

Unit F: Spatial Geometry

Half Course I

HALF COURSE I

Unit F: Spatial Geometry

Hours: 13

General Learning Outcome:

Analyze and find relationships among 2-D figures and 3-D objects.

This unit is intended to give students experience with geometric visualization and drawing in a motivational context with hands-on learning experiences.

General Outcome

Analyze and find relationships among 2-D figures and 3-D objects.

Specific Outcomes

- F-1 Sketch 90° , 180° , and 270° clockwise rotations for 2-dimensional figures.
- F-2 Sketch the reflections of 2-dimensional figures over vertical, horizontal, left diagonal, and right diagonal lines of reflection.
- F-3 Draw top, front, and side views for both 3-dimensional rod/block objects and their sketches.
- F-4 Sketch 3-dimensional rod/block objects using isometric dot paper.

SPATIAL GEOMETRY

Instructional Materials

- Teachers could use Cuisenaire rods or interlocking cubes in this unit. However, the unit has been written for Cuisenaire rods.
- One copy of the textbook, *Spatial Problem Solving with Cuisenaire Rods* per teacher. There are limited reproduction rights. This text is available from the Manitoba Text Book Bureau. See “Suggested Learning Resources” column on page I-F-5 for bibliographic information.
- One classroom set of Cuisenaire rods, available from the Manitoba Text Book Bureau, can be divided into 24 student packages containing the following: one yellow, two purple, two green, four red, and three white rods.
- Square dot paper and isometric dot paper (see Blackline Masters, pp. I-F-27 and I-F-28)
- Miras (or mirrors)
- *Essentials of Mathematics 10*

Student Materials

- pencil crayons or crayons

Connections with Problem Analysis and Analysis of Games and Numbers

Any of the Problem Analysis and Analysis of Games and Numbers activities may be interspersed with problems from the Spatial Geometry unit.

Teachers should find activities that involve visualization and patterns themes.

Travelling Knight
Check It out
Face Lift

**PRESCRIBED LEARNING
OUTCOMES**

General Outcome

Analyze and find relationships among 2-D figures and 3-D objects.

Specific Outcome(s)

F-1 sketch 90° , 180° , and 270° clockwise rotations for 2-dimensional figures

SUGGESTIONS FOR INSTRUCTION

The following is a possible sequence of learning experiences. You might consider presenting this unit in two separate parts throughout the semester.

Part I: 2-D images (specific outcomes F-1 and F-2)
Rotations and reflections
Time allotment: approximately 5 hours

Part II: 3-D images (specific outcomes F-3 and F-4)
Plans and elevations
Time allotment: approximately 8 hours

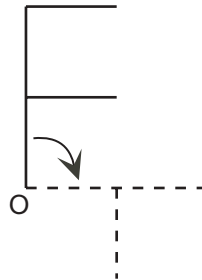
F-1-1 Manipulative materials are required for this unit. Cuisenaire rods or interlocking blocks should be used. *Spatial Problem Solving with Cuisenaire Rods* by Patricia S. Davidson and Robert E. Willcutt is a strongly recommended resource. The rods and this book are available through the Manitoba Text Book Bureau.

Teachers can make coloured overhead transparencies to demonstrate rotations. Students use the rods to create the shape, and then rotate either the structure itself or the grid paper underneath.

Rotation: A transformation in which the points of a figure are rotated about a fixed point.

Examples

The figure F below is rotated 90° around the point O.



Communications	Patterns
✓ Connections	Problem Solving
Number Sense	Reasoning
Organization and Structure	✓ Technology
	✓ Visualization

(continued)

SUGGESTIONS FOR ASSESSMENT

Students need to demonstrate their understanding of this concept through the use of manipulatives.

One suggestion is to have the teacher check a student's creation of the transformation that was made using rods. Another suggestion is to have the student colour in the new image on grid paper. Students should be encouraged to create their own questions and then present them to the class. A student's creation would go into his or her portfolio.

**SUGGESTED LEARNING
RESOURCES**

Print

Senior 2 Consumer Mathematics (25S) Part I: A Course for Distance Learning. Winnipeg, MB: Manitoba Education and Training, 2000.
— Module 3, Lesson 5

Baartmans, B.G., and S.A. Sorby. "Making Connections: Spatial Skills and Engineering Drawings." *The Mathematics Teacher* 89.4. Reston, VA: NCTM, April 1996.

Davidson, Patricia S., and Robert E. Willcutt. *Spatial Problem Solving with Cuisenaire Rods.* New York, NY: Cuisenaire Co. of America, 1983. (This booklet contains blackline masters which may be copied.)

Baron, Celia, Rick Wunderlich, and Leanne Zorn. *Essentials of Mathematics 10.* Vancouver, BC: British Columbia Ministry of Education, 2002.
Chapter 5
ISBN 0-7726-4675-9

The Math Survival Kit. Pilmer Publishing. ISBN 0-9730067-0-6

Cuisenaire rods are available through the Manitoba Text Book Bureau. Six sets are ample for one class. Rods may also be available from your local elementary schools.

