

Unit E: Trigonometry

Half Course I

HALF COURSE I

Unit E: Trigonometry

Hours: 5

General Learning Outcome:

Demonstrate an understanding of ratio and proportion and apply these concepts in solving triangles.

This unit provides students with the necessary skills to solve right triangle problems. Students are given the opportunity to use similarity and proportion as well as the basic trigonometric ratios to solve problems.

Specific Outcomes

E-1 Apply ratio and proportion in similar triangles.

E-2 Use the trigonometric ratios sine, cosine, and tangent in solving right triangles.

Extension: Solve problems involving two right triangles.

TRIGONOMETRY

Instructional Materials

- *Essentials of Mathematics 10*
- protractor
- ruler
- scientific calculator

Connections with Problem Analysis and Analysis of Games and Numbers

Any of the Problem Analysis and Analysis of Games and Numbers activities may be interspersed with problems from the Trigonometry unit.

PRESCRIBED LEARNING OUTCOMES

General Outcome

Demonstrate an understanding of ratio and proportion and apply these concepts in solving triangles.

Specific Outcome(s)

E-1 apply ratio and proportion in similar triangles

SUGGESTIONS FOR INSTRUCTION

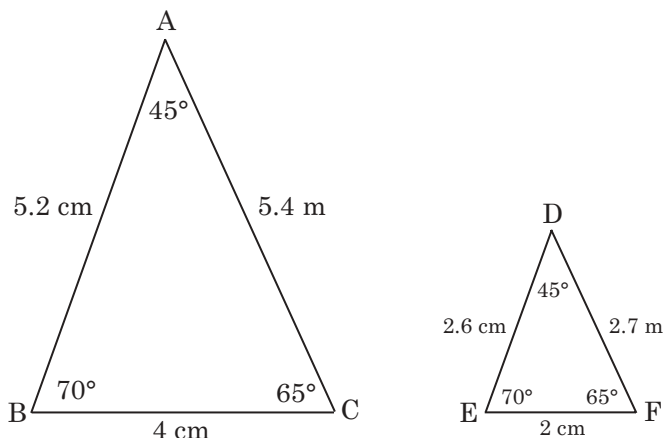
In *Senior 1 Mathematics* (10F), students worked with similar triangles and trigonometric ratios.

E-1.1 Introduce students to the concept of similar triangles.

Note: Similar triangles have the same shape. They will have the same shape if their corresponding angles are equal.

Example 1

ΔABC is similar to ΔDEF (can be written as $\Delta ABC \sim \Delta DEF$).

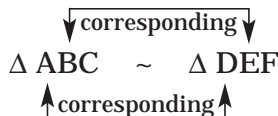


$\angle A = \angle D, \therefore \angle A$ and $\angle D$ are corresponding angles
 $\angle B = \angle E, \therefore \angle B$ and $\angle E$ are corresponding angles
 $\angle C = \angle F, \therefore \angle C$ and $\angle F$ are corresponding angles

In this example, the corresponding angles in ΔABC and ΔDEF are equal, therefore, $\Delta ABC \sim \Delta DEF$.

Have students use a ruler and a protractor to draw several more triangles similar to ΔABC .

Note: When using the notation $\Delta ABC \sim \Delta DEF$, be sure to write the pairs of corresponding angles in the same position.



(continued)

Communications	Patterns
✓ Connections	✓ Problem Solving
Number Sense	✓ Reasoning
✓ Organization and Structure	✓ Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

Assess performance on activities and problems while they are in progress.

While standard pencil-and-paper tests and quizzes can be used in this unit, performance tasks and the posing of problems provide alternatives.

Project work such as indoor and outdoor activities should be included in the assessment of this unit.

For example, “Use similar triangles or trigonometry to measure the height of a flagpole or other tall structure.”

Journal Entry

Two triangles are similar.

- Explain what the above statement means.
- Draw and label a diagram of two similar triangles.
- What is true about the measures of the corresponding angles of similar triangles?
- What is true about the measure of the corresponding sides of the triangles?

Mental Math

Given $\triangle MNO \sim \triangle XYZ$:

- Identify the pairs of equal corresponding angles.
- Write the ratios of the corresponding sides.

Solutions

a) $\angle M = \angle X, \angle N = \angle Y, \angle O = \angle Z$

b) $\frac{m}{x} = \frac{n}{y} = \frac{o}{z}$

or

$$\frac{MN}{XY} = \frac{NO}{YZ} = \frac{MO}{XZ}$$

SUGGESTED LEARNING RESOURCES
Print

Senior 2 Consumer Mathematics (25S) Part I: A Course for Distance Learning. Winnipeg, MB: Manitoba Education and Training, 2000.
— Module 3, Lessons 1,2

Baron, Celia, Rick Wunderlich, and Leanne Zorn. *Essentials of Mathematics 10.* Vancouver, BC: British Columbia Ministry of Education, 2002.
Chapter 5
ISBN 0-7726-4675-9

Pilmer, David. *The Math Survival Kit.* Lake Fletcher, NS: Pilmer Publishing, 2001.
ISBN 0-9730067-0-6

**PRESCRIBED LEARNING
OUTCOMES**

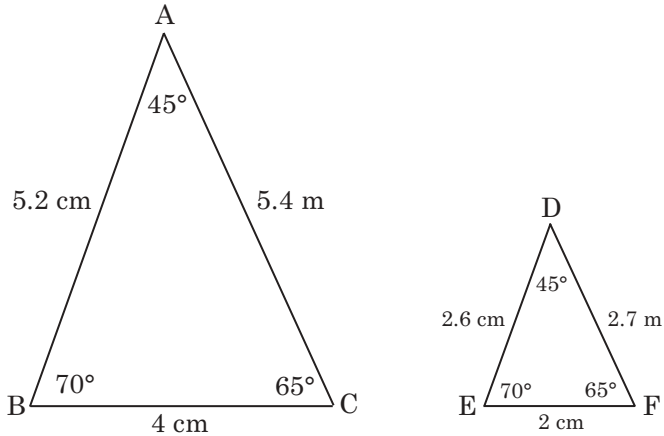
E-1 apply ratio and proportion in similar triangles
– continued

SUGGESTIONS FOR INSTRUCTION

Note: The ratios of the corresponding sides of similar triangles are equivalent.

