Unit B: Analysis of Games and Numbers

Half Course I

HALF COURSE I

Unit B: Analysis of Games and Numbers

Hours: 5

General Learning Outcome:

Develop, use, and justify mathematical strategies by analyzing a variety of puzzles and games; develop an awareness of how numbers are used in society.

The material provided for this unit should be used throughout the course to provide a change of pace in a context which is enjoyable, and calls for mathematical and logical thinking.

Specific Outcomes

B-1 Demonstrate the use of an appropriate strategy in solving puzzles and playing games involving patterns.

B-2 Demonstrate how numbers are used descriptively throughout society.

ANALYSIS OF GAMES AND NUMBERS

Instructional Materials

- *Essentials of Mathematics 10*
- See Appendix I for possible activities.
- See Appendix II for additional resources.

PRESCRIBED LEARNING OUTCOMES

General Outcome

Develop, use, and justify mathematical strategies by analyzing a variety of puzzles and games; develop an awareness of how numbers are used in society.

Specific Outcome(s)

B-1 demonstrate the use of an appropriate strategy in solving puzzles and playing games involving patterns

Communications	✔ **Patterns**
Connections	Problem Solving
✔ **Number Sense**	✔ **Reasoning**
✔ **Organization and Structure**	Technology
	✔ **Visualization**

SUGGESTIONS FOR INSTRUCTION

Consideration should be given to interspersing Analysis of Games and Numbers throughout the course, i.e., you may wish to spend a few days on this early in the course for motivational reasons and then use the remaining or other activities to provide a break between other units, or in the middle of a long unit.

The objective of the following collection of activities is to have students play the games and find winning strategies. Students are expected to find the strategies and be able to explain the strategies by oral or written demonstration.

Sufficient time should be devoted to playing and enjoying a game before analysis begins. Allow students time to discuss the game and articulate their “winning” strategies.

Finding the strategy is the first step. The subsequent steps are of equal importance. Students will be examining their own thinking processes. Such examinations are not easy and probably an uncommon experience. The actual results may be of little consequence, but finding the results is the real objective. Another key objective is the communication of strategies and thought processes.

Teachers should try the activities before giving them to students. These activities are independent of grade and could be repeated. An introduction to an activity could be at Senior 2 and then the variations could be introduced at a later grade or later in the course.

It is not intended that these activities be taught in a block of time, but, rather, dealt with periodically during the year. For sample tasks and games, see Appendix I of this unit.

SUGGESTIONS FOR ASSESSMENT

The willingness to accept the challenge of the learning experiences is important. Keep a daily record.

You may wish to keep anecdotal notes on how students develop their strategies.

Analysis of Games and Numbers activities are an appropriate context for journal writing on both content and attitudinal factors.

When a game is played more than once, students may keep dated journal entries for recording their thinking about the strategy. These could become part of a portfolio.

SUGGESTED LEARNING RESOURCES

Print

Senior 2 Consumer Mathematics (25S) Part I: A Course for Distance Learning. Winnipeg, MB: Manitoba Education and Training, 2000.
— Cover Assignments Modules 1-5

Ash, Russell. “The Top 10 of Everything.” Montreal, QC: *Reader's Digest*, n.d. (published annually)

Blocksma, Mary. *Reading the Numbers: A Survival Guide to the Measurements, Numbers, and Sizes Encountered in Everyday Life*. New York, NY: Penguin Books, 1989.

Giblin, P., and I. Porteous. *Challenging Mathematics.* Toronto, ON/New York, NY: Oxford University Press, 1990.

McFarlan, Donald (ed.). *Guiness Book of World Records.* New York, NY: Bantam, n.d. (published annually)

Mathematics Teacher (various issues). Reston, VA: NCTM, n.d.

Sutcliffe, Andrea. *Numbers: How Many, How Long, How Far, How Much . . . All the Numbers You'll Ever Need.* New York, NY: Harper Perennial, 1996

Verhille, C., and R. Blake. “The Peg Game.” *Mathematics Teacher* (January, 1982): 39-43.

Dell Logic Problems (various issues)

Penny Press Logic Problems (various issues)

See Appendix II for a list of additional resources.