Online

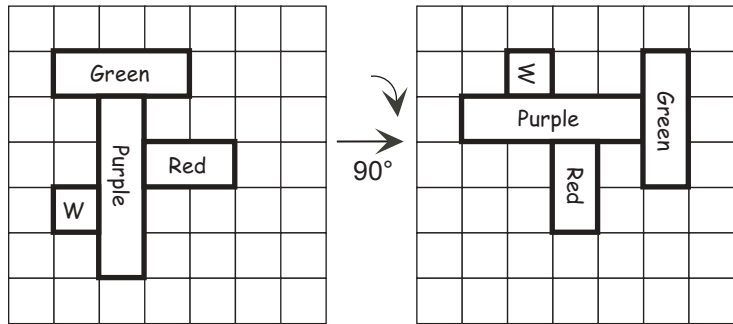
<www.eternity-puzzle.co.uk>

**PRESCRIBED LEARNING
OUTCOMES**

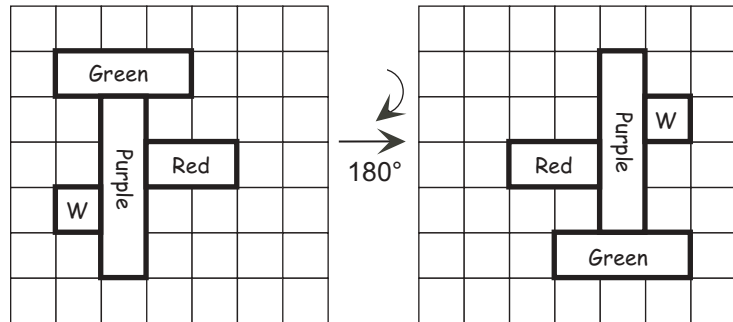
F-1 sketch 90° , 180° , and 270° clockwise rotations for 2-dimensional figures
– *continued*

SUGGESTIONS FOR INSTRUCTION

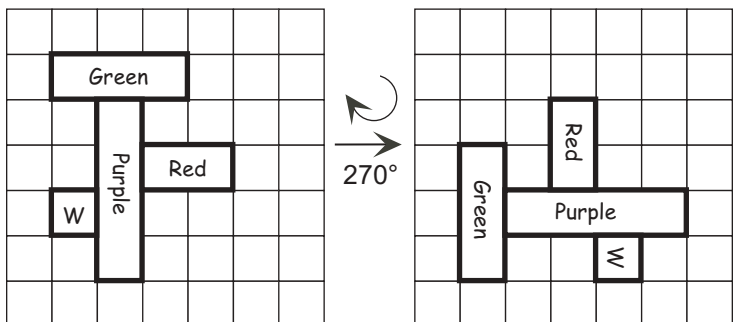
Below is an example of a 90° clockwise rotation.



Below is an example of a 180° clockwise rotation.



Below is an example of a 270° clockwise rotation.



Communications	Patterns
✓ Connections	Problem Solving
Number Sense	Reasoning
Organization and Structure	✓ Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

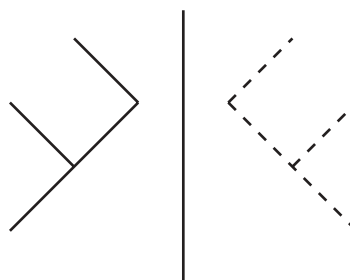
PRESCRIBED LEARNING OUTCOMES

F-2 sketch the reflections of 2-dimensional figures over vertical, horizontal, left diagonal, and right diagonal lines of reflection

SUGGESTIONS FOR INSTRUCTION

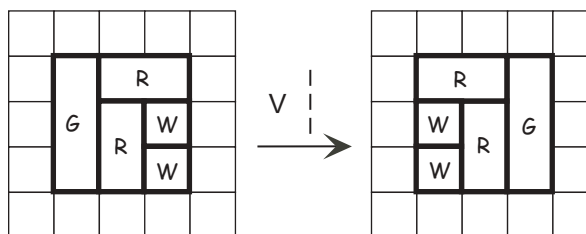
F-2-1 Miras help students see reflections. Mirrors can also be used effectively. Teachers are encouraged to make overhead transparencies, and colour in the shapes as per the rod colours. Bringing students up to the overhead projector to actually see the reflection on the mira can be helpful. Students should be encouraged to “guess” where the image will go before actually using the mira to see it.

Reflection: A transformation where a figure is “flipped” across a line.

