunter, J.A.H. *Entertaining Mathematical Teasers and How to Solve Them*. Dover Publications, Inc. ISBN 0-486-24500-4.

Matt-Smith, Geoffrey. *Mathematical Puzzles for Beginners and Enthusiasts*. Dover Publications, Inc. ISBN 0-486-20198-8.

Nash, Helen, and Dorothy Masterson. *Humorous Cryptograms*. Sterling Publishing Company, Inc. ISBN 0-8069-3982-6.

National Council of Teachers of Mathematics. *How to Evaluate Progress in Problem Solving*. National Council of Teachers of Mathematics. ISBN 0-87353-241-4.

Sloane, Paul, and Des MacHale. *Improve Your Lateral Thinking*. Sterling Publishing Company, Inc. ISBN 0-8069-1374-6.

Weber, Ken. *Five Minute Mysteries for the Armchair Detective*. Stoddart Publishing Co., Ltd. ISBN 0-7737-5210-2.

Internet

There are many sites on the Internet with problems and puzzles. If you are using a search engine to find these sites, search using the words "Mathematics Puzzles Problems."

As of February 2003, the following sites were available:

AAA Math

<<http://www.aaamath.com>>

This site has games and practice sheets for various grade levels and topics. There are links to other sites on the web with games and puzzles.

Algebra Story and Word Problems

<http://www2.hawaii.edu/suremath/intro_algebra.html>

There are word problems for various subjects on this site. As well, there are helpful hints to assist in problem solving. Some of the problems may be too algebraic for Senior 3 Consumer Mathematics students.

Breaking Away from the Mathbook

<<http://www.math.nmsu.edu/breakingaway/main.html>>

Although the site is subtitled **Creative Projects for K-8**, some of the projects may be suitable for Senior 3 Consumer Mathematics students.

Math Forum

<<http://mathforum.org>>

This is a good site to begin searching for problems and puzzles. One feature is **Problems of the Week**. New problems are available as well as a library of previous problems. Students can submit their answers and get some feedback. There are links to other math sites and several departments that are useful.

Word Problems for Kids

<<http://www.stfx.ca/special/mathproblems/welcome.html>>

This is a Canadian site with word problems, hints, and solutions from previous mathematics competitions. The problems are sorted by grade level. Choosing problems from Grades 5 through 9 will lead to a wealth of non-algebraic problems.