Example 2

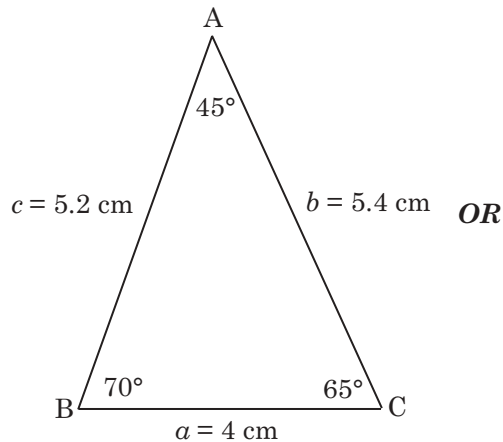
$\Delta ABC \sim \Delta DEF$



Note: The sides of a triangle can be labelled in one of two ways:

- a) using lower case letters, e.g.,
 - opposite $\angle A$ is side 'a'
 - opposite $\angle B$ is side 'b'
 - opposite $\angle C$ is side 'c'
- b) using the vertices of the triangle, e.g.,
 - opposite $\angle A$ is side BC
 - opposite $\angle B$ is side AC
 - opposite $\angle C$ is side AB

Therefore, in ΔABC ,



(continued)

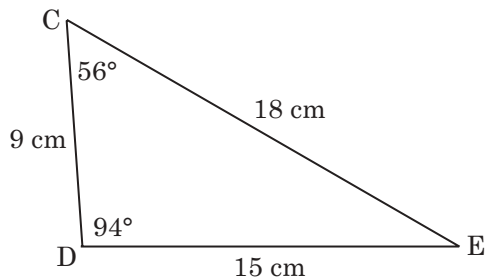
Communications	Patterns
✓ Connections	✓ Problem Solving
Number Sense	✓ Reasoning
✓ Organization and Structure	✓ Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Mental Math

Given $\triangle CDE$,



State the measure of each of the following:

- | | | |
|--------------------|--------------------|--------------------|
| $d =$ _____ | $e =$ _____ | $c =$ _____ |
| $\angle C =$ _____ | $\angle D =$ _____ | $\angle E =$ _____ |
| $CE =$ _____ | $CD =$ _____ | $DE =$ _____ |

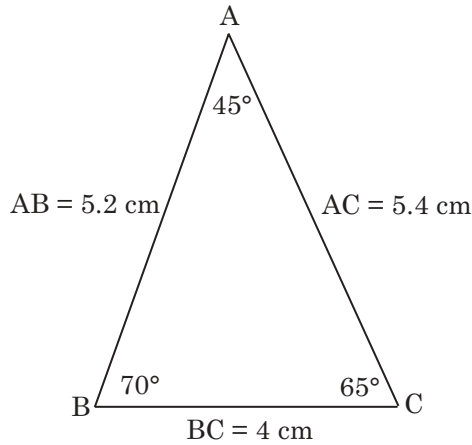
Solution

- | | | |
|-----------------------|-----------------------|-----------------------|
| $d = 18$ m | $e = 9$ cm | $c = 15$ cm |
| $\angle C = 56^\circ$ | $\angle D = 94^\circ$ | $\angle E = 30^\circ$ |
| $CE = 18$ cm | $CD = 9$ cm | $DE = 15$ cm |

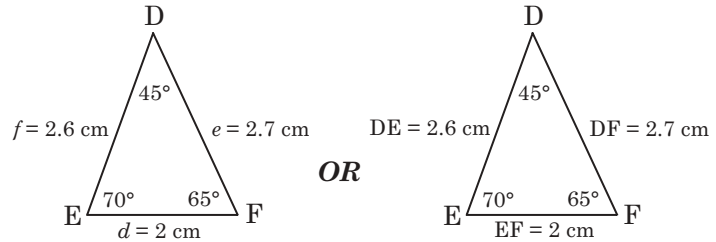
**PRESCRIBED LEARNING
OUTCOMES**

E-1 apply ratio and proportion in similar triangles
– *continued*

SUGGESTIONS FOR INSTRUCTION



Therefore, $\triangle DEF$,



Since $\triangle ABC \sim \triangle DEF$, the ratios of the corresponding sides of the two triangles will be equivalent. Therefore,

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f} \text{ or } \frac{BC}{EF} = \frac{AC}{DF} = \frac{AB}{DE}$$

Now substitute the known values:

$$\frac{4}{2} = \frac{5.4}{2.7} = \frac{5.2}{2.6}$$

$$2 = 2 = 2$$

Note: This ratio represents the scalar between the two triangles. That is, $\triangle ABC$ is twice the size of $\triangle DEF$.

Note: A ratio of sides within the same triangle can also be compared to the corresponding ratio in the other triangles. For example,

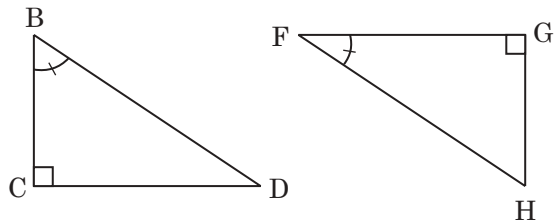
$$\frac{a}{b} = \frac{d}{e} \text{ or } \frac{f}{e} = \frac{c}{b}$$

Have students determine the equivalent ratios for the similar triangles that they drew previously.

Communications	Patterns
✓ Connections	✓ Problem Solving
Number Sense	✓ Reasoning
✓ Organization and Structure	✓ Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

 SUGGESTED LEARNING
RESOURCES

Mental Math
 $\triangle BCD \sim \triangle FGH$


- State the pairs of corresponding angles.
- Write the ratios of corresponding sides.