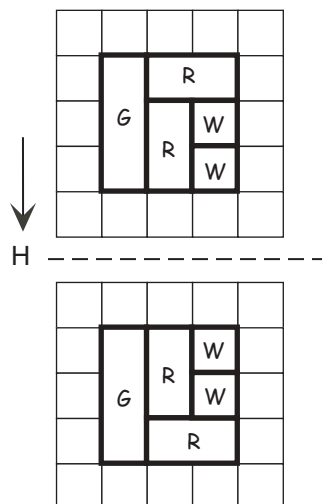


Examples

Below is an example of a vertical reflection.



Below is an example of a horizontal reflection.



Communications	Patterns
✓ Connections	Problem Solving
Number Sense	Reasoning
Organization and Structure	✓ Technology
	✓ Visualization

(continued)

SUGGESTIONS FOR ASSESSMENT

Students need to demonstrate their understanding of this concept through the use of manipulatives.

One suggestion is to have the teacher check a student's creation of the transformation that was made using rods. Another suggestion is to have the student colour in the new image on grid paper. Students should be encouraged to create their own questions and then present them to the class. A student's creation would go into his or her portfolio.

**SUGGESTED LEARNING
RESOURCES**

Print

*Senior 2 Consumer
Mathematics (25S) Part I:
A Course for Distance
Learning.* Winnipeg, MB:
Manitoba Education and
Training, 2000.
— Module 5, Lesson 2

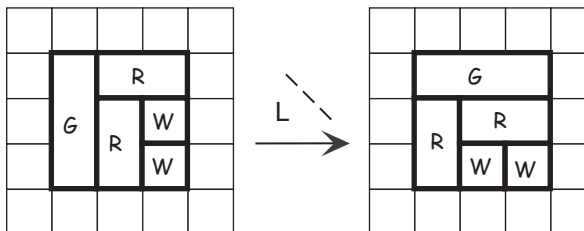
PRESCRIBED LEARNING OUTCOMES

F-2 sketch the reflections of 2-dimensional figures over vertical, horizontal, left diagonal, and right diagonal lines of reflection
 – *continued*

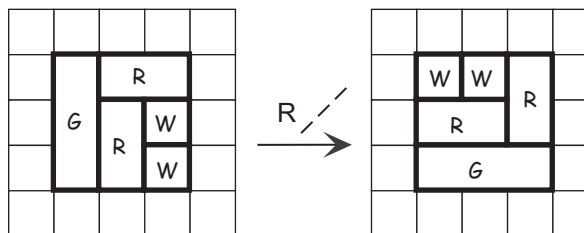
Communications	Patterns
✓ Connections	Problem Solving
Number Sense	Reasoning
Organization and Structure	✓ Technology
	✓ Visualization

SUGGESTIONS FOR INSTRUCTION

Below is an example of a left diagonal reflection.



Below is an example of a right diagonal reflection.



F-3 draw top, front, and side views for both 3-dimensional rod/block objects and their sketches

Communications	Patterns
✓ Connections	Problem Solving
Number Sense	✓ Reasoning
Organization and Structure	Technology
	✓ Visualization

F-3-1 Students need to demonstrate their understanding of this concept through the use of manipulatives. They should build the given structure with the rods or interlocking cubes.

There are three views we will consider (top, front, side). Generally, the side view demonstrated would be a side view from the right. Students should be encouraged to leave their desks, and actually stand above, beside, or directly down in front of the structure in order to see the exact views.

Square dot paper or grid paper should be provided for the students to draw 2-D images of each view. Some students will prefer to colour in the squares, while others will simply write in the letter representing the colour.

The sequence should be to draw the views from the given structure and then to draw or build the structure from the given views. This last activity can be demanding for some students.

(continued)

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Students should be observed moving about their desks as they position themselves to see the object from the three different views. The actual drawings of the views can be easily graded for accuracy. Teachers might want to provide the students with the three views, and then ask that the students use the rods to construct the object from these views. Again, this can be challenging for some students.

Encourage students to create their own questions and present them to the class. This attaches meaning to the activity and the questions can be included in the portfolio assessment.

Print

*Senior 2 Consumer
Mathematics (25S) Part I:
A Course for Distance
Learning.* Winnipeg, MB:
Manitoba Education and
Training, 2000.
— Module 5, Lesson 5

A master sheet of square dot paper for drawing views is provided on page I-F-27.

