

Unit B: Analysis of Games and Numbers

Half Course II

HALF COURSE II

Unit B: Analysis of Games and Numbers

Hours: 5

General Learning Outcome:

Develop, use, and justify mathematical strategies by analyzing a variety of puzzles and games; develop an awareness of how numbers are used in society.

The material provided for this unit should be used throughout the course to provide a change of pace in a context which is enjoyable, and calls for mathematical and logical thinking.

Specific Outcomes

- B-1 Demonstrate the use of an appropriate strategy in solving puzzles and playing games involving patterns.
- B-2 Demonstrate how numbers are used descriptively throughout society.

ANALYSIS OF GAMES AND NUMBERS

Instructional Materials

- *Essentials of Mathematics 10*
- See Appendix I for possible activities.
- See Appendix II for additional resources.

**PRESCRIBED LEARNING
OUTCOMES**

General Outcome

Develop, use, and justify mathematical strategies by analyzing a variety of puzzles and games; develop an awareness of how numbers are used in society.

Specific Outcome(s)

B-1 demonstrate the use of an appropriate strategy in solving puzzles and playing games involving patterns

SUGGESTIONS FOR INSTRUCTION

Consideration should be given to interspersing Analysis of Games and Numbers throughout the course, i.e., you may wish to spend a few days on this early in the course for motivational reasons, and then use the remaining or other activities to provide a break between other units or in the middle of a long unit.

The objective of the following collection of activities is having students play the games and find winning strategies. Students are expected to find the strategies and be able to explain the strategies by oral or written demonstration.

Devote sufficient time to playing and enjoying a game before analysis begins. Allow students time to discuss the game and articulate their “winning” strategies.

Finding the strategy is the first step, although the subsequent steps are of equal importance. The actual results may be of less consequence, but a discussion of the results is important. Throughout the activity, students should be encouraged to examine their own thinking processes, something that is not easy and perhaps not a common experience for many. The examination should lead to communication and exploration of the strategies.

Teachers will find a wide variation among their students and in the problems that students find easy or difficult.

These exercises could be thought of as “problem solving.” It is not intended that they be taught in a block of time, but, rather, dealt with periodically during the year.

Communications	✓ Patterns
Connections	Problem Solving
✓ Number Sense	✓ Reasoning
✓ Organization and Structure	Technology
	✓ Visualization

SUGGESTIONS FOR ASSESSMENT

The willingness to accept the challenge presented in the learning experiences is important. Keep a daily record.

Keep anecdotal notes on how students develop their strategies.

Analysis of Games and Numbers activities are an appropriate context for journal writing on both content and attitudinal factors.

When a game is played more than once, students may keep dated journal entries on their thinking about the strategy.

These could become part of a portfolio.

**SUGGESTED LEARNING
RESOURCES**

Print

Senior 2 Consumer Mathematics (25S) Part II: A Course for Distance Learning. Winnipeg, MB: Manitoba Education and Training, 2000.

— Cover Assignments
Modules 6-10

Blocksma, Mary. *Reading the Numbers: A Survival Guide to the Measurements, Numbers, and Sizes Encountered in Everyday Life.* New York, NY: Penguin Books, 1989.

McFarlan, Donald (ed.). *Guinness Book of World Records.* New York, NY: Bantam, n.d. (published annually)

Sutcliffe, Andrea. *Numbers: How Many, How Long, How Far, How Much . . . All the Numbers You'll Ever Need.* New York, NY: Harper Perennial, 1996

Verhille, C., and R. Blake. "The Peg Game." *Mathematics Teacher* (January, 1982): 39-43.

Mathematics Teacher (various issues). Reston, VA: NCTM, n.d.

Dell Logic Problems (various issues)

Penny Press Logic Problems (various issues)

See Appendix II for a list of additional resources.

**PRESCRIBED LEARNING
OUTCOMES**

B-2 demonstrate how numbers are used descriptively throughout society

SUGGESTIONS FOR INSTRUCTION

Students should present a topic showing how numbers are used in society. Have students ask questions of the presenter. Some topics may allow for further exploration.

Topic Examples:

Describe and define the numbers used in:

- | | |
|----------------------------|----------------------|
| circles | steel wool |
| volume | nails |
| land measures | plywood |
| length | consumer price index |
| metric system | rain |
| property land descriptions | cheques |
| sandpaper | insulation |
| screws and bolts | |

Present information from the newspaper and have the students answer questions based on the information. A good source for this type of activity is the Media Clips in the NCTM publication, *Mathematics Teacher*.

