

Unit F: Probability and Sampling

Half Course II

HALF COURSE II

Unit F: Probability and Sampling

Hours: 18

General Learning Outcome:

Use probability and sampling to make and justify decisions in real-life situations.

The intent of the unit is to have students learn about probability and sampling techniques in an experimental environment.

Specific Outcomes

- F-1 Estimate population sizes through a variety of sampling methods.
- F-2 Use sample data to make decisions.
- F-3 Apply probability and sampling techniques to real-life situations.

PROBABILITY AND SAMPLING

Instructional Materials

- calculator
- ruler
- graph paper
- additional
 - Pepperidge Farm orange and white crackers
 - dice
 - pennies
- *Essentials of Mathematics 10*

Connections with Problem Analysis and Analysis of Games and Numbers

Any of the Problem Analysis and Analysis of Games and Numbers activities may be interspersed with problems from the Probability and Sampling unit.

**PRESCRIBED LEARNING
OUTCOMES**

General Outcome

Use probability and sampling to make and justify decision in real-life situations.

Specific Outcome(s)

F-1 estimate population sizes through a variety of sampling methods

SUGGESTIONS FOR INSTRUCTION

It is recommended that students:

- work in groups to complete the activities and record results
- make and justify their predictions.
- be risk-takers when making predictions.
- use concrete materials such as dice, cards, coins, blocks, et cetera, to make probability and sampling more realistic.
- design projects around real-life situations.

F-1.1 The essential idea of sampling is to learn about the whole (population) by studying a part (sample). Using good data collection strategies is critical to being able to draw meaningful conclusions from the data once it has been collected and analyzed. Each of the following random sampling methods can be used to determine the size and/or characteristics of a specific population.

1. *Grid Method*—this method uses a rectangular grid to sample a specific population.
2. *Capture-Tag-Recapture Method*—this method assumes that if enough members of a wildlife population are captured, tagged, and released, then any sufficiently large random sample of that population will contain about the same fraction of tagged animals as the entire population.

Important terms to emphasize:

- *Population*—the entire group of people/animals/objects about which information is desired.
- *Sample*—refers to a (typically small) part of the population that is actually examined to gain information about the population.
- *Random Sample*—is one in which all members of the population have an equal chance of being selected. A random sample is usually representative of the population.
- *Biased Sample*—a sample that is not representative of the entire population; that is, it over represents certain segments of the population or under represents certain segments of the population.
- *Unbiased Sample*—a sample that is as varied as the entire population; that is, each segment of the population is represented in the sample.

✓ Communications	Patterns
✓ Connections	Problem Solving
Number Sense	✓ Reasoning
Organization and Structure	Technology
	✓ Visualization

(continued)

SUGGESTIONS FOR ASSESSMENT

Problem

A biologist was trying to determine the effectiveness of a certain brand of herbicide. To do this, she first needed to determine the number of weeds in the plot of land to be studied. She divided the plot into 50 equal-sized sections and randomly chose 10 plots to sample. The number of weeds counted in the sampled sections were: 16, 35, 18, 6, 22, 19, 31, 22, 4, 0.

- a) Determine total number of weeds found in sampled sections. (173)
- b) Determine the estimated total number of weeds found in the plot of land.

$$\frac{173}{10} = \frac{n}{50}$$

$$\frac{(173)(50)}{10} = n$$

$$865 = n$$

Therefore, the estimated number of weeds in this plot of land is 865.

- c) What might be the effect on the estimated population of weeds if a smaller sample size was chosen? (The smaller the sample size, the less likely the population will be adequately represented. Under- or overestimated population sizes might be determined.)

Mental Math

Determine the missing value in each sampling proportion.

a) $\frac{8}{5} = \frac{n}{20}$ (32)

b) $\frac{15}{8} = \frac{n}{40}$ (75)

c) $\frac{25}{10} = \frac{n}{200}$ (500)

SUGGESTED LEARNING RESOURCES

Print

Senior 2 Consumer Mathematics (25S) Part II: A Course for Distance Learning. Winnipeg, MB: Manitoba Education and Training, 2000.
— Module 6, Lesson 1

Montesanto, Ralph, and David Zimmer. *Dealing with Data: Probability and Sampling.* New York, NY: D.C. Heath, 1996.

Baron, Celia, Rick Wunderlich, and Leanne Zorn. *Essentials of Mathematics 10.* Vancouver, BC: British Columbia Ministry of Education, 2002.
Chapter 6
ISBN 0-7726-4675-9

Alexander, Robert, and Brendan Kelly. *Mathematics 11.* Don Mills, ON: Addison Wesley Longman Ltd., 1998.

Knill, George, et al. *Mathpower 11.* Toronto, ON: McGraw-Hill Ryerson Ltd., 1999.