Solution

a) $\angle B = \angle F$, $\angle C = \angle G$, $\angle D = \angle H$

b) $\frac{b}{f} = \frac{c}{g} = \frac{d}{h}$

or

$$\frac{CD}{GH} = \frac{BD}{FH} = \frac{BC}{FG}$$

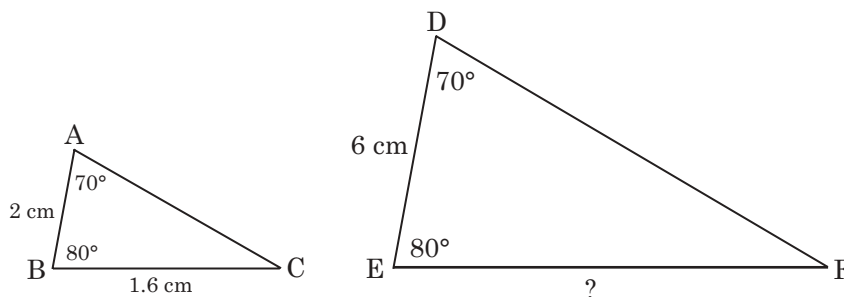
**PRESCRIBED LEARNING
OUTCOMES**

E-1 apply ratio and proportion in similar triangles
– *continued*

SUGGESTIONS FOR INSTRUCTION

Example 3

The two triangles are similar. Determine the length of side d .



Solution

Since $\triangle ABC \sim \triangle DEF$

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$

$$\frac{1.6}{d} = \frac{b}{e} = \frac{2}{6}$$

$$\frac{1.6}{d} = \frac{2}{6}$$

$$\frac{1.6 \times 6}{2} = \frac{2 \times d}{2}$$

$$4.8 = d$$

\therefore side d is 4.8 cm

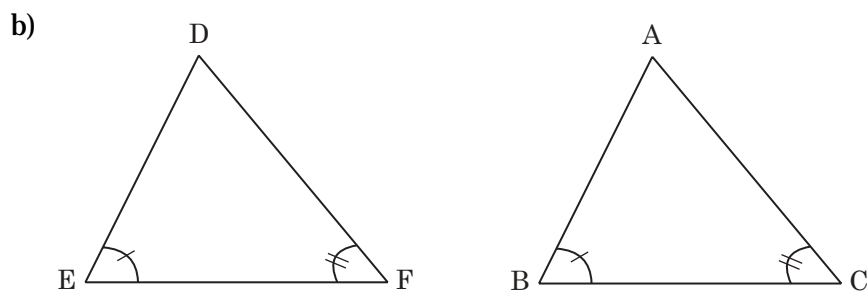
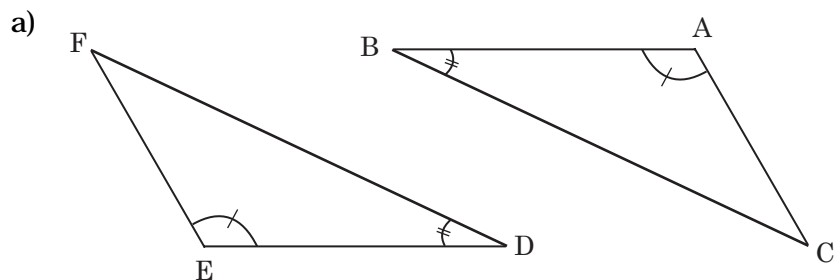
Communications	Patterns
✓ Connections	✓ Problem Solving
Number Sense	✓ Reasoning
✓ Organization and Structure	✓ Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Problem

In each diagram the triangles are similar. For each pair of triangles, write the ratio of sides that is equal to $\frac{FD}{DE}$.



Solution

a) $\frac{FD}{DE} = \frac{BC}{BA}$

b) $\frac{FD}{DE} = \frac{AC}{AB}$

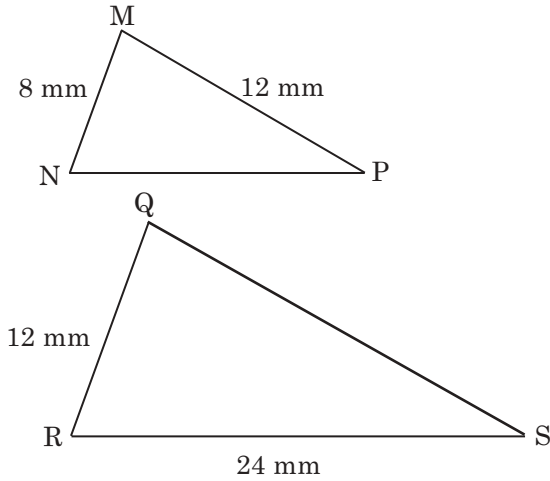
**PRESCRIBED LEARNING
OUTCOMES**

E-1 apply ratio and proportion in similar triangles
– *continued*

SUGGESTIONS FOR INSTRUCTION

Example 4

Since $\triangle MNP \sim \triangle QRS$, determine the lengths of the missing sides.