PRESCRIBED LEARNING OUTCOMES

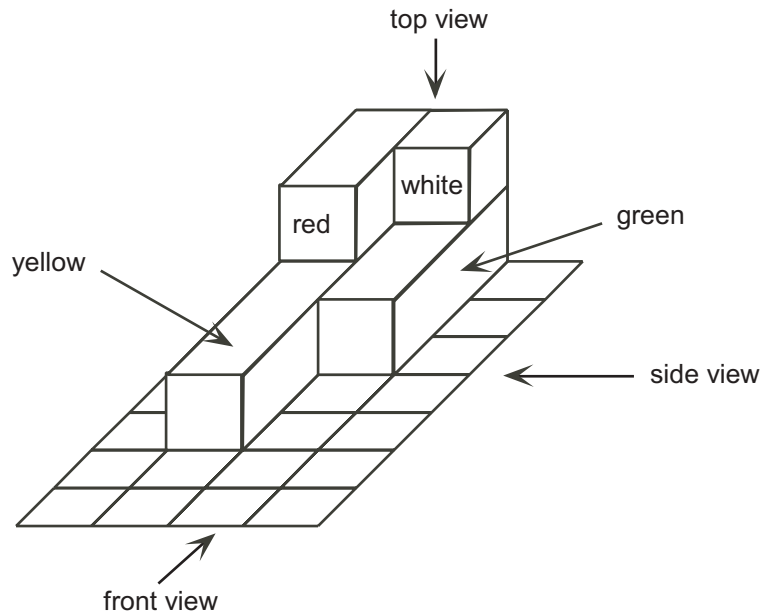
F-3 draw top, front, and side views for both 3-dimensional rod/block objects and their sketches
 – *continued*

SUGGESTIONS FOR INSTRUCTION

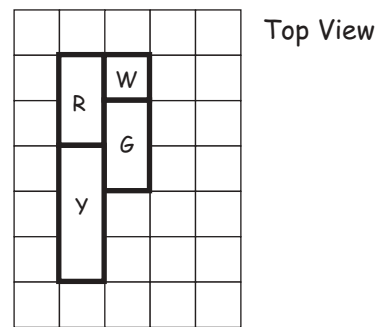
Example 1

Seeing 3-dimensional rod designs in two dimensions

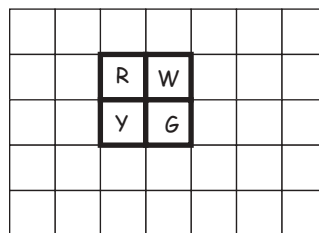
Build the 3-dimensional rod design shown using one yellow, one light green, one red, and one white rod.



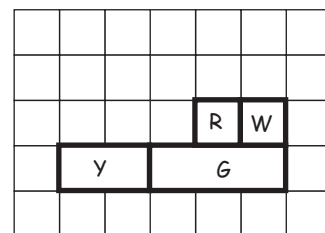
Pictured below are the 2-dimensional top, front, and side views of the rod or cube design. Observe carefully how they are obtained.



Front View



Side View



Communications	Patterns
✓ Connections	Problem Solving
Number Sense	✓ Reasoning
Organization and Structure	Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

**PRESCRIBED LEARNING
OUTCOMES**

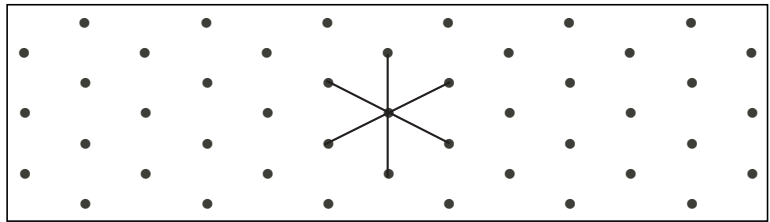
F-4 sketch 3-dimensional rod/ block objects using isometric dot paper

SUGGESTIONS FOR INSTRUCTION

F-4-1 Isometric dot paper is used to draw 3-dimensional figures. Note how the dots are arranged. This pattern of arranging dots on isometric dot paper allows one to draw 3-dimensional figures more easily.

Choose any dot on a piece of isometric dot paper. Draw a line segment from it to the next nearest dot. Note that one can draw the line segment either

- up
- down
- slanted up and to the right
- slanted up and to the left
- slanted down and to the right
- slanted down and to the left

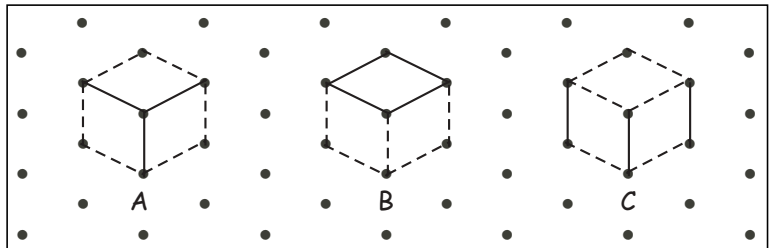


The line segments you have drawn are all the same length. The word “isometric” means equal measure. The prefix “iso” means “equal” and “metric” means measure.

F-4-2 Introductory Investigation

Draw the white Cuisenaire rod using isometric dot paper. (**Note:** Ensure that the orientation of the isometric dot paper is correct. See samples below.) Some students may find it helpful to position the rod so that they are looking directly at one of its vertical edges. They can then represent the rod on paper as they see it.

Some students find it helpful to draw a Y and then complete the figure as in Diagram A. Some draw a diamond and then draw the lines down as shown in Diagram B. Others find it easier to draw the three vertical lines first and then complete the figure as shown in Diagram C.