**PRESCRIBED LEARNING
OUTCOMES**

F-1 estimate population sizes through a variety of sampling methods
– *continued*

SUGGESTIONS FOR INSTRUCTION

F-1.2 Random sampling methods

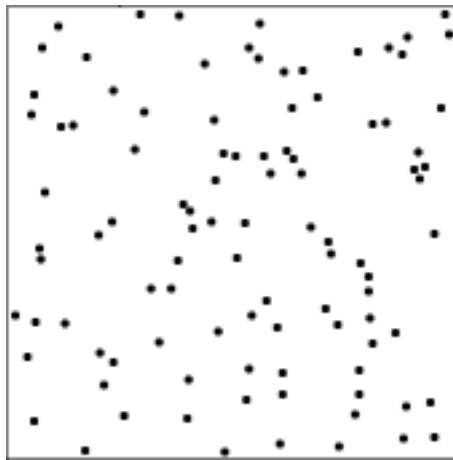
Grid Method—can be used to estimate the population of wildlife in a specific area.

Method

1. Identify the population and the area in which it is to be studied.
2. Divide this area into equal-sized and numbered and/or lettered sections (squares or rectangles). **Note:** Long, narrow rectangles may produce a more unbiased sample.
3. Randomly choose the sections to be studied. This could be done using a random number generated by a computer spreadsheet, by using a table of previously generated random numbers, or small cards that are numbered and/or lettered to match the grid.
4. Counting the number of animals in each randomly chosen section.
5. Determining the total number of animals counted in all random sections.
6. Use proportional reasoning to calculate the estimated size of the population in the specific area.

Example

The diagram shows an aerial view of a woodland marsh. Each dot represents a duck swimming in the marsh. Use a grid and random sampling to estimate the total number of ducks in this marsh.



- | | |
|-------------------------|------------------------|
| ✓ Communications | Patterns |
| ✓ Connections | Problem Solving |
| Number Sense | ✓ Reasoning |
| Organization and | Technology |
| Structure | ✓ Visualization |

(continued)

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Problem

A Senior 2 student was trying to determine the number of butterflies that visit her backyard on any given summer day. To do this, she captured 20 butterflies and marked (tagged) them with a harmless white dot on their left wing. Several days later, she captured 10 more and found that four of them were “tagged.”

- a) Use a proportion to determine the estimated number of butterflies in her yard on that day,

Solution

$$\frac{4}{10} = \frac{20}{n}$$

$$4 \times n = 10 \times 20$$

$$n = \frac{10 \times 20}{4}$$

$$n = 50$$

Therefore, the estimated number of butterflies is 50.

- b) Is this number exactly equal to the true number of butterflies on her property?

Solution

No, this will only be an estimate. On any given day, butterflies would fly in and out of her property. Samples taken on many different days over a period of time could give a better estimate.

- c) What practical difficulties arise when using this method?

Solution

There are no boundaries to her property that would “contain” the butterflies. Capturing the butterflies might prove difficult. Butterfly populations might vary depending on weather conditions and time of day.

Mental Math

Determine the missing value in each sampling proportion.

a) $\frac{2}{20} = \frac{30}{n}$ (300)

b) $\frac{5}{40} = \frac{30}{n}$ (240)

c) $\frac{6}{15} = \frac{60}{n}$ (150)

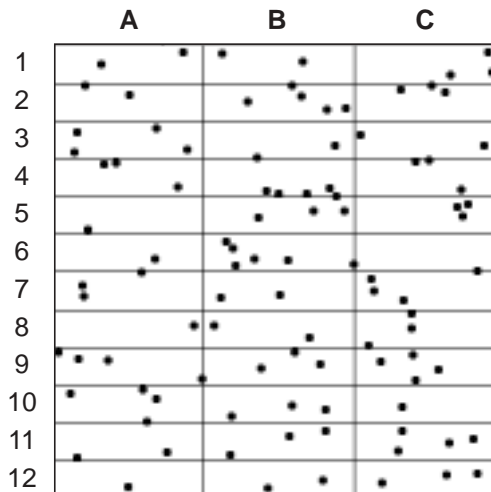
**PRESCRIBED LEARNING
OUTCOMES**

F-1 estimate population sizes through a variety of sampling methods
– *continued*

SUGGESTIONS FOR INSTRUCTION

Solution

1. Divide this area into equal-sized and numbered sections (squares or rectangles). **Note:** Long, narrow rectangles may produce a more unbiased sample.