Solution

$$\frac{m}{q} = \frac{n}{r} = \frac{p}{s}$$

$$\frac{m}{24} = \frac{12}{r} = \frac{8}{12}$$

$$\frac{m}{24} = \frac{8}{12}$$

$$m = \frac{8 \times 24}{12}$$

$$m = 16$$

$$\frac{12}{r} = \frac{8}{12}$$

$$\frac{12 \times 12}{8} = \frac{8 \times r}{8}$$

$$18 = r$$

\therefore side m is 16 mm

\therefore side r is 18 mm

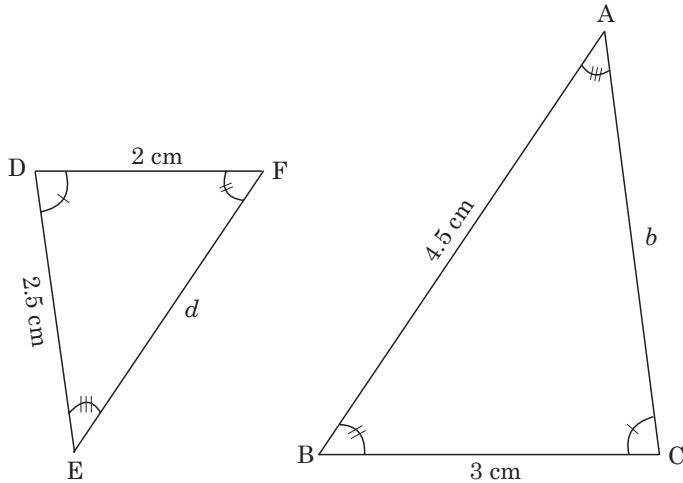
Communications	Patterns
✓ Connections	✓ Problem Solving
Number Sense	✓ Reasoning
✓ Organization and Structure	✓ Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Problem

Find the missing sides of the following triangles using equivalent ratios.



Solution

$$\triangle DFE \sim \triangle CBA$$

$$\frac{d}{c} = \frac{f}{b} = \frac{e}{a}$$

$$\frac{d}{4.5} = \frac{2.5}{b} = \frac{2}{3}$$

$$\frac{d}{4.5} = \frac{2}{3}$$

$$d = \frac{2 \times 4.5}{3}$$

$$d = 3 \text{ cm}$$

$$\frac{2.5}{b} = \frac{2}{3}$$

$$\frac{2.5 \times 3}{2} = b$$

$$3.75 \text{ cm} = b$$

**PRESCRIBED LEARNING
OUTCOMES**

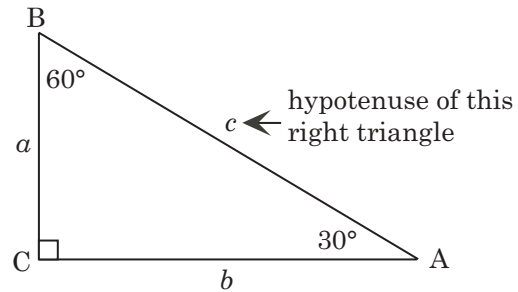
E-2 use the trigonometric ratios sine, cosine, and tangent in solving right triangles

SUGGESTIONS FOR INSTRUCTION

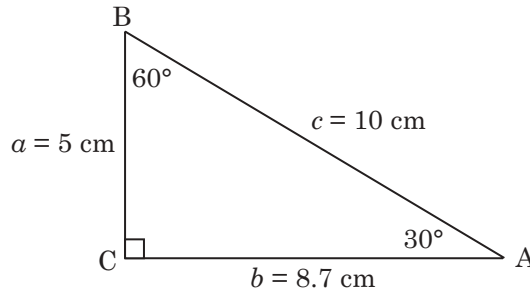
E-2-1 Complete an activity that introduces the basic trigonometric ratios.

Example

1. Instruct students to draw and label a 30-60-90 right triangle of any size as shown below.



2. Have students measure each side of their triangles to the nearest tenth of a centimetre and place their measurements on their diagrams.



3. Have students determine the following ratios for the sides of their triangles.

$$\frac{a}{c}, \frac{b}{c}, \frac{a}{b}$$

Solution

$$\frac{a}{c} = \frac{5}{10} = 0.5$$

$$\frac{b}{c} = \frac{8.7}{10} = 0.87$$

$$\frac{a}{b} = \frac{5}{8.7} = 0.57$$

Note: Regardless of the size of the 30-60-90 right triangle drawn, all students should get ratio values roughly equivalent to these.

(continued)

Communications	Patterns
✓ Connections	✓ Problem Solving
Number Sense	✓ Reasoning
✓ Organization and Structure	✓ Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT**SUGGESTED LEARNING
RESOURCES****Problem**

In a right triangle, the leg adjacent to the angle of 25° is 12 cm long.

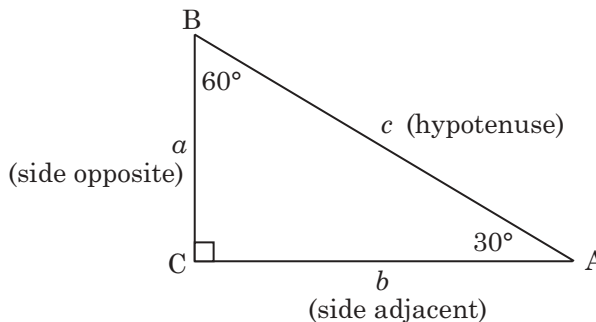
- a) Draw a diagram of the triangle indicating the given angle and side.
- b) What are the measures of the other angles?
- c) How long is the side opposite the 25° angle to the nearest tenth of a centimetre?
- d) How long is the hypotenuse of the triangle?

**PRESCRIBED LEARNING
OUTCOMES**

E-2 use the trigonometric ratios sine, cosine, and tangent in solving right triangles
– *continued*

SUGGESTIONS FOR INSTRUCTION

4. Have students label the sides of their triangles using $\angle A$ as the reference angle and using the words side opposite, side adjacent, and hypotenuse.



5. Have students rewrite the earlier determined ratios using these words and relate to sine, cosine, and tangent.

Ratio 1: $\frac{a}{c} = \frac{\text{side opposite } \angle A}{\text{hypotenuse}}$, called sine of $\angle A$

Ratio 2: $\frac{b}{c} = \frac{\text{side adjacent } \angle A}{\text{hypotenuse}}$, called cosine of $\angle A$

Ratio 3: $\frac{a}{b} = \frac{\text{side opposite } \angle A}{\text{side adjacent } \angle A}$, called tangent of $\angle A$

6. Have students write these ratios using $\angle B$ (60°) as the reference angle.

Solution

$$\sin \angle B = \frac{b}{c}, \quad \cos \angle B = \frac{a}{c}, \quad \tan \angle B = \frac{b}{a}$$

7. Relate these ratios to the calculator-determined values.

Using sides: $\sin \angle A = \frac{a}{c}, = \frac{5}{10} = 0.5$

Using calculator: $\sin 30^\circ = 0.5$

Note: It may be necessary to review how to use the trig keys on a scientific calculator.