(continued)

Communications	Patterns
✓ Connections	Problem Solving
Number Sense	✓ Reasoning
Organization and Structure	Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

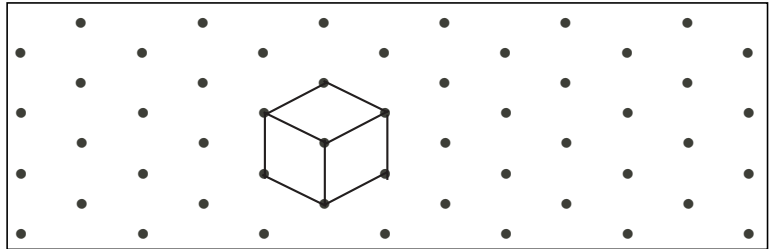
**SUGGESTED LEARNING
RESOURCES**

**PRESCRIBED LEARNING
OUTCOMES**

F-4 sketch 3-dimensional rod/ block objects using isometric dot paper
– *continued*

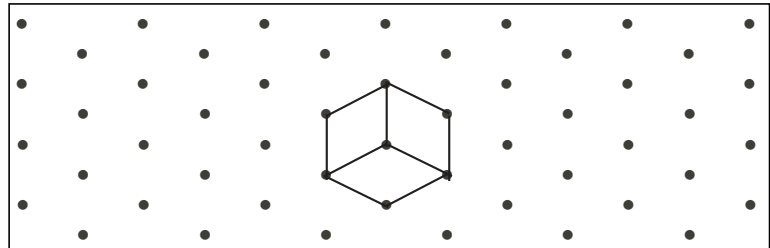
SUGGESTIONS FOR INSTRUCTION

Your figure should look like the following.

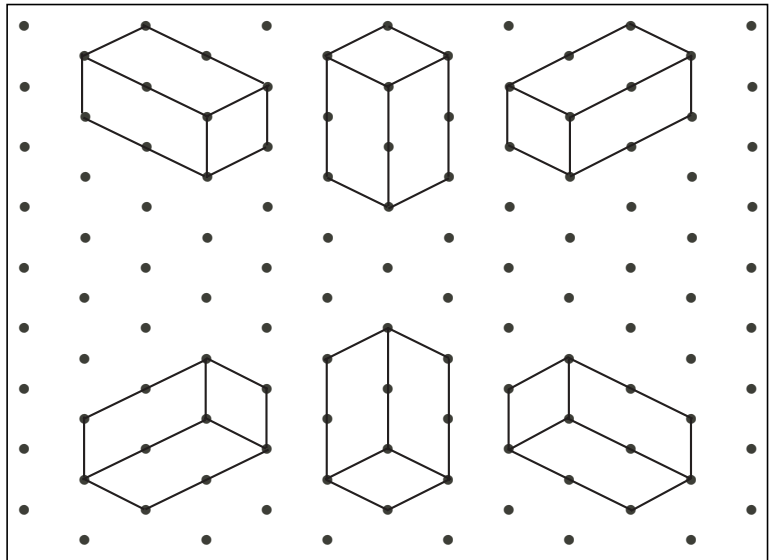


Choose the way that works best for you.

Try to make your figure look like this.



Now take the red rod and draw it. Although as a solid figure it has only one shape, you can represent it on isometric dot paper in six ways. Can you draw all six representations?



On isometric dot paper you are able to draw vertical line segments but unable to draw horizontal line segments. Instead of horizontal lines, you draw lines that are at an angle of 30° to the horizontal.

(continued)

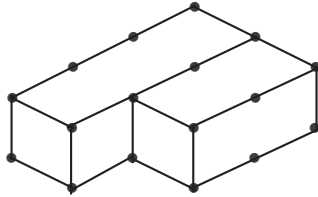
Communications	Patterns
✓ Connections	Problem Solving
Number Sense	✓ Reasoning
Organization and Structure	Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

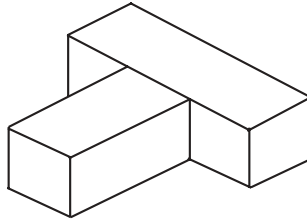
Assignment

Copy the following six figures onto isometric paper.