PRESCRIBED LEARNING OUTCOMES	SUGGESTIONS FOR INSTRUCTION

B-2 demonstrate how numbers are used descriptively throughout society

Present a topic or application area showing how numbers are used and have students discuss the use of numbers. Some topics may allow for further exploration.

Topic Examples:

shoes	socks
computers	humidity
social insurance number	snow
calendar	temperature
sunscreen	military time
sound	astronomical prefixes
clothing sizes	miniscule prefixes
type size	

Teachers could present information from the newspaper and have the students answer questions based on the information. A good source for this type of activity is the Media Clips Department in the NCTM publication, *Mathematics Teacher*.

It is not intended that students memorize the use of numbers and be able to reproduce that information on a test.

Students could form groups and be responsible for researching a topic and presenting their findings to the class.

B-2.1 Backburner by Mary McIver

It's So Simple, Only a Child Can Do It
(Reprinted from *Homemaker's Magazine*.)

See if you can answer the skill-testing question that the Island Shell Aerocentre, a posh aircraft refueling facility in Toronto, required entrants to answer during a recent contest: $35 + 12 \times 12 \div 2 = \square$. Everybody get 282? Well, you're wrong. According to new math procedures, you're required to perform the multiplication and division chores before you add and subtract. So the right answer is 107. However, the Aerocentre got so many 282 answers — roughly half the entries — it decided both were acceptable. Excellent! As satirist Tom Lehrer once pointed out, the trouble with new math is that it seems to be more important to know what you're doing than to get the right answer.

Communications	✔ **Patterns**
✔ **Connections**	Problem Solving
✔ **Number Sense**	✔ **Reasoning**
Organization and Structure	Technology
	Visualization

SUGGESTIONS FOR ASSESSMENT

SUGGESTED LEARNING RESOURCES

PRESCRIBED LEARNING OUTCOMES

B-2 demonstrate how numbers are used descriptively throughout society
– *continued*

SUGGESTIONS FOR INSTRUCTION

1. Explain how entrants obtained an answer of 282. What steps should be followed to arrive at the correct answer of 107?

2. Consider the following generalized version of the skill-testing question used by the Island Shell Aerocentre.

   $a + b \times b \div c$

   Find all values of *a*, *b*, and *c* for which entrants would get the same answer using either the correct or the incorrect procedure. (*Mathematics Teacher*, 90.3 March, 1997.)

Answers:

1. Here are the steps to get an answer of 282: Add 35 to 12, multiply the result by 12, and then divide the new result by 2. Here are the steps to get an answer of 107: Multiply 12 by 12, divide the answer by 2, and then add 35.

2. $a + (b \times b \div c) = (a + b) \times b \div c$

$$a + \frac{b^2}{c} = \frac{(a+b)b}{c}$$

$$ac + b^2 = ab + b^2$$

$$ac = ab$$

Case1: If *a* is equal to 0, then *b* and *c* can be anything. The following is one possible skill-testing question:

$0 + 7 \times 7 \div 2$

Case 2: If *a* is not equal to 0, then *b* and *c* are equal to each other. The value of *a* can be anything. The following is one possible skill-testing question:

$17 + 5 \times 5 \div 5$

Communications	✔ **Patterns**
✔ **Connections**	Problem Solving
✔ **Number Sense**	✔ **Reasoning**
Organization and Structure	Technology
	Visualization

SUGGESTIONS FOR ASSESSMENT

SUGGESTED LEARNING RESOURCES

PRESCRIBED LEARNING OUTCOMES

B-2 demonstrate how numbers are used descriptively throughout society
– *continued*

Communications	✔ **Patterns**
✔ **Connections**	Problem Solving
✔ **Number Sense**	✔ **Reasoning**
Organization and Structure	Technology
	Visualization

SUGGESTIONS FOR INSTRUCTION

B-2.2 **Snow**

If you've just lived through a blizzard which meteorologists report dropped 40 cm of snow on your sidewalk, just how much actual precipitation is that? Snow, in its melted form, is figured into the annual precipitation figures. The rule of thumb for estimating this is that 10 cm of snow equals 10 mm of water.* But anyone who has shovelled a few sidewalks knows that there's snow and there's snow — 10 cm of light fluffy stuff is less heavy — and contains less water — than the wet, backbreaking kind. So measuring snow depth won't work for the statistical record.