Have students use their calculators to determine $\cos 30^\circ$ and $\tan 30^\circ$ and compare the results to the previously written ratios.

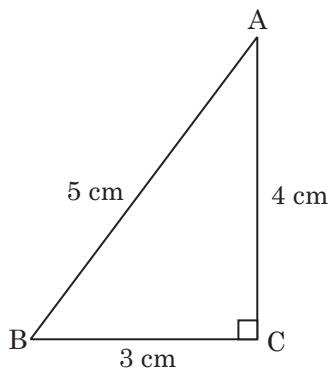
Communications	Patterns
✓ Connections	✓ Problem Solving
Number Sense	✓ Reasoning
✓ Organization and Structure	✓ Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Problem

Given $\triangle ABC$ is a right triangle.



Determine each of the following ratios:

- | | |
|----------------------|----------------------|
| a) $\sin \angle B =$ | b) $\cos \angle B =$ |
| c) $\tan \angle B =$ | d) $\sin \angle A =$ |
| e) $\cos \angle A =$ | f) $\tan \angle A =$ |

Solution

- | | |
|------------------|------------------|
| a) $\frac{4}{5}$ | b) $\frac{3}{5}$ |
| c) $\frac{4}{3}$ | d) $\frac{3}{5}$ |
| e) $\frac{4}{5}$ | f) $\frac{3}{4}$ |

**PRESCRIBED LEARNING
OUTCOMES**

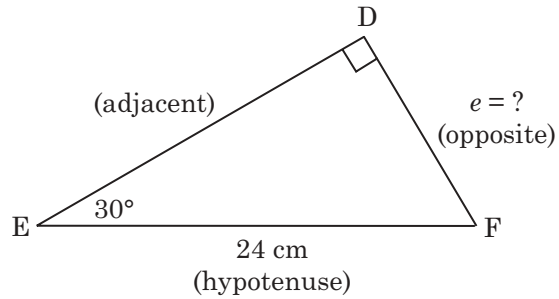
E-2 use the trigonometric ratios sine, cosine, and tangent in solving right triangles
– *continued*

SUGGESTIONS FOR INSTRUCTION

E-2-2 Solve right triangle questions using trigonometric ratios.

Example 1

Find the missing side in $\triangle DEF$.



Solution

$$\sin 30^\circ = \frac{e}{24}$$

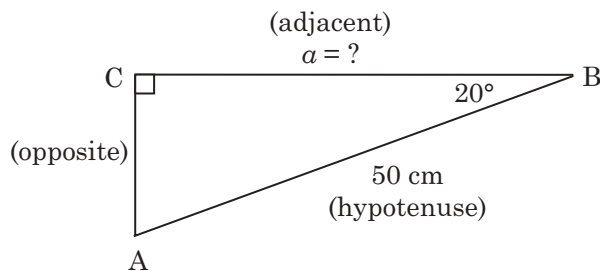
$$\sin 30^\circ \times 24 = e$$

$$12 \text{ cm} = e$$

Note: Have students label the sides of the triangles using the given reference angle and the words opposite, adjacent, and hypotenuse. Choose the trig ratio that contains the given side and the unknown side (sine).

Example 2

Find the missing side in $\triangle ABC$.



Solution

$$\cos 20^\circ = \frac{a}{50}$$

$$\cos 20^\circ \times 50 = a$$

$$46.98 \text{ cm} = a$$

Communications	Patterns
✓ Connections	✓ Problem Solving
Number Sense	✓ Reasoning
✓ Organization and Structure	✓ Technology
	✓ Visualization

(continued)

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Problem

Use your calculator to determine the value of the following trig ratios rounded to four decimal places.

- | | |
|--------------------|--------------------|
| a) $\sin 45^\circ$ | b) $\cos 60^\circ$ |
| c) $\tan 70^\circ$ | d) $\sin 60^\circ$ |
| e) $\cos 45^\circ$ | f) $\tan 45^\circ$ |

Solution

- | | |
|-----------|-----------|
| a) 0.7071 | b) 0.5000 |
| c) 2.7475 | d) 0.8660 |
| e) 0.7071 | f) 1.0000 |

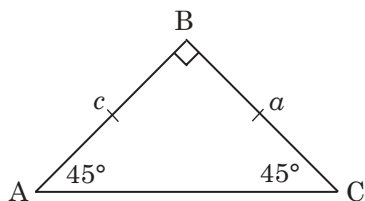
Enrichment Question

Why is the $\tan 45^\circ = 1$?

Solution

The triangle must have two angles equal to 45° . Therefore, it is an isosceles triangle and has two sides equal. The $\tan 45^\circ$ is the ratio of these two equal sides. Therefore, the ratio will equal 1.

Example:



side $a =$ side c

$$\tan A = \frac{a}{c} = 1 \text{ or } \tan C = \frac{c}{a} = 1$$

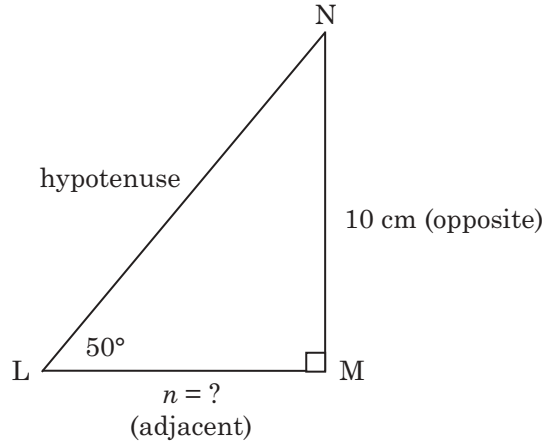
**PRESCRIBED LEARNING
OUTCOMES**

E-2 use the trigonometric ratios sine, cosine, and tangent in solving right triangles
– *continued*

SUGGESTIONS FOR INSTRUCTION

Example 3

Find the missing side in $\triangle LMN$.