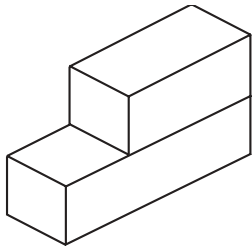
a)



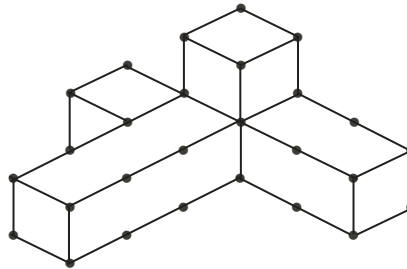
b)



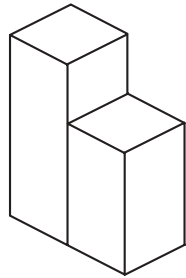
c)



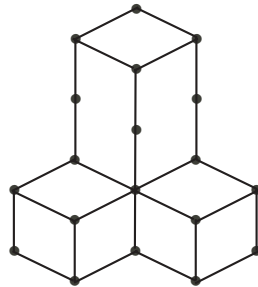
d)



e)



f)



SUGGESTED LEARNING RESOURCES

Print

Senior 2 Consumer Mathematics (25S) Part I: A Course for Distance Learning. Winnipeg, MB: Manitoba Education and Training, 2000.
— Module 5, Lesson 4

A copy of isometric dot paper is found at the end of this unit on page I-F-28.

**PRESCRIBED LEARNING
OUTCOMES**

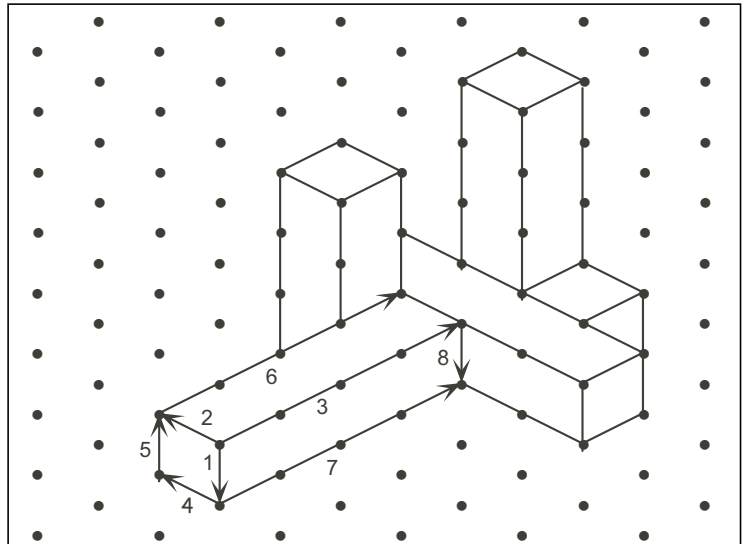
F-4 sketch 3-dimensional rod/ block objects using isometric dot paper
– *continued*

SUGGESTIONS FOR INSTRUCTION

Suggestions for Drawing 3-D Isometric Diagrams

Note: Be sure to have your paper in portrait position if you are using the master found on I-F-28. This keeps vertical lines vertical (see diagram below).

1. View the model from either the left or right side. Each person in the group can have his/her own view.
2. Begin by drawing the vertical edge (#1) of the rod that is closest to the viewer.
3. Now draw the edges along the top of this rod (#2 and #3). It may be helpful to look for a letter “Y” to begin each drawing.
4. Add the lines that are needed to give this rod some depth (#4 – #8). (One, two, . . . , five units, depending on the size of this rod.)
5. Continue by drawing the rods that touch this rod and complete the drawing until all rods have been drawn.



Teachers can have students practise various rod designs on isometric dot paper.

Communications	Patterns
✓ Connections	Problem Solving
Number Sense	✓ Reasoning
Organization and Structure	Technology
	✓ Visualization

(continued)

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

**PRESCRIBED LEARNING
OUTCOMES**

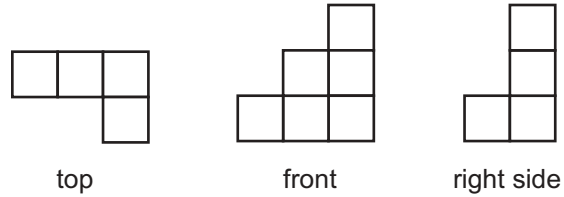
F-4 sketch 3-dimensional
rod/ block objects using
isometric dot paper
– *continued*

SUGGESTIONS FOR INSTRUCTION

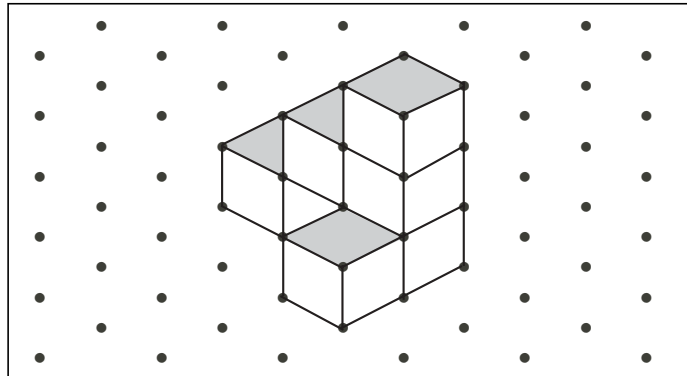
Example 2

Construct an object using cubes that has the views given below. Then, sketch the object in 3-D using isometric paper.