2. Randomly choose the sections to be studied. This could be done using a random number generated by a computer spreadsheet, by using a table of previously generated random numbers, or numbered and/or lettered cards
3. Randomly chosen sections: A5, A7, B11, B1, B9, C4.
4. Count the number of ducks in each randomly chosen section.
5. A5(1), A7(3), B11(3), B1(2), B9(3), C4(3)
6. Determine the total number of ducks counted in all random sections. Total = 15.
7. Use proportional reasoning to calculate the estimated size of the population in the specific area.

$$\frac{\# \text{ of ducks in sample}}{\# \text{ of sections sampled}} = \frac{\text{total \# of ducks in marsh}}{\text{total \# of sections}}$$

$$\frac{15}{6} = \frac{n}{36}$$

$$\frac{(15)(36)}{6} = n$$

$$90 = n$$

Therefore, the estimated number of ducks in the marsh is 90 (actual count = 100).

Note: Students/groups can decide whether to count a duck that is “on a line” between rectangles.

(continued)

✓ Communications	Patterns
✓ Connections	Problem Solving
Number Sense	✓ Reasoning
Organization and Structure	Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Additional Capture/Recapture Problems

1. Suppose that naturalists catch, tag, and release 40 deer in a forest. Several weeks later they catch a sample of 100 deer, eight of which have tags. What is the estimate for the number of deer in the forest? (500)
2. Suppose that the high school in a small town has 900 students (tagged sample). A random survey of 100 people in the town finds 15 high school students. What is the estimate for the number of people in the town? (6000)
3. In the wild some animals are “trap happy,” that is, they are easier to capture and tag the first time and easier to recapture than others. A “trap happy” animal tagged the first time is also likely to be in the second sample. What do you think this behaviour will do to the estimate of the population size? Show calculations to support your answer.

Possible solution

If a number of “trap happy” animals were tagged and then recaptured, the proportion of tagged animals in the recapture would be higher than expected. This would result in an underestimate of the animal population.

For example, 50 tagged animals, a sample of 20 recaptured
 Few “trap happy” animals Many “trap happy” animals

$$\frac{2}{20} = \frac{50}{n} \quad (500)$$

$$\frac{8}{20} = \frac{50}{n} \quad (125)$$

4. If, when using the capture-recapture method, some animals lose their marks during the study, how will this affect the estimate of the population size? Show calculations to support your answer.

Possible solution

If a number of the animals that were tagged lose their marks, and are then recaptured, the proportion of tagged animals in the recapture would be smaller than expected. This would result in an overestimate of the animal population.

For example, 50 tagged animals, a sample of 20 recaptured
 Few animals lose their marks Many animals lose their marks

$$\frac{8}{20} = \frac{50}{n} \quad (125)$$

$$\frac{2}{20} = \frac{50}{n} \quad (500)$$

**PRESCRIBED LEARNING
OUTCOMES**

F-1 estimate population sizes through a variety of sampling methods
– *continued*

SUGGESTIONS FOR INSTRUCTION

Notes

- Have different student groups use different random numbers. Collect class data and determine the “average” estimated population.
- Experiment with different-sized or different-shaped grids.
- Discuss the effect of varying the sample size; that is, the number of sections sampled.

F-1-3 Capture-Tag-Recapture Method

Simulation Activity

Materials: Pepperidge Farm fish—orange coloured, Pepperidge Farm fish—white coloured (there are about 300 fish per bag), 1 large bowl (the pond), 1 scoop

Method

1. Divide students into groups of 3 or 4.
2. Give each group a “pond” containing the same number of orange fish. (from 50 to 100)
3. Give each group a container of 15 white fish.
4. Scoop out 15 orange fish and “tag” them by replacing each orange fish with white fish and return them to the pond. This is the “capture” step.
5. Mix the pond well.
6. Recapture the fish by scooping out a sample of about 10 fish. Count the fish in the sample that are tagged and not tagged.
7. Repeat the experiment four more times to see how these numbers vary. Determine the “average” number of tagged and untagged fish in the samples chosen.
8. Set up a proportion to determine the estimated population of fish.
9. Collect the class results and average these to determine the average estimated number of fish in the pond.
10. Have each group count the fish in their pond and compare this number to their group estimate and to the class estimate. Discuss results.
11. Enjoy a snack!

✓ Communications	Patterns
✓ Connections	Problem Solving
Number Sense	✓ Reasoning
Organization and	Technology
Structure	✓ Visualization

(continued)

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Problem

A coin manufacturing company is minting a special commemorative coin. The coin is being produced on several coin press machines over a period of several weeks. They plan to produce 100 000 such coins. To check the quality of the coins being produced, quality control specialists took a sample of 80 coins as they came off one press machine. They placed each of the coins on a scale and determined its mass. The mass distributions are shown in the given table.

Mass of Coin (g)	Frequency	Percentage Frequency
4.00	8	
4.25	18	
4.50	28	
4.75	16	
5.00	10	
Total	80	

- Calculate the percentage frequency for each of the given masses.
- Construct a histogram of coin mass versus percentage frequency.
- Do you think that this would be a valid sample that would accurately represent the masses of the 100 000 coins that will be produced? Explain.
- What factors should the quality control specialists consider when trying to select an unbiased sample that would be representative of the entire 100 000 coins?