It is neither intended that students memorize the use of numbers nor reproduce that information on a test. The aim is to develop a general knowledge of the use of numbers.

Students could work individually or in groups, and be responsible for researching a topic and presenting their findings to the class.

B-2.1 Enough Peanut Butter to Fill the Grand Canyon

Americans are expected to eat enough peanut butter this year to fill the Grand Canyon . . . per capita consumption has climbed to 3.36 pounds, which should boost total U.S. consumption to 857 million pounds this year, said the newsletter, published by the Peanut Advisory Board, an industry group in Atlanta.

— Copyright © 1993 by *Associated Press*.

Discussion

Watertown Daily Times, 21 January 1993. Submitted by Meg Clemens, Northwest Tech, Ogdensburg, New York.

After reading this item in her local newspaper, Meg Clemens sent a letter to the newspaper, which was published on 8 February 1993. Part of her letter is given.

From Media Clips. *Mathematics Teacher* 87.6. Copyright © 1994 by National Council of Teachers of Mathematics.

Communications	✓ Patterns
✓ Connections	Problem Solving
✓ Number Sense	✓ Reasoning
Organization and Structure	Technology
	Visualization

SUGGESTIONS FOR ASSESSMENT

Journal Question

Describe how numbers are used in forecasting weather.

Portfolio

Students can research a topic and place their reports in their portfolios.

Anecdotal Notes

Analysis of Games and Numbers activities are generally not appropriate on pencil-and-paper timed tests.

**SUGGESTED LEARNING
RESOURCES**

**PRESCRIBED LEARNING
OUTCOMES**

B-2 demonstrate how numbers are used descriptively throughout society
– *continued*

SUGGESTIONS FOR INSTRUCTION

“I flinched when I read the Associated Press article about the peanut butter lovers fan club which claims “Americans . . . (will) eat enough peanut butter this year to fill the Grand Canyon.” No way — the Grand Canyon is immense. I estimate this year’s peanut butter consumption, 857 million pounds, would take up about 12 million cubic feet. That is about the size of a 25-storey hotel whose ground floor is as big as a football field. Granted that’s a lot of peanut butter, but it’s not even close to the size of the Grand Canyon. The Grand Canyon is 217 miles long, from 4 to 13 miles wide and between 4,000 to 5,500 feet deep. Using conservative estimates, if we keep eating peanut butter at this year’s rate, we could fill the Grand Canyon in about 5 million years, less if we eat more peanut butter.”

Ask students to verify the claims made by Clemens in her letter.

Rain

There is hardly a more misunderstood weather forecast than the prediction of rain. If you read that there’s a 50 percent chance of rain for today, it sounds as if the weather people were guessing that there’s half a chance that the area covered by the forecast (yours) will get rained on, as if they were betting 50-50 odds. Although many local forecasters still hang on to this idea, Environment Canada insists that the forecast percentage is not a probability, but the percentage of your area that will be wet by the end of the day. One could argue that odds are still 50-50 that you won’t need your umbrella.

Rain reports — or the measure of rainfall in millimetres — represent the amount of rain that would have remained on the ground if it did not run off or soak in. To calculate this, rain gauges — straight-sided containers 20 cm in diameter — are scattered throughout the forecast area, each one often representing many square kilometres. Measurements of rain collected in rain gauges are taken daily, either by hand or by electronic devices, and reported.

Communications	✓ Patterns
✓ Connections	Problem Solving
✓ Number Sense	✓ Reasoning
Organization and Structure	Technology Visualization

Rain: From Blocksma, M. *Reading the Numbers*. Copyright © 1989 Mary Blocksma. Reprinted by permission of Mary Blocksma.

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Portfolio

Students can add to their portfolios work samples based on the topic.

Students can do research on a topic and place their reports in their portfolios.

Anecdotal Notes

Analysis of Games and Numbers tasks are generally not appropriate on pencil-and-paper timed tests.