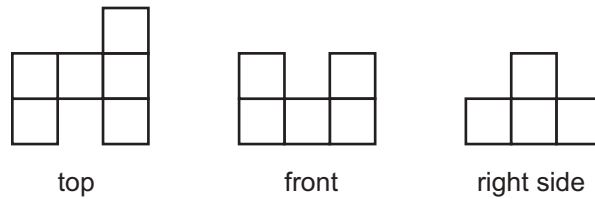
a)



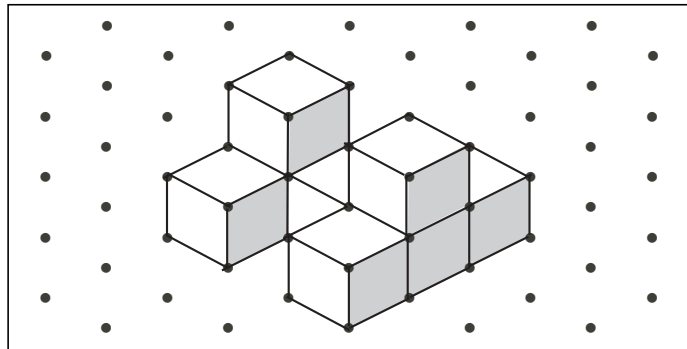
Solution



b)



Solution



Communications	Patterns
✓ Connections	Problem Solving
Number Sense	✓ Reasoning
Organization and Structure	Technology
	✓ Visualization

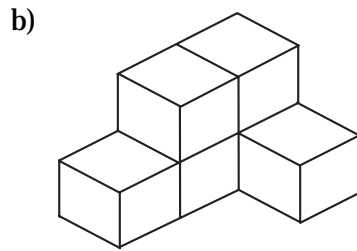
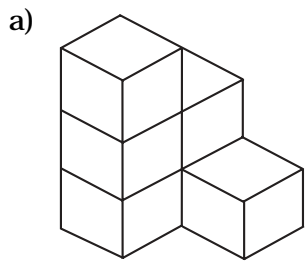
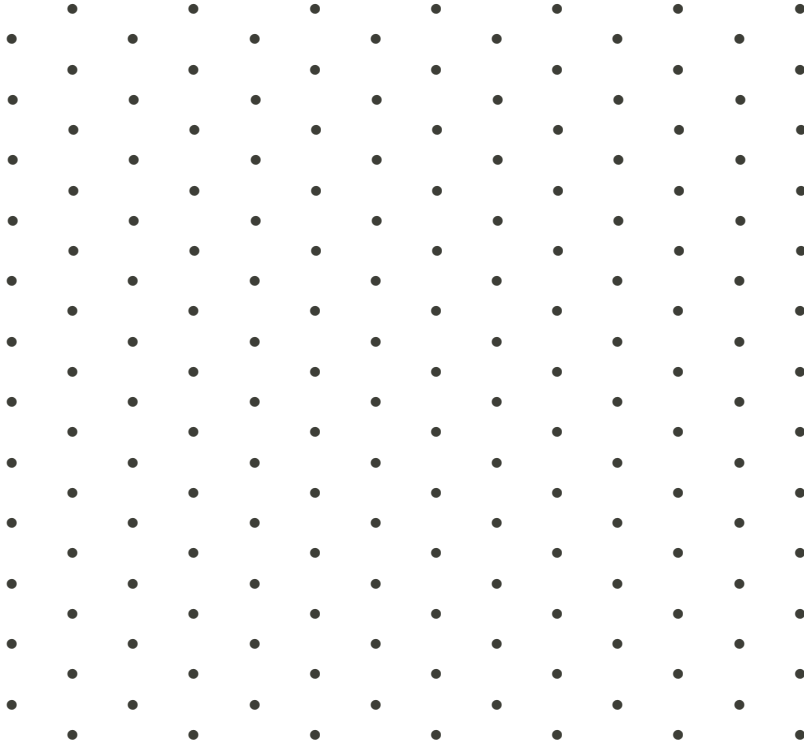
(continued)

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Assignment

Sketch each figure onto isometric paper.



**PRESCRIBED LEARNING
OUTCOMES**

F-4 sketch 3-dimensional rod/ block objects using isometric dot paper
– *continued*

SUGGESTIONS FOR INSTRUCTION

For this activity, teachers could provide students with the following set of instructions.

Extensions

- A. Given the three views:
1. Draw reflection or rotation images.
 2. Draw the isometric view of the transformed object.
 3. Construct the 3-D object.
- B. Given a model or isometric sketch of a 3-D object, draw the views of a rotated or reflected object.
1. Reflect the lower-level floor plan (top view) through a vertical line. Draw the new front and side views, and the new isometric sketch.
 2. Reflect the front view through a vertical line and then complete drawings of the new top and side views, and the new isometric sketch.
 3. Reflect the top view through a left diagonal and complete the other three sketches.
 4. Rotate the top view 90° counterclockwise, and draw the new views and isometric sketch.