Solution

a)

Mass of Coin (g)	Frequency	Percentage Frequency
4.00	8	10.0%
4.25	18	22.5%
4.50	28	35.0%
4.75	16	20.0%
5.00	10	12.5%
Total	80	100%

(continued)

**PRESCRIBED LEARNING
OUTCOMES**

F-1 estimate population sizes through a variety of sampling methods
– *continued*

SUGGESTIONS FOR INSTRUCTION

Sample solution (100 fish in pond)

Sample	# Tagged Fish
1	1
2	2
3	1
4	0
5	3
Average	1.4

$$\frac{\text{\# of tagged fish in sample}}{\text{\# of fish in the sample}} = \frac{\text{total \# of tagged fish in population}}{\text{\# of fish in the population}}$$

$$\frac{1.4}{10} = \frac{15}{n}$$

$$1.4 \times n = (10)(15)$$

$$n = \frac{(10)(15)}{1.4}$$

$$n = 107$$

The estimated number of fish in the pond is 107.

Notes

- This simulation could be varied by using alternate 'fish' such as: two different-coloured beads, M&Ms or Skittles tagged using a non-toxic marker.
- A colourful computer simulation can be found at: http://www.gis.psu.edu/geog121/takecens/cap_recap.html (Java).
- Group and class results could be graphed.

✓ Communications	Patterns
✓ Connections	Problem Solving
Number Sense	✓ Reasoning
Organization and Structure	Technology
	✓ Visualization

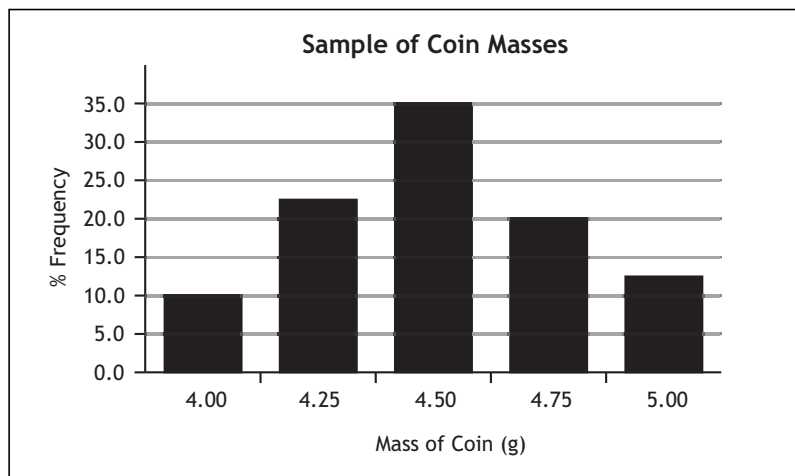
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SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Solution — continued

b)



c) This sample is not a very good sample. The sample was taken at one time from only one machine. The sample may also be too small.

d) Some factors the quality control specialists should consider are:

- Taking samples from several machines
- Taking samples at different time periods
- Taking a larger sample

**PRESCRIBED LEARNING
OUTCOMES**

F-1 estimate population sizes through a variety of sampling methods
– *continued*

SUGGESTIONS FOR INSTRUCTION

Example 1

A game warden nets and tags 200 trout in a lake in northern Manitoba. Two months later, he nets 50 trout and finds four tagged trout. Use a proportion to determine an estimate of the number of trout in the lake.

Solution

$$\frac{4}{50} = \frac{200}{n}$$

$$4 \times n = 50 \times 200$$

$$n = \frac{50 \times 200}{4}$$

$$n = 2500$$

Therefore, the estimated number of trout in the lake is 2500.

Example 2

A game warden captures and tags 80 elk in a provincial park in Manitoba. Two months later, she captures 20 elk and finds two tagged elk. Use a proportion to determine an estimate of the number of elk in the park.

Solution

$$\frac{2}{20} = \frac{80}{n}$$

$$2 \times n = 20 \times 80$$

$$n = \frac{20 \times 80}{2}$$

$$n = 800$$

Therefore, the estimated number of elk in the park is 800.

✓ Communications	Patterns
✓ Connections	Problem Solving
Number Sense	✓ Reasoning
Organization and Structure	Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Problem

A Senior 2 math class prepared a survey that asked this question, “Do you think that the legal drinking age should be raised to 21 years?” They selected a random sample of students in their school and obtained the results shown in the following table.

Raise the Age to 21			
	Yes	No	% Yes
Senior 2	25	20	
Senior 3	15	20	
Senior 4	10	20	
Totals	50	60	

- How many students at each grade level were surveyed?
- If the total sample size represents 20% of all students in the school, determine the total student population.
- Determine the % of students sampled that responded “yes.”
- What is the probability that a Senior 3 student chosen at random in this school would answer “no” to this question?
- Determine the estimated number of Senior 2 students in this school that would answer “yes” to this question.
- What inferences can you make from the data collected?
- Do you think this sample would represent the feelings of students in your school? Conduct a survey of your own to compare results.