**PRESCRIBED LEARNING
OUTCOMES**

B-2 demonstrate how numbers are used descriptively throughout society
– *continued*

SUGGESTIONS FOR INSTRUCTION

Since rain rarely falls evenly, this method of calculating rainfall might be likened to measuring the height of one meteorologist in each state and then saying that Missouri meteorologists are taller than Maryland meteorologists. Nevertheless, even a general idea of rainfall is valuable. A rain gauge is accurate for one place — your house, for example, or your garden or farm. You can make a rain gauge from any straight-sided, flat-bottomed container — a large juice can, for example. Ideally, it's set one metre above the ground and away from buildings, trees, or other interference.

The meteorological year is often calculated to run from April through March, so that winter statistics won't be split into two years, as would happen in a January-December year. Cumulative, year-to-date rainfall reported in most newspapers usually begins April 1, which is why late-summer precipitation statistics may appear surprisingly low.

- a) Why must the rain gauge be a straight-sided container as opposed to, for example, a funnel-shaped container?

Answer:

Consider horizontal cross-sections.

- b) Will a 15-cm diameter rain gauge give the same result as a 10-cm diameter rain gauge?

Answer:

Yes, because the volume of water is not being measured. A horizontal cross-section of the gauge is used to measure the depth of water. However, a larger diameter is preferable to a smaller one.

Communications	✓ Patterns
✓ Connections	Problem Solving
✓ Number Sense	✓ Reasoning
Organization and Structure	Technology Visualization

SUGGESTIONS FOR ASSESSMENT

**SUGGESTED LEARNING
RESOURCES**

Appendix I

Teacher Information: Bonus Dots

Materials

- pen or pencil
- gameboard

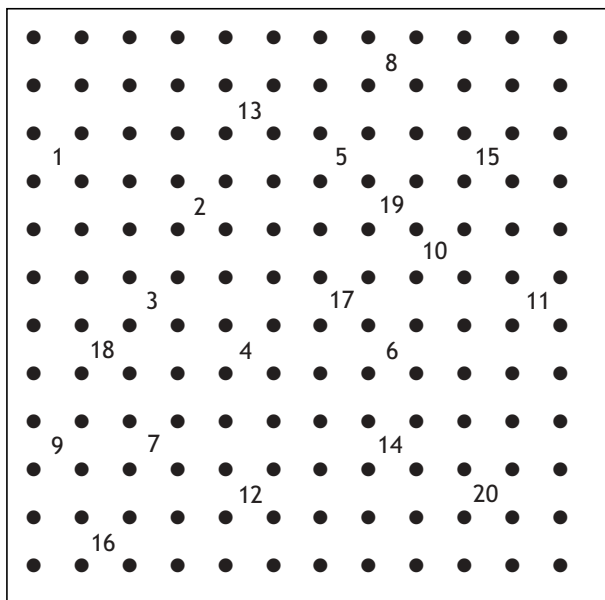
Number of Players

2 to 4

Rules

1. The object of the game is to score the most points by enclosing squares on the gameboard. Bonus points can be scored by enclosing squares with numbers in them.
2. Each player chooses a symbol to write inside completed squares.
3. Before play begins, players take turns writing the numbers 1 to 20, in order, in potential squares on the gameboard.

Example:



4. The first player connects two adjacent dots vertically and horizontally.
5. When a player encloses a square, he/she writes his/her symbol in the square. Completing a square lets the player have another turn. The player continues as long as squares are completed.
6. The game is over when all squares on the gameboard have been enclosed. Each player counts the number of squares that contain his/her symbol, then add all the bonus points he/she has enclosed to the point total. The player with the highest score is the winner.

Teacher Information: Bonus Dots (continued)