Extension or Project Possibility

- C. Bonus Activity
1. Build a “complex” model using all the rods that you are given.
 2. Draw and colour the three views.
 3. Draw and colour a 3-D isometric picture of this design.
 4. On a separate sheet of paper, draw an “uncoloured” set of views.
 5. Give this set of uncoloured views to a classmate and have him/her try to duplicate your design.
 6. Check the accuracy of your classmate’s construction using your coloured views and 3-D picture.
 7. Peer and self-evaluation may be used as assessment.
 8. This work sample could be included in your portfolio.

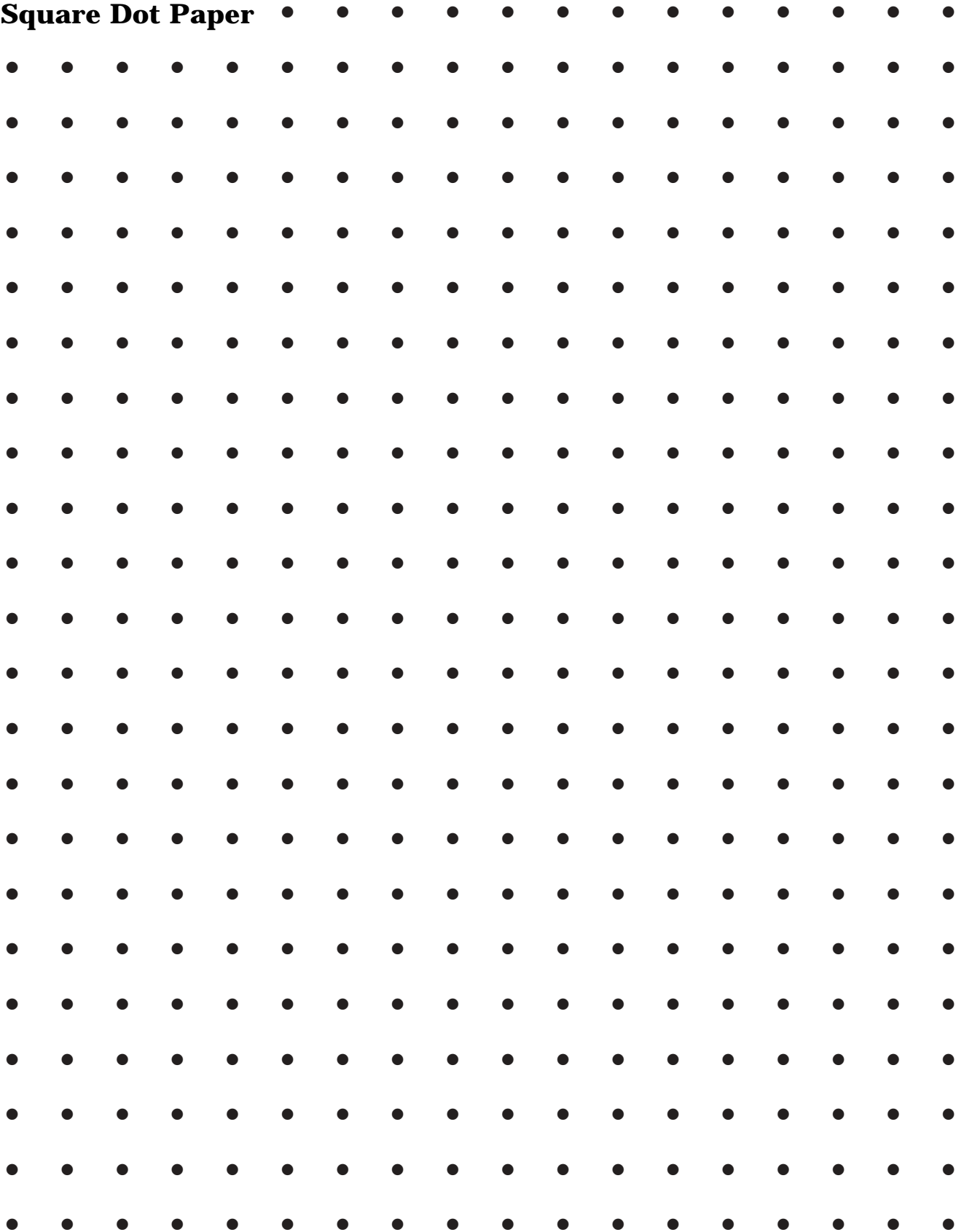
Communications	Patterns
✓ Connections	Problem Solving
Number Sense	✓ Reasoning
Organization and Structure	Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Appendix

Square Dot Paper



Isometric Dot Paper