**PRESCRIBED LEARNING
OUTCOMES**

F-2 use sample data to make decisions

SUGGESTIONS FOR INSTRUCTION

F-2.1 Gather data from a variety of different sources to assist with decision making and applications.

Example

An athletic shoe manufacturer wishes to market shoes to the student body of your school. The company has asked your class to collect data related to shoe size. Your class collected the data by measuring the foot size of all students from two Senior 2 math classes. The data are given in the following charts.

Required Shoe Size (cm)	Girls Frequency	Percentage Frequency
22.0	1	
22.5	2	
23.0	2	
23.5	3	
24.0	4	
24.5	6	
25.0	4	
25.5	5	
26.0	2	
26.5	2	
27.0	1	
Total	32	

Required Shoe Size (cm)	Boys Frequency	Percentage Frequency
25.0	1	
25.5	1	
26.0	2	
26.5	2	
27.0	5	
27.5	7	
28.0	4	
28.5	3	
29.0	2	
29.5	2	
30.0	1	
Total	30	

- ✓ **Communications** Patterns
- ✓ **Connections** Problem Solving
- Number Sense ✓ **Reasoning**
- Organization and Technology
- Structure ✓ **Visualization**

(continued)

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Solutions

a) Senior 2 — 45, Senior 3 — 35, Senior 4 — 30

b) total # in sample: $45 + 35 + 30 = 110$

$$20\% \times n = 110 \quad \text{or} \quad \frac{20}{100} = \frac{110}{n}$$

$$n = \frac{110}{20\%} \quad 20 \cdot n = 100 \cdot 110$$

$$n = 550 \text{ students} \quad n = \frac{100 \cdot 110}{20}$$

$$n = 550 \text{ students}$$

c)

Raise the Age to 21			
	Yes	No	% Yes
Senior 2	25	20	55.6%
Senior 3	15	20	42.9%
Senior 4	10	20	33.3%
Totals	50	60	45.5%

d) # of Senior 3s answering “no” = 57.1%

e) # of Senior 2s in this school answering “yes” = 125

$$\frac{45}{110} = \frac{n}{550}$$

$$n = 225 \text{ Senior 2 students}$$

$$225 \times 55.6\% = 125$$

f) Many inferences can be made from the sample data including: the majority of Senior 2 students support the idea, the majority of Senior 3 and 4 students do not. Generally, the older the student, the less likely she or he is to support this idea.

g) Answers will vary.

**PRESCRIBED LEARNING
OUTCOMES**

F-2 use sample data to make decisions
– *continued*

SUGGESTIONS FOR INSTRUCTION

1. Calculate the percentage frequency for each shoe size in each chart.
2. Construct a histogram showing the distribution of shoe sizes for female students and one for male students.

Solution

1.

Required Shoe Size (cm)	Girls Frequency	Percentage Frequency
22.0	1	3.1%
22.5	2	6.2%
23.0	2	6.2%
23.5	3	9.4%
24.0	4	12.5%
24.5	6	18.8%
25.0	4	12.5%
25.5	5	15.6%
26.0	2	6.2%
26.5	2	6.2%
27.0	1	3.1%
Total	32	100%

Required Shoe Size (cm)	Boys Frequency	Percentage Frequency
25.0	1	3.3%
25.5	1	3.3%
26.0	2	6.7%
26.5	2	6.7%
27.0	5	16.7%
27.5	7	23.3%
28.0	4	13.3%
28.5	3	10.0%
29.0	2	6.7%
29.5	2	6.7%
30.0	1	3.3%
Total	30	100%

✓ **Communications** Patterns
 ✓ **Connections** Problem Solving
 Number Sense ✓ **Reasoning**
 Organization and Technology
 Structure ✓ **Visualization**

(continued)

SUGGESTIONS FOR ASSESSMENT**SUGGESTED LEARNING
RESOURCES****Assorted Problems**

1. A company makes bicycle helmets. The quality control department finds that 8% of the helmets have at least one defect. How many defected helmets would you expect to find in a random sample of each size?
 - a) 40 helmets (3)
 - b) 120 helmets (10)
 - c) 450 helmets (36)
2. Provincial government data suggests that the probability of a young driver between the ages of 16-18 years being involved in one accident this year is 15%. In a school population of 520 young drivers in this age range, calculate the number who most likely will be involved in an accident this year. (78 students)
3. A random sample of students was asked the question, "Should extra credits be given for participating in extracurricular high school sports?" The responses were: Yes — 52, No — 36, No Opinion — 12
 - a) How many students were sampled?
 - b) What % of students responded to each answer?
 - c) If the sample represented 10% of the students in the school, determine the school population.
 - d) If the sample was taken as students left the gym following a basketball game, would the sample be representative of the total school population? Explain.