Variation

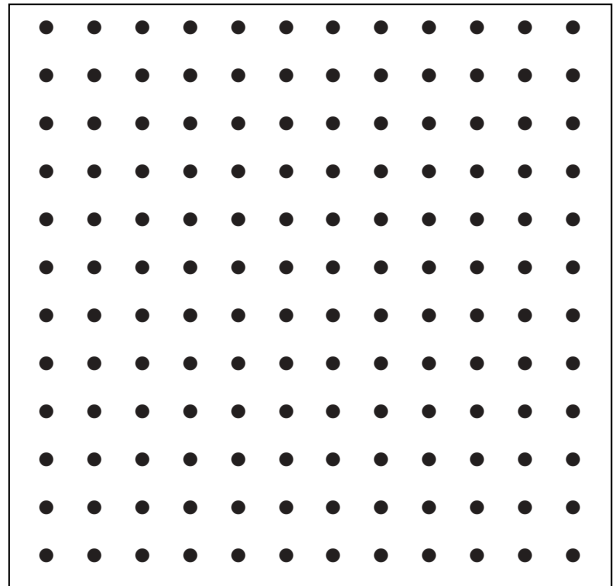
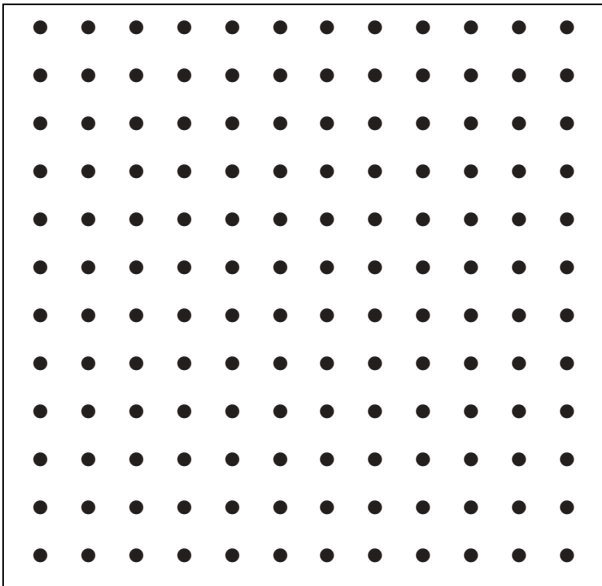
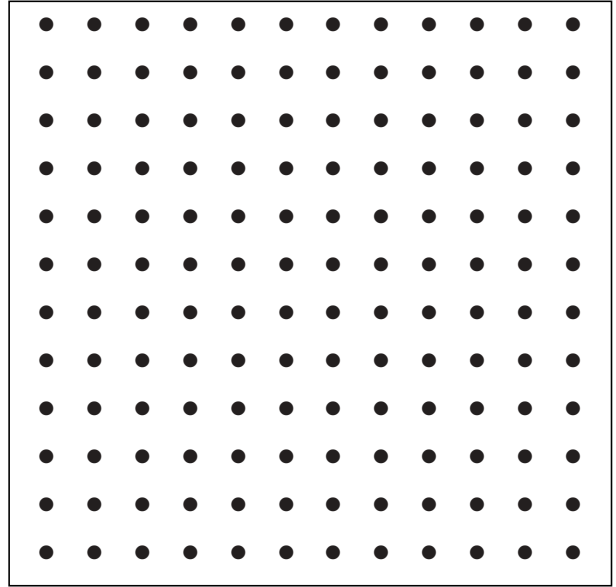
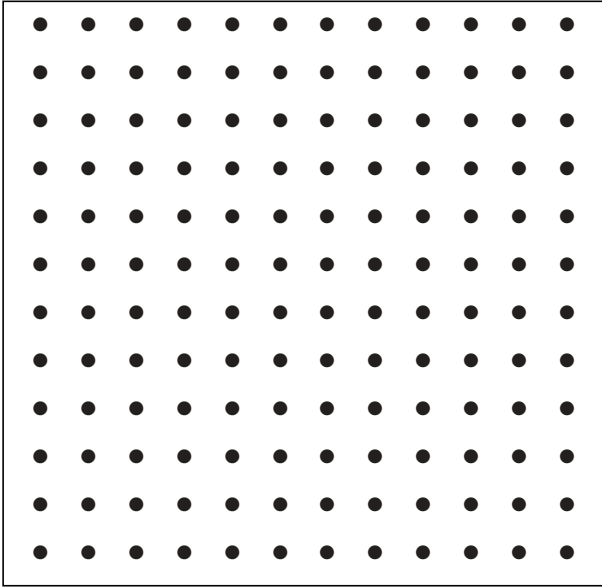
- Use a smaller gameboard and the numbers from 1 to 10 so the game will be shorter.
- Before play begins, each player marks five squares with a unique symbol (different from the claiming symbol). Players gain 10 additional bonus points for each of their own unique symbols they enclose in a square. They lose five points if they enclose their opponent's unique symbol.

Suggestions

- Encourage students to describe their strategy.
- Have students find other variations and describe the advantages and disadvantages.
- Is going first an advantage?