Meteorologists solve this problem by collecting snow in rain gauges — straight-sided 20-cm containers — and in snow gauges, which are slightly larger. These measure not only the snowfall but the amount of water generated when it melts. In mountainous areas where the snowpack (deep layers of accumulated snowfalls) is an important source of water, the snow depth is not only measured, but core samples are taken to see what kind of snow is involved. Snow that melts or is rained on does not always run off immediately — the water can filter to the bottom and freeze into ice, as much as doubling the snow/water ratio. This kind of information is important not just to skiers, but to water- and flood-control agencies.

***Note:** Snow is measured in centimetres. 10 cm of snow = 1 cm or 10 mm of water, a ratio of 10:1.

a) If 10 cm of snow equals 10 mm of water, how much water should result from the following snowfalls?
   - i) 15 cm
   - ii) 7 cm
   - iii) 5.5 cm

b) If 10 cm of snow equals 10 mm of water, how much snow would have to fall to get the following amounts of water?
   - i) 2 mm
   - i) 1.5 mm
   - iii) 10 mm

SUGGESTIONS FOR ASSESSMENT

Journal Question

Describe how numbers are used to determine the amount of water in a snowfall.

Portfolio

Students can add work samples based on the topic to their portfolios.

Students can do research on a topic and place their reports in their portfolios.

Anecdotal Notes

Analysis of Games and Numbers tasks are generally not appropriate on pencil-and-paper timed tests.

SUGGESTED LEARNING RESOURCES

PRESCRIBED LEARNING OUTCOMES

B-2 demonstrate how numbers are used descriptively throughout society
– *continued*

SUGGESTIONS FOR INSTRUCTION

c) Research the last three major snowfalls in your area and calculate the amount of water that would have resulted.

Answers:

a) i) 15 mm
ii) 7 mm
iii) 5.5 mm
(cm become mm)

b) i) 2 cm
ii) 1.5 cm
iii) 10 cm
(mm become cm)

c) Answers will vary.

Communications	✔ **Patterns**
✔ **Connections**	Problem Solving
✔ **Number Sense**	✔ **Reasoning**
Organization and Structure	Technology
	Visualization

SUGGESTIONS FOR ASSESSMENT

SUGGESTED LEARNING RESOURCES

Appendix I

Teacher Information: Tridots

Materials

- pen or pencil
- gameboard

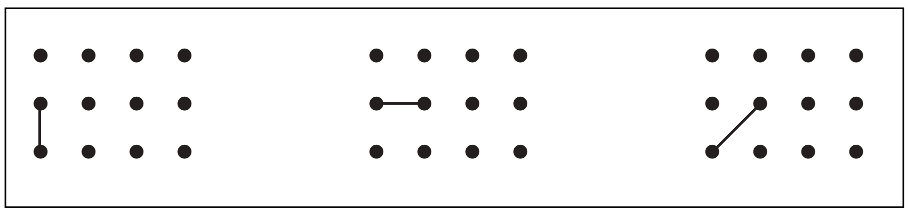
Number of Players

2 to 3

Rules

1. The object of the game is to enclose the largest number of triangles by drawing lines between adjacent dots on the gameboard.
2. Each player creates a symbol to write inside completed triangles to claim them.
3. Players take turns connecting adjacent dots with straight lines. A player may connect any two adjacent dots on the gameboard.

   Examples:



4. Whenever a player encloses a triangle, that player writes his or her symbol inside to claim the triangle. The player gets to go again. The player continues as long as he/she encloses triangles.
5. When all triangles on the gameboard have been enclosed and claimed, the game is over. Players count the number of triangles that contain their symbols. The player who has claimed the most triangles is the winner.

Variation

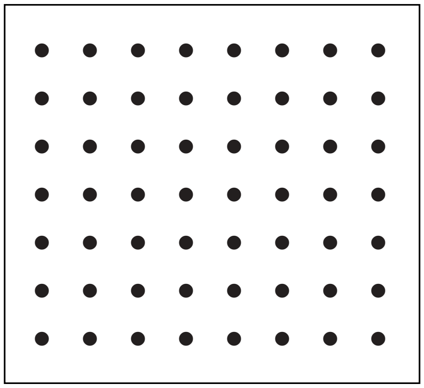
Any size gameboard can be used.