Solution

$$\tan 50^\circ = \frac{10}{n}$$

$$\tan 50^\circ \times n = 10 \quad \text{cross product}$$

$$\frac{\tan 50^\circ \times n}{\tan 50^\circ} = \frac{10}{\tan 50^\circ} \quad \text{divide both sides by } \tan 50^\circ$$

$$n = 8.39 \text{ cm} \quad \text{calculator steps}$$

$$\boxed{10} \boxed{\div} \boxed{\text{TAN}} \boxed{50} \boxed{=}$$

or

$$\boxed{10} \boxed{\div} \boxed{50} \boxed{\text{TAN}} \boxed{=}$$

Note: Encourage students to do this simplification before determining the value of $\tan 50^\circ$.

Communications	Patterns
✓ Connections	✓ Problem Solving
Number Sense	✓ Reasoning
✓ Organization and Structure	✓ Technology
	✓ Visualization

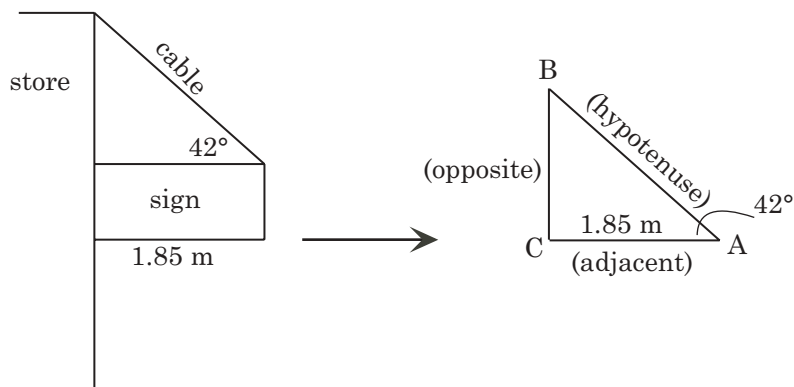
SUGGESTIONS FOR ASSESSMENT

SUGGESTED LEARNING RESOURCES

Problem

A sign at a hardware store hangs out 1.85 m from the storefront and is supported by a cable. The angle between the sign and the cable is 42° . What length must the cable be?

Solution



$$\cos 42^\circ = \frac{1.85}{c}$$

$$\cos 42^\circ \times c = 1.85$$

$$c = \frac{1.85}{\cos 42^\circ}$$

$$c = 2.49$$

Therefore, the cable must be 2.49 m in length.

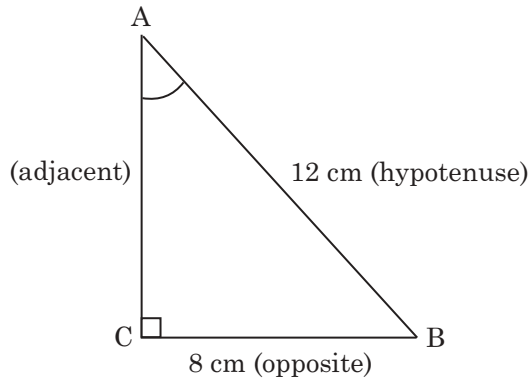
**PRESCRIBED LEARNING
OUTCOMES**

E-2 use the trigonometric ratios sine, cosine, and tangent in solving right triangles
– *continued*

SUGGESTIONS FOR INSTRUCTION

Example 4

Determine the measure of the missing angles in $\triangle ABC$ rounded to the nearest degree.



Solution

Using $\angle A$ as the reference angle:

$$\sin \angle A = \frac{8}{12}$$

$$\sin \angle A = 0.666 \dots$$

$$\angle A = 41.8^\circ \text{ or } 42^\circ$$

Calculator steps:

$$\boxed{8} \boxed{\div} \boxed{12} \boxed{=} \boxed{\text{INV}} \boxed{\text{SIN}}$$

or

$$\boxed{2\text{ND}} \text{ or } \boxed{\text{SHIFT}}$$

$$\angle B = 180 - (90 + 42)$$

$$\angle B = 48^\circ$$

Note: Have students determine the size of $\angle B$ using a trig ratio. For example:

$$\cos \angle B = \frac{8}{12}$$

$$\cos^{-1}\left(\frac{8}{12}\right) = 48.2^\circ$$

Communications	Patterns
✓ Connections	✓ Problem Solving
Number Sense	✓ Reasoning
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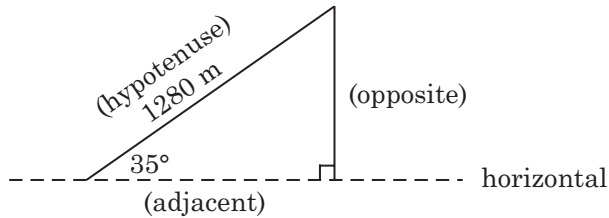
SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Problem

Skiers ski down a slope that is inclined at a 35° angle to the horizontal. If the skiers reach ground level after travelling a distance of 1280 m, how high is the slope where they began their run?

Solution



$$\sin 35^\circ = \frac{\text{opposite}}{1280}$$

$$\sin 35^\circ \times 1280 = \text{opposite}$$

$$734.18 = \text{opposite}$$

Therefore, the slope was 734.18 m high.