Blackline Master: Bonus Dots

Gameboard



Teacher Information: Super Nim

Materials

- 72 game tokens (bingo chips or pennies)
- gameboard

Number of Players

2

Rules

1. The object of the game is to score points by picking up the last piece in each section of the board.
2. Place one token in each space of the gameboard. In turn, each player picks up from one to all of the tokens on any one of the three rows in any one section of the gameboard.
3. A player scores one point each time he/she takes the last token in each of the six sections of the board. The player to take the last token in the last remaining section scores two points. The player to score the most points is the winner.

Teaching Suggestions

- Have students suggest variations and try them out.
- Is there an advantage to going first?

Blackline Master: Super Nim

Gameboard

Teacher Information: Crossing the River

Skills Required

- logical reasoning

When To Do

Any time.

Teaching Suggestions

Assign four students to play each role. Have students act out the situation with suggestions from the class. This might help students visualize the situation.

Solution

1. Take goat to the other side.
2. Return “empty.”
3. Take wolf over.
4. Bring goat back.
5. Take cabbage over.
6. Return “empty.”
7. Take goat over — task completed.

Blackline Master: Crossing the River

A man has a goat, a wolf, and a head of cabbage. He has to cross a river to get to the market. There is a boat which will hold the man and either the goat or the wolf or the cabbage at one time. The problem is that, if left alone, the goat will eat the cabbage. If left alone, the wolf will eat the goat.

How does he transport himself and the three items across the river without losing any of them? What is the minimum number of trips he can make to transport himself and all items across the river?



Teacher Information: The Travelling Knight

Skills Required

- visualization
- deductive reasoning

When To Do

This activity may be introduced at any time. It may provide a break during the Spatial Geometry unit.

Teaching Suggestions

Have a copy of the chessboard grid on the overhead and try to move the knight to help students visualize how the knight moves.

Solution

It takes six moves to move a knight from one corner of a chessboard to the opposite corner.

K							
		1					
			2				
				3		5	
					4		6

Other solutions are possible.

Blackline Master: The Travelling Knight

From any position on a chessboard, the Knight can move to any of the eight squares shown.

		3		2			
	4				1		
			K				
	5				8		
		6		7			

Challenge

What is the minimum number of moves to take a Knight from one corner of a chessboard to the opposite corner?

Extension

Is it possible to find a path so that the Knight touches all squares

- a) at least once?
- b) exactly once?

Teacher Information: Multiple Discounts

Skills Required

- percentage

When To Do

This activity may be introduced at any time. It may provide a break during the Consumer Decisions unit.

Teaching Suggestions

This set of questions investigates the effect of applying several series of percent discounts to an item. Students should realize that taking 25% off an item already discounted by 25% is not equivalent to taking 50%.

Solutions

1. a) Discount: $\$24.95 \times 25\% = \6.24
Sale Price: $\$24.95 - \$6.24 = \$18.71$

or

Sale Price: $\$24.95 \times 75\% = \18.71

b) Discount: $\$18.71 \times 25\% = \4.68
Sale Price: $\$18.71 - \$4.68 = \$14.03$

or

Sale Price: $\$18.71 \times 75\% = \14.03

c) $\$24.95 \times 50\% = \12.48

d) no

1. a) Discount: $\$69.99 \times 25\% = \14.50
Sale Price: $\$69.99 - \$14.50 = \$55.49$

b) Discount: $\$55.49 \times 15\% = \8.32
Sale Price: $\$55.49 - \$8.32 = \$47.17$

c) Discount: $\$69.99 \times 40\% = \28.00
Sale Price: $\$69.99 - \$28.00 = \$41.99$

d) You are taking 40% off the original price in part (c). In part (b), you are taking 15% off of a price that is lower than the price you used when you took 25% off in part (a).