Solutions

- a) 100
- b) Yes — 52%, No — 36%, No Opinion — 12%
- c) 1000 students
- d) This sample might not be truly representative of the total population because the students leaving the gym would most likely support the earning of extra credits for extracurricular sports. Students not interested in sports and those who simply do not support the extra credits might not be fairly represented in this sample.

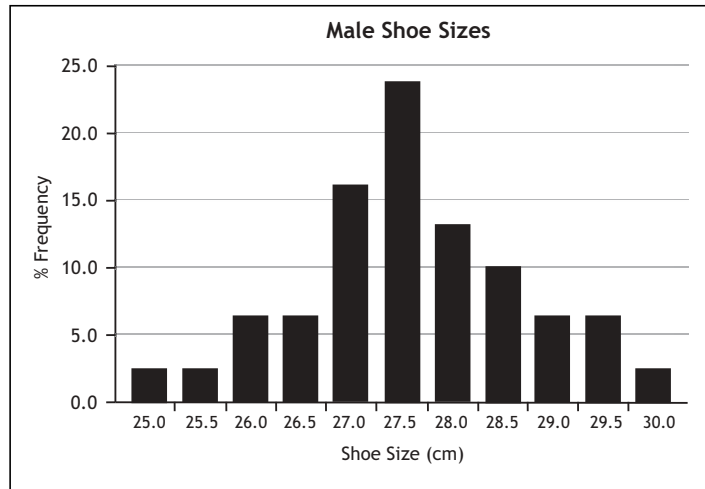
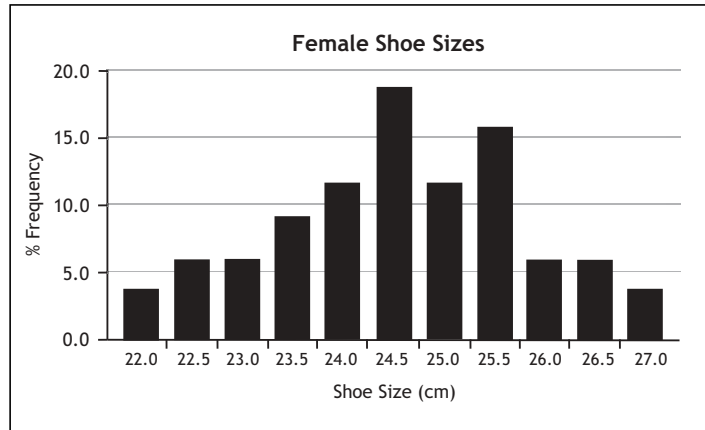
**PRESCRIBED LEARNING
OUTCOMES**

F-2 use sample data to make decisions
– *continued*

SUGGESTIONS FOR INSTRUCTION

Solution — continued

2.



Possible questions related to these data:

1. Do you think that these sample data represent the target population of all students in your school? Explain why or why not.
2. How could you sample the students of your school so that the data would be more representative of the target population?
3. Suppose you sampled 60 students at each grade level in your school. Would this be a more representative sample?
4. What factors would you have to consider to ensure that your sample accurately represented the shoe size distribution throughout your school?

(continued)

✓ Communications	Patterns
✓ Connections	Problem Solving
Number Sense	✓ Reasoning
Organization and Structure	Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

**PRESCRIBED LEARNING
OUTCOMES**

F-2 use sample data to make decisions
– *continued*

SUGGESTIONS FOR INSTRUCTION

5. Suppose the percentage frequency of your sample data accurately represents the target population in your school. Estimate the number of students in your school for each shoe size. **Note:** A school population of 600 (300 females and 300 males) is used for all calculations.

Solutions

1. This sample does not accurately represent the target population of all students in the school because the sample only represents students in a Senior 2 class. Most Senior 3 and Senior 4 students would not be enrolled in this course and would not be represented in the sample.
2. A more representative sample would have data collected from Senior 3 and Senior 4 students. The number of students chosen could be based on the % of students at each grade level. For example, if 35% of all students were in Senior 3 and wanting a sample size of 60 boys and 60 girls, then 21 Senior 3 boys and 21 Senior 3 girls should be chosen ($60 \times 35\%$)
3. This sample would be more representative, providing that there were an equal number of students in all three grades.
4. Several factors to consider: What percentage of students are found at each grade? What percentage of females and males are there at each grade level? How big should the sample size be? How would the students be chosen at random? Where and when would the data be collected?
5. Sample answer based upon a school population of 600 students.