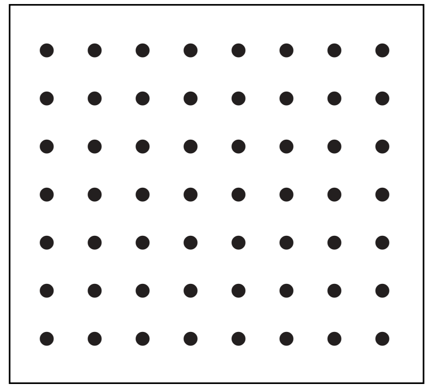
Suggestions

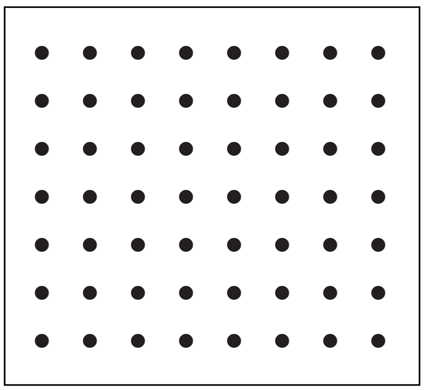
- Start with a gameboard made up of an 8 x 8 array of dots.
- Increase and decrease the size of the board to change the game.
- Introduce the game on an overhead projector and then allow students various opportunities to play.
- Ask students to come up with strategies for winning.
- Ask students to consider whether going first is an advantage.