Teacher Information: Multiple Discounts (continued)

3. The amount does not go back to \$10.00 because you are taking 7% of different amounts.

4. $\$299.95 \times 14\% = \41.99
 $\$299.95 - \$41.99 = \$257.96$
 Tax on $\$257.96 \Rightarrow 257.94 \times 14\% = \36.11
 $\$257.96 + 36.11 = \294.07

5. PST: $\$29.99 \times 7\% = \2.10
 Discount for GST: $\$29.99 \times 7\% = \2.10
 $\$29.99 - \$2.10 = \$27.89$
 GST on $\$27.89 = \$27.89 \times 0.07 = \$1.95$
 Total GST: $\$27.89 + \$1.95 + 2.10 = \$31.94$

6. Increase: $\$14.95 \times 1.10 = \16.45
 Decrease: $\$16.45 \times 0.90 = \14.81

7. Decrease: $\$14.95 \times 0.90 = \13.46
 Increase: $\$13.46 \times 1.10 = \14.81

8. The answers are the same. To earn more money, I'd prefer the increase first. In the first year I'd earn \$3.00 per hour when compared to the second case.

9. Answers will vary.

One solution: I would prefer the gain first. If I invest \$100, and I gain 10% the first year, I'd have \$110 after the first year. The 10% decrease will mean the investment is now at \$99. But for the one year, I would have had \$110.

Blackline Master: Multiple Discounts

When you walk into a store and an article of clothing is marked 25% off and the store is advertising a further 25% discount on all items in the store, does that mean you are getting the article of clothing at 50% off? Try the following:

1. You purchase a shirt that regularly sells for \$24.95. It is marked 25% off.
 - a) What is the new sale price?
 - b) The store offers a further 25% off. Using your answer from part (a), take off a further 25% and state the new price of the shirt.
 - c) What is 50% off the \$24.95 regular price?
 - d) Do your answers to parts (b) and (c) agree?
2. Another store is selling all merchandise at 15% off its last marked price. You find a pair of shoes originally priced at \$69.99, marked down by 25% (last marked price).
 - a) What is the price of the shoes with just the 25% sale?
 - b) What is the price of the shoes with the 15% off the price in part (a)?
 - c) What would be the cost of the shoes if the store gave a 40% discount off the original price?
 - d) Why are the answers to parts (b) and (c) different?

Another promotion stores often offer is a “Don’t pay the GST.” For items on which there is no PST, most stores who offer this promotion take 7% off the cost of a product and then add back the tax. For example, a \$10.00 item would have \$0.70 taken off for a price of \$9.30, then 7% of \$9.30 (\$0.65) is added back and the customer pays a total of \$9.95.

3. Why does the amount not go back to \$10.00?
4. The store offers a “No Tax” day. Nothing you buy is subject to either PST or GST. You buy a stereo for \$299.95. What is the total you pay, if the tax is dealt with as above?
5. A store offers a “Don’t Pay the GST” day. You buy a pair of jeans for \$29.99. You have to pay the PST and the GST is dealt with as above. What is your final cost?
6. You are working for \$14.95 an hour. One year, you have a 10% raise in wages. The following year, you have a 10% decrease. What is your hourly wage after both adjustments?
7. You are working for \$14.95 an hour. One year, you have a 10% decrease in wages. The following year, you have a 10% raise. What is your hourly wage after both adjustments?
8. Compare your answers to questions 6 and 7. What do you notice? In which situation would you rather be?
9. If you have an investment that loses 10% one year and gains 10% in another year, in which order would you prefer the gain and loss? Use calculations to support your answer.

Teacher Information: Patterns

Skills Required

- arithmetic
- logical reasoning

When To Do

This activity may be introduced at any time.

Teaching Suggestions

Go through the first example with the students.

Have students explain their answers to the class. The descriptions may differ from the ones given. If the pattern described by the student is logical, the sequence may be different but still correct.

Solutions

1. Description: Add “5” to the previous term
Next 3 terms: 24, 29, 34
2. Description: Add “1” to the first term, add “2” to the second term, add “3” to the third term, and so on
Next 3 terms: 22, 29, 37
3. Description: Subtract “6” from the previous term
Next 3 terms: -3, -9, -15
4. Description: Add “1” to the previous term, then multiply this new term by “3.”
Next 3 terms: 31, 93, 94
5. Description: Multiply the first term by “4,” then divide this new term by “2”
Next 3 terms: 48, 24, 96
6. Description: Multiply the first term by “2,” then multiply this new term by “4,” then multiply this new term by “6,” and so on
Next 3 terms: 1920, 19 200, 230 400
7. Description: Add “5” to the first term, add “7” to the next term, add “9” to the next term, and so on.
Next 3 terms: 48, 63, 80
8. Description: Starting from “1”, alternate multiplying each consecutive whole number by either “6” or “7”
Next 3 terms: 42, 42, 56
9. Description: Starting from “1”, subtract “3” from each consecutive whole number and then square
Next 3 terms: 9, 16, 25