Female: 24.0 cm shoe size: $300 \times 12.5\% = 37.5$ (37 or 38 females)

Note: A spreadsheet could be used to calculate the percentage frequency and the estimated number of students with each shoe size and to construct the histograms.

✓ Communications	Patterns
✓ Connections	Problem Solving
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Organization and Structure	Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

**PRESCRIBED LEARNING
OUTCOMES**

F-3 apply probability and sampling techniques to real-life situations

SUGGESTIONS FOR INSTRUCTION

Students can be given data from which to draw inferences or they can design surveys to collect data of their own.

F-3-1 Problem With Data Given

A production manager for the CDs-R-Us Company is concerned about the quality of CDs that the company is producing. The company is expected to produce 5000 CDs per day. The manager randomly chooses a sample of 250 CDs and finds that 10 of them have defects.

1. What percent of the sample CDs had defects?
2. If this sample is a valid representation of the total number of CDs produced on that day, what is the probability that a CD chosen at random from the day's production will be defective?
3. If this sample is a valid representation of the total number of CDs produced on that day, how many of the 5000 CDs produced that day would most likely be defective?
4. If this sample is a valid representation of the total number of CDs produced by the company, how many CDs in a shipment of 25 000 CDs would most likely be defective?
5. The company instituted improvements in its production line. Later a sample of 250 CDs found only four defective ones. Determine the defective rate and calculate how many of the 5000 CDs produced on that day would most likely be defective.

Solution

1. $\frac{10}{250} \times 100 = 4\%$
2. $P(\text{defective}) = \frac{\# \text{ defective}}{\# \text{ in sample}}$, $P(\text{defective}) = \frac{10}{250}$ or $\frac{1}{25}$
or 4%
3. 4% of 5000 or $4\% \times 5000 = 200$ defective CDs
4. 4% of 25 000 or $4\% \times 25\ 000 = 1000$ defective CDs
5. $\frac{4}{250} \times 100 = 1.6\%$, $1.6\% \times 5000 = 80$
or
 $\frac{(4)(5000)}{250} = n$
 $80 = n$

(continued)

✓ Communications	Patterns
✓ Connections	Problem Solving
Number Sense	✓ Reasoning
Organization and Structure	Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

**PRESCRIBED LEARNING
OUTCOMES**

F-3 apply probability and sampling techniques to real-life situations
– *continued*

SUGGESTIONS FOR INSTRUCTION

F-3-2 Problem With Data Collected By Students

Students should be encouraged to collect data from which to make inferences. Data could be collected within the school, the community, newspapers, or the Internet.

If the data is collected by way of a survey, care must be taken to design it properly. A well-designed survey involves several steps:

- Identify the population to be studied
- Select method of choosing an unbiased sample
- Prepare the survey question(s) or item
- Collect the data
- Calculate the results and display them graphically
- Make inferences (conclusions) about the population

Example

Some educators are concerned about the number of hours per week that high school students spend working at part-time jobs. Many feel that these jobs have a negative impact on the success that students have at school. The first step in studying this problem is to determine how many students actually have part-time jobs and, if so, how many hours they work in a week. Follow the steps necessary to create a well-designed survey.

Sample Solution

1. Identify the population to be studied. (*All students in the school.*)
2. Select a method of choosing an unbiased sample. (*Each grade level should be represented in the sample according to their proportion in the school. For example, if 30% of the student population are Senior 2 students, then 30% of the sample should be Senior 2 students. The sample could be made random by choosing every 5th student from each class, every 5th student that enters the school, et cetera*)
3. Prepare the survey question(s) or item, for example, What grade are you in? Do you have a part-time job? On average, how many hours per week do you work at this job? Additional questions related to the impact that this job may or may not have on success at school could also be asked.

✓ Communications	Patterns
✓ Connections	Problem Solving
Number Sense	✓ Reasoning
Organization and Structure	Technology
	✓ Visualization

(continued)

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

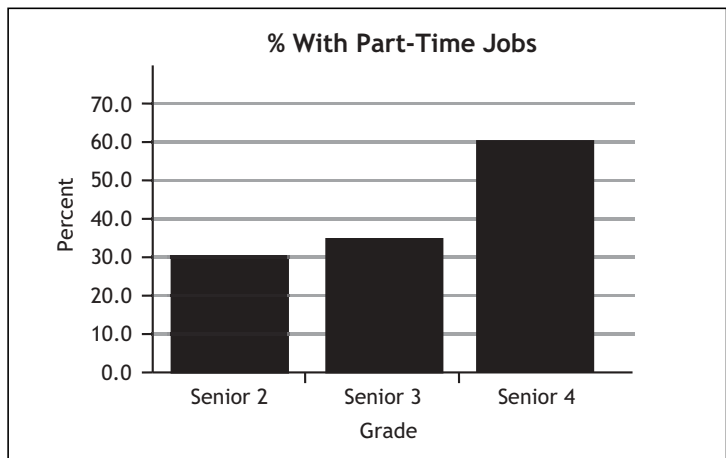
**PRESCRIBED LEARNING
OUTCOMES**

F-3 apply probability and sampling techniques to real-life situations
– *continued*

SUGGESTIONS FOR INSTRUCTION

4. Collect the data and calculate the results. The following data are hypothetical based on the following population size: Senior 2 — 163, Senior 3 — 175, Senior 4 — 162. Sample size — 80.