**PRESCRIBED LEARNING
OUTCOMES**

E-2 use the trigonometric ratios sine, cosine, and tangent in solving right triangles
– *continued*

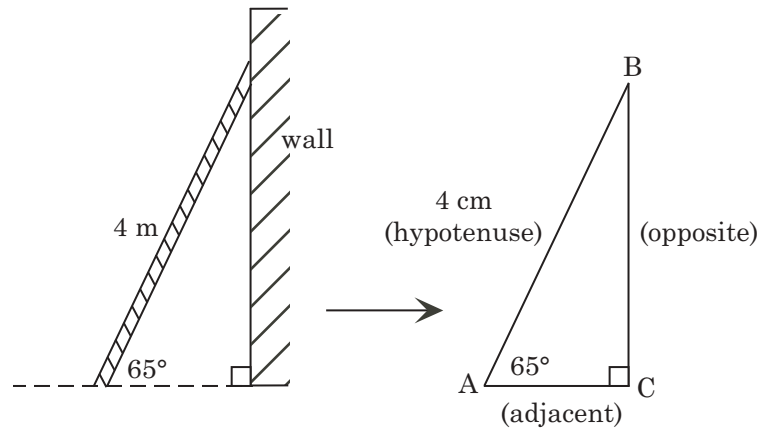
SUGGESTIONS FOR INSTRUCTION

E-2-3 Have students solve various types of trigonometry problems involving right triangles.

Example 1

A 4 m ladder leaning against a house makes a 65° angle with the ground. How high up the wall of the house does the ladder reach?

Solution



$$\sin 65^\circ = \frac{\text{opposite}}{4}$$

$$\sin 65^\circ = \text{opposite}$$

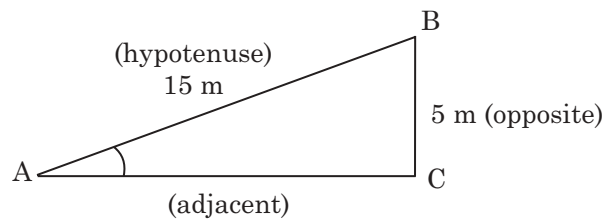
$$3.63 = \text{opposite}$$

Therefore, the ladder reaches 3.63 metres up the wall.

Example 2

A 15 m ramp is built to the loft door of the barn. The loft door is 5 m above ground level. Find the angle that the ramp makes with the ground (angle of elevation).

Solution



Communications	Patterns
✓ Connections	✓ Problem Solving
Number Sense	✓ Reasoning
✓ Organization and Structure	✓ Technology
	✓ Visualization

(continued)

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

**PRESCRIBED LEARNING
OUTCOMES**

E-2 use the trigonometric ratios sine, cosine, and tangent in solving right triangles
– *continued*

SUGGESTIONS FOR INSTRUCTION

$$\sin \angle A = \frac{5}{15}$$

$$\sin \angle A = 0.333 \dots$$

$$\angle A = 19.47^\circ$$

Therefore, the ramp makes an angle of 19.47° with the ground.

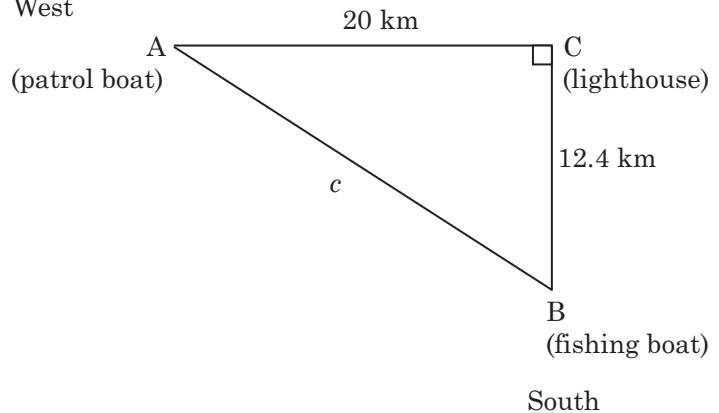
Example 3

A patrol boat is 20 km west of a lighthouse. A small fishing boat is disabled and is 12.4 km south of the lighthouse.

- At what angle south of due east must the patrol boat travel to reach the fishing boat (nearest tenth of a degree)?
- How far is the patrol boat from the fishing boat (to the nearest tenth of a kilometre)?

Solution

- a) West



$$\tan \angle A = \frac{12.4}{20}$$

$$\tan \angle A = 0.62$$

$$\angle A = 31.8^\circ$$

Therefore, the patrol boat should travel at 31.8° south of east in order to reach the fishing boat.

Communications	Patterns
✓ Connections	✓ Problem Solving
Number Sense	✓ Reasoning
✓ Organization and Structure	✓ Technology
	✓ Visualization

(continued)

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

**PRESCRIBED LEARNING
OUTCOMES**

E-2 use the trigonometric ratios sine, cosine, and tangent in solving right triangles
– *continued*

SUGGESTIONS FOR INSTRUCTION

$$\begin{aligned} \text{b) } \sin 31.8^\circ &= \frac{12.4}{c} \\ c &= \frac{12.4}{\sin 31.8^\circ} \\ c &= 23.5 \end{aligned}$$

Therefore, the patrol boat is 23.5 km away from the fishing boat.

Note: Other applications include safety specifications that state:

- The angle between the ramp and the horizontal of a wheelchair ramp should be no greater than 4.7° (or 1 m vertical rise to 12 m horizontal).
- The horizontal distance from base of ladder to building should not exceed one-third of the length of the ladder nor should it be less than one-quarter of the length of the ladder.

$$\text{H.D.} \leq \frac{1}{3} \text{ length of the ladder}$$

$$\text{H.D.} \geq \frac{1}{4} \text{ length of the ladder}$$

Communications	Patterns
✓ Connections	✓ Problem Solving
Number Sense	✓ Reasoning
✓ Organization and Structure	✓ Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

**PRESCRIBED LEARNING
OUTCOMES**

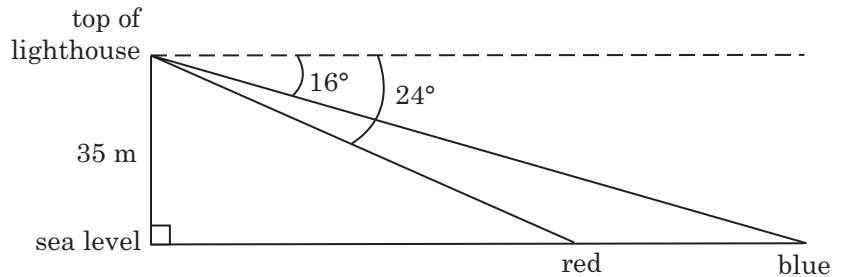
Extension:
Solve problems involving two right triangles.