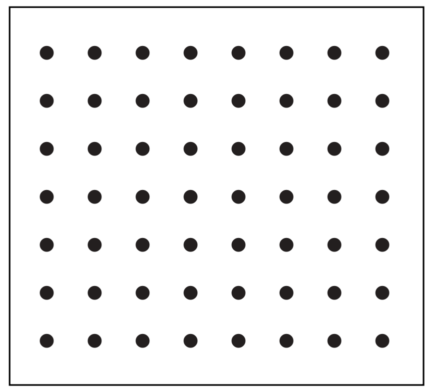
Blackline Master: Tridots

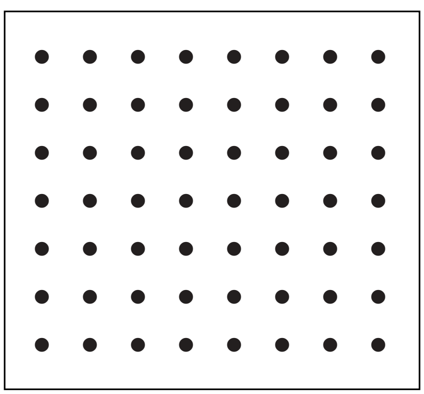
Gameboard

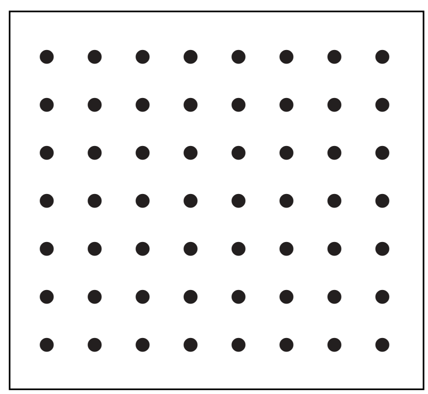






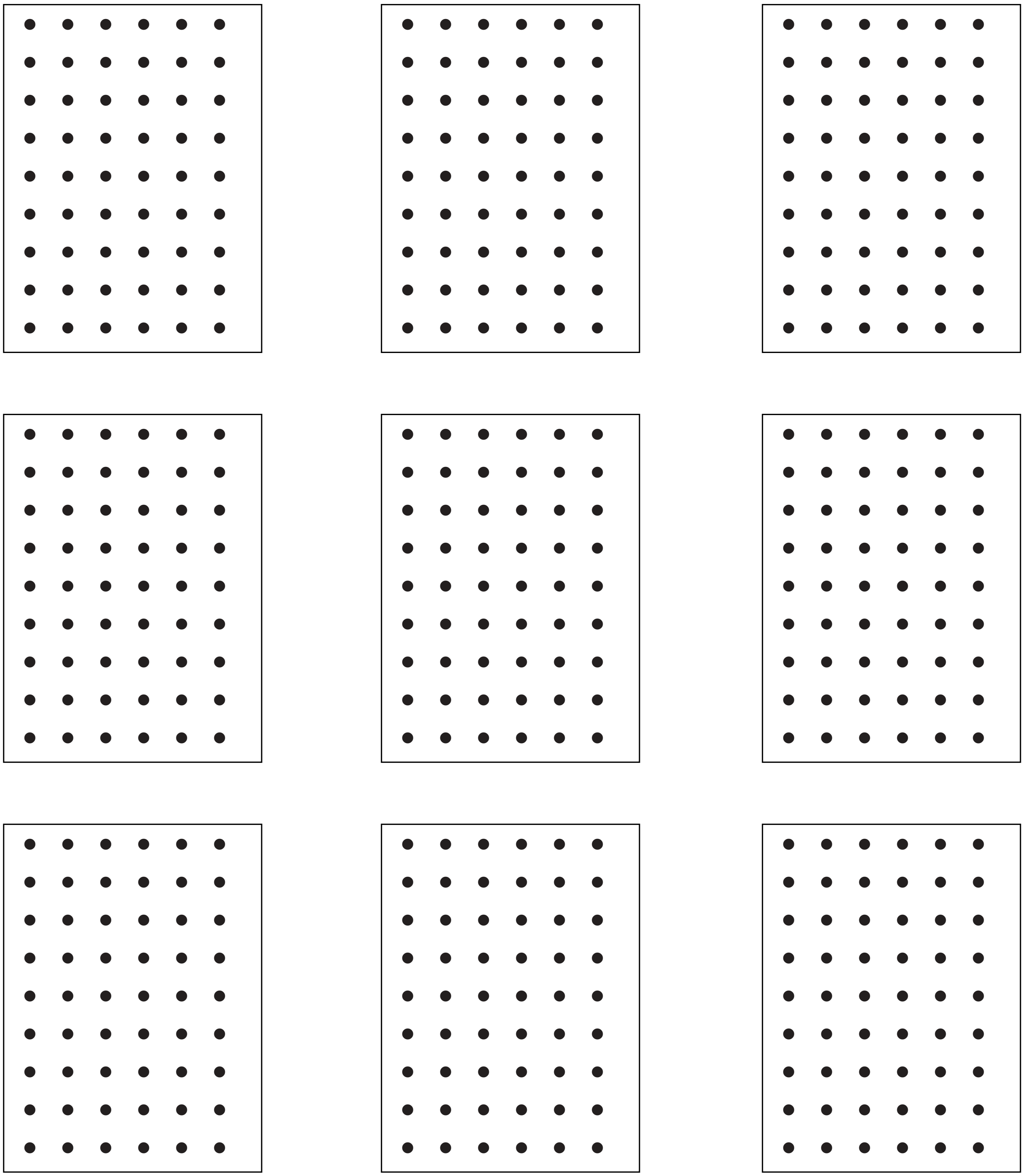






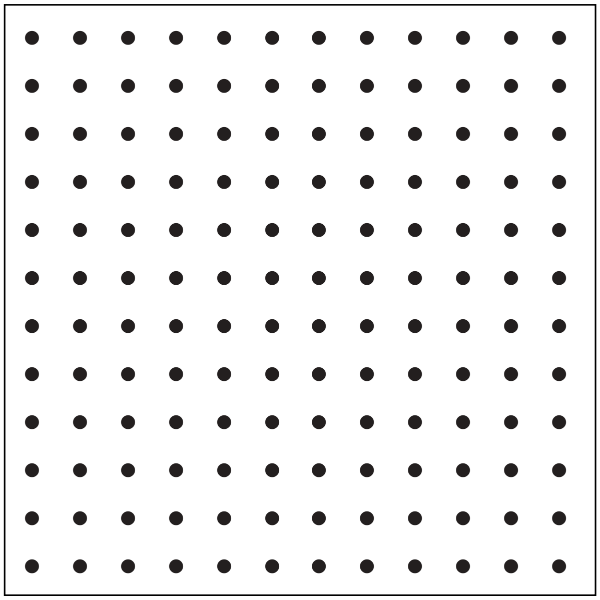
Blackline Master: Tridots (continued)