Teacher Information: Patterns (continued)

Solutions (continued)

10. Description: These are powers of “3” minus “1”
Next 3 terms: 215, 342, 511
11. Description: These are multiples of “5” with the digits “1” or “2” alternately placed in front of each
Next 3 terms: 230, 135, 240
12. Description: These are the multiples of “2” alternately interspersed with the multiples of “7”
Next 3 terms: 21, 8, 28
13. Description: Starting with the number “1,” add or subtract “2” from each consecutive whole number
Next 3 terms: 8, 5, 10
14. Description: Each number is doubled and then has “3” added to it, to produce the next term
Next 3 terms: 125, 253, 509

Extension

Have students create their own patterns.

Blackline Master: Patterns

Some people say that mathematics is the science of patterns. Patterns take many forms and can be found in all branches of mathematics.

Patterns in number sequences such as 3, 6, 9, 12, ... are familiar to us since they are among the patterns we first learn as young students.

- Study the following number patterns.
- Briefly describe the pattern.
- Write the next three terms in the pattern.

Example: 3, 6, 9, 12, ...

Description: *"The next term is found by adding "3" to the previous term*

Next 3 terms: *15, 18, 21*

1. 4, 9, 14, 19, ...

Description: _____

Next 3 terms: _____ , _____ , _____

2. 1, 2, 4, 7, 11, 16, ...

Description: _____

Next 3 terms: _____ , _____ , _____

3. 15, 9, 3, ...

Description: _____

Next 3 terms: _____ , _____ , _____

4. 2, 3, 9, 10, 30, ...

Description: _____

Next 3 terms: _____ , _____ , _____

5. 3, 12, 6, 24, 12, ...

Description: _____

Next 3 terms: _____ , _____ , _____

6. 5, 10, 40, 240, ...

Description: _____

Next 3 terms: _____ , _____ , _____

7. 3, 8, 15, 24, 35, ...

Description: _____

Next 3 terms: _____ , _____ , _____

Blackline Master: Patterns (continued)

8. 6, 14, 18, 28, 30, ...

Description: _____

Next 3 terms: _____, _____, _____

9. 4, 1, 0, 1, 4, ...

Description: _____

Next 3 terms: _____, _____, _____

10. 2, 8, 26, 80, 242, ...

Description: _____

Next 3 terms: _____, _____, _____

11. 15, 210, 115, 220, 125, ...

Description: _____

Next 3 terms: _____, _____, _____

12. 2, 7, 4, 14, 6, ...

Description: _____

Next 3 terms: _____, _____, _____

13. -1, 4, 1, 6, 3, ...

Description: _____

Next 3 terms: _____, _____, _____

14. 1, 5, 13, 29, 61, ...

Description: _____

Next 3 terms: _____, _____, _____

Appendix II

Additional Resources

Print

The Diagram Group. *The Little Giant Encyclopedia of Games for 1 or 2*. Sterling Publishing Company Inc. ISBN 0-8069-0981-1.

Fleisher, Paul. *Brain Food: Games That Make Kids Think*. Zephyr Press. ISBN 1-56976-072-1.

Nasht, Helen, and Dorothy Masterson. *Humorous Cryptograms*. Sterling Publishing Company Inc. ISBN 0-8069-3982-6.

Tuller, Dave, and Michael Rios. *Mensa Math and Logic Puzzles*. Sterling Publishing Company Inc. ISBN 0-8069-4199-5.

Internet

There are many sites on the Internet with problems and puzzles. When searching for problems and puzzles, use the words “mathematical games.”

As of February 2002, the following sites were available:

Fun Brain

<<http://www.learningnetwork.funbrain.com>>

This site offers some interactive games. Some of the games could be adapted to pencil-and-paper games.

Puzzlemaker

<<http://www.puzzlemaker.school.discovery.com>>

This site allows teachers to create their own puzzles. One of the puzzle types is cryptograms. When using the site, be sure to print the answer with the puzzle as all puzzles are created new with each visit.

This Is Mega Mathematics

<<http://www.c3.lanl.gov/mega-math/>>

There are a variety of activities here. One of the activities involves map colouring. Another activity involves games with graphs.