Part-Time Job?			
	Yes	No	% Working
Senior 2	8	18	30.8%
Senior 3	10	18	35.7%
Senior 4	16	10	61.5%
Totals	34	46	42.5%



Hours per Week	Grade		
	Senior 2	Senior 3	Senior 4
<5	1	2	0
5-10	2	2	3
10-15	3	3	5
15-20	2	2	6
>20	0	1	2
Total	8	10	16

- ✓ **Communications** Patterns
- ✓ **Connections** Problem Solving
- Number Sense ✓ **Reasoning**
- Organization and Technology
- Structure ✓ **Visualization**

(continued)

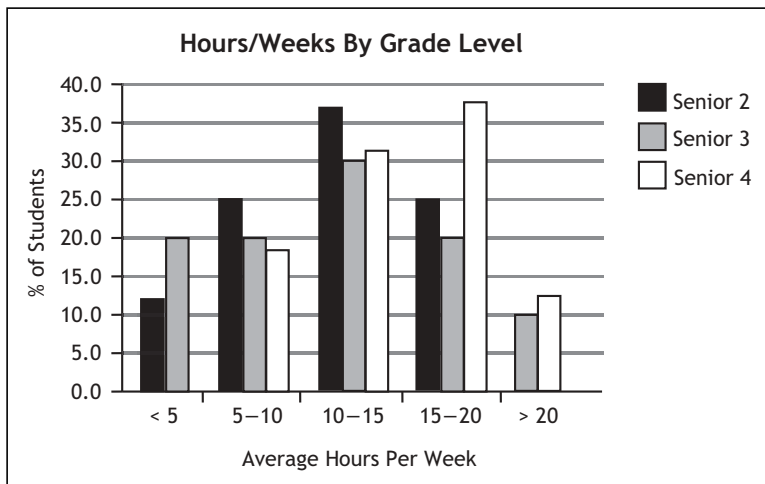
SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
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OUTCOMES**

F-3 apply probability and sampling techniques to real-life situations
– *continued*

SUGGESTIONS FOR INSTRUCTION



F-3-3 Make inferences (conclusions) about the population (assuming the sample is a valid sample).

Sample Questions

1. Determine the probability that a student chosen at random from any grade level will have a part-time job.

Senior 2: $\frac{8}{26}$ or 30.8%

Senior 3: $\frac{10}{28}$ or 35.7%

Senior 4: $\frac{16}{26}$ or 61.5%

2. Determine the probability that a student chosen at random from any grade level will work between 10 and 15 hours per week.

Senior 2: $\frac{3}{8}$ or 37.5%

Senior 3: $\frac{3}{10}$ or 30.0%

Senior 4: $\frac{5}{16}$ or 31.2%

3. Determine an estimate of the total number of students at each grade level that have part-time jobs.

Senior 2: $30.8\% \times 163 = 50$ students

Senior 3: $35.7\% \times 175 = 62$ students

Senior 4: $61.5\% \times 162 = 100$ students

✓ Communications	Patterns
✓ Connections	Problem Solving
Number Sense	✓ Reasoning
Organization and Structure	Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
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**PRESCRIBED LEARNING
OUTCOMES**

F-3 apply probability and sampling techniques to real-life situations
– *continued*

SUGGESTIONS FOR INSTRUCTION

4. Do you feel that the results from your school would be typical of all high school students in Manitoba? In Canada? Explain.

F-3-4 Other possible survey topics

- # hours/week spent on homework
- # of students who travel to school by car, bus, et cetera.
- Should cyclists be required to wear helmets?
- Should the legal drinking age be raised to 21 years?
- Do you agree with the province’s decision to institute a graduated licence system for beginning drivers?

F-3-5 Design a one-question survey about a topic of interest to the student. Word the survey in the form of a yes or no question. Design and conduct the survey with a random sample of people. Display the data in charts and graphs and use the results to make inferences.

F-3-6 Collect survey data from newspapers, magazines, or the Internet. Determine the target population and describe how the survey was conducted. What inferences were made from the survey data? Do you agree or disagree with these inferences? Explain. Survey your school population on the same topic and prepare a report of your findings. Groups of students could choose different survey topics and then present their results to the class.

✓ Communications	Patterns
✓ Connections	Problem Solving
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SUGGESTIONS FOR ASSESSMENT

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RESOURCES**