SUGGESTIONS FOR INSTRUCTION

Introduce students to angles of depression and elevation.

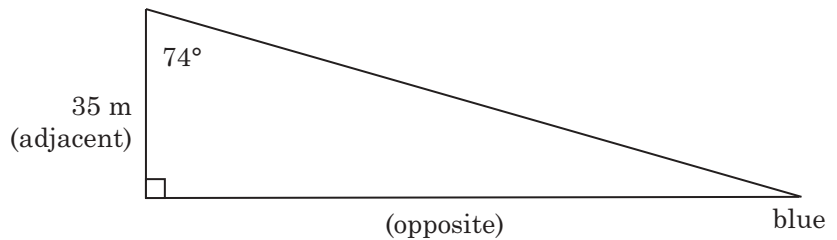
Example 1

A lighthouse sits at the top of a cliff and the top of the lighthouse is 35 m above sea level. The angle of depression to sight a red fishing boat, which is between the cliff and the blue boat, is 24° . The angle of depression to sight a blue sailboat is 16° . How far apart are the boats?



Solution

Triangle 1



$$\tan 74^\circ = \frac{\text{opposite}}{35}$$

$$\tan 74^\circ \times 35 = \text{opposite}$$

$$122.06 = \text{opposite}$$

\therefore The blue boat is 122.06 m from the base of the cliff.

- | | |
|-------------------------------------|--------------------------|
| Communications | Patterns |
| ✓ Connections | ✓ Problem Solving |
| Number Sense | ✓ Reasoning |
| ✓ Organization and Structure | ✓ Technology |
| | ✓ Visualization |

(continued)

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

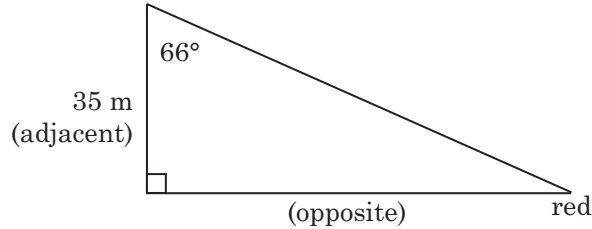
**PRESCRIBED LEARNING
OUTCOMES**

Extension:
Solve problems involving two
right triangles.
– *continued*

SUGGESTIONS FOR INSTRUCTION

Solution — continued

Triangle 2



$$\tan 66^\circ = \frac{\text{opposite}}{35}$$

$$\tan 66^\circ \times 35 = \text{opposite}$$

$$78.61 = \text{opposite}$$

$$122.06 - 78.61 = 43.45$$

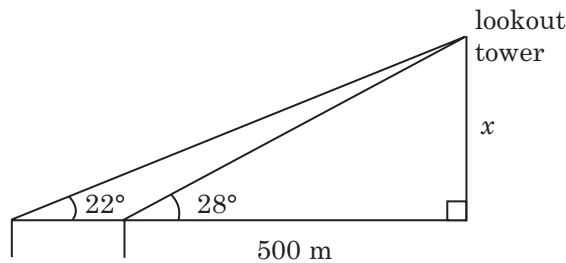
∴ The red boat is 78.61 m from the base of the cliff.

Therefore, the boats are 43.45 m apart.

Example 2

A backpacker finds that the angle of elevation to the top of a lookout tower is 22° . As she walks closer to the tower in a straight line, the angle of elevation changes to 28° . If, at this point in her journey, she is 500 m from the base of the tower, calculate how far away she was from the tower when she began.

Solution



- | | |
|-------------------------------------|--------------------------|
| Communications | Patterns |
| ✓ Connections | ✓ Problem Solving |
| Number Sense | ✓ Reasoning |
| ✓ Organization and Structure | ✓ Technology |
| | ✓ Visualization |

(continued)

SUGGESTIONS FOR ASSESSMENT

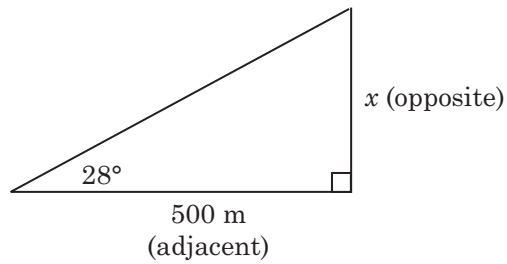
**SUGGESTED LEARNING
RESOURCES**

**PRESCRIBED LEARNING
OUTCOMES**

Extension:
Solve problems involving two
right triangles.
– *continued*

SUGGESTIONS FOR INSTRUCTION

Triangle 1

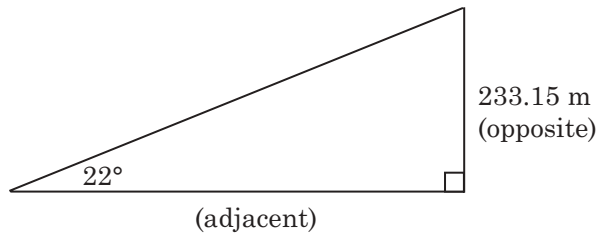


$$\tan 28^\circ = \frac{x}{500}$$

$$\tan 28^\circ \times 500 = x$$

Therefore, the tower is 233.15 m tall.

Triangle 2



$$\tan 22^\circ = \frac{233.15}{\text{adjacent}}$$

$$\tan 22^\circ \times \text{adjacent} = 233.15$$

$$\text{adjacent} = \frac{233.15}{\tan 22^\circ}$$

Therefore, the backpacker was 577.07 m away from the tower
when she began.

Communications	Patterns
✓ Connections	✓ Problem Solving
Number Sense	✓ Reasoning
✓ Organization and	✓ Technology
Structure	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**