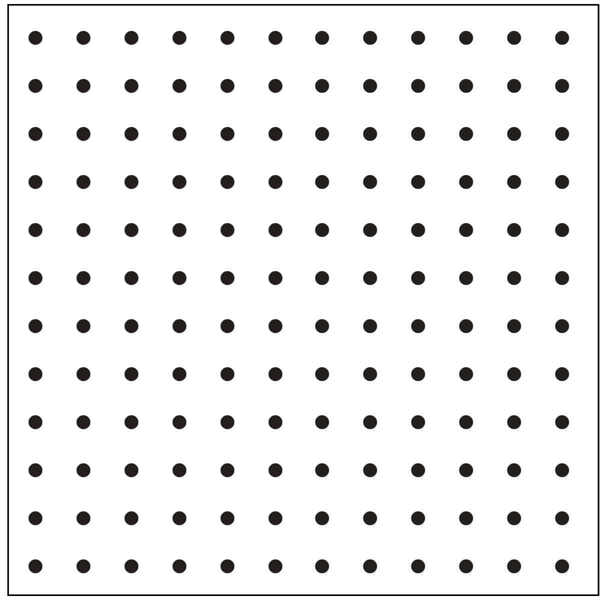
Gameboard

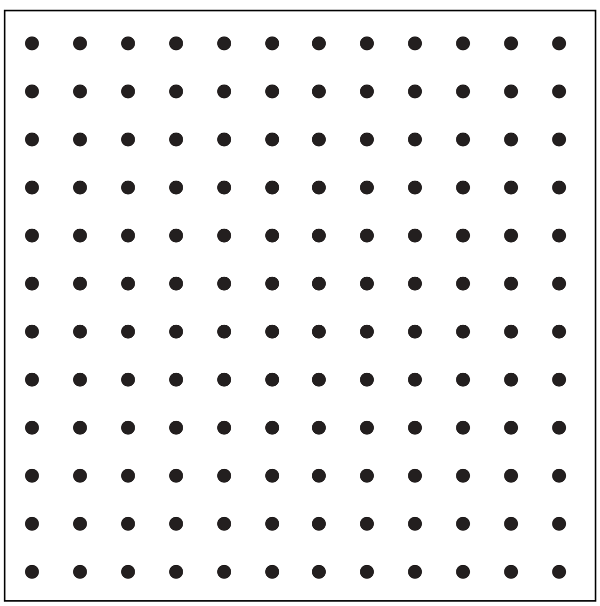


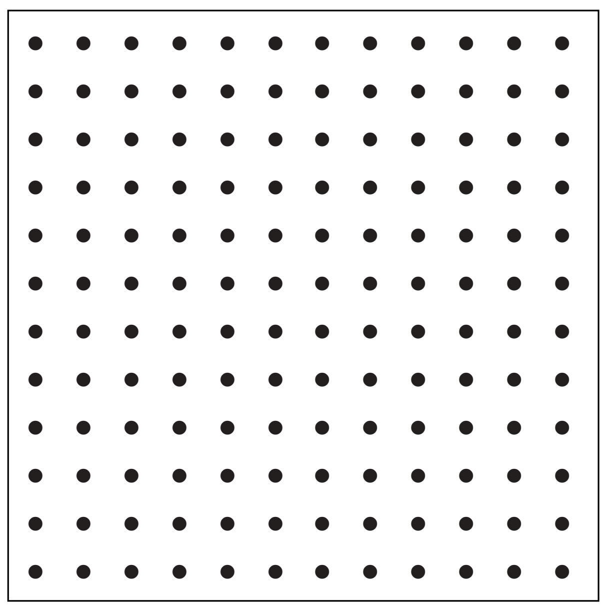
Blackline Master: Tridots (continued)

Gameboard









Teacher Information: Snake

Materials

- pen or pencil (two colours may be useful)
- gameboard

Number of Players

2

Rules

1. The object of the game is to be the last player to add a line to the snake without connecting to another part of the snake.
2. The first player starts the snake by connecting two adjacent dots either horizontally or vertically on the gameboard.
3. The second player connects an adjacent dot to either end of the snake with a horizontal or a vertical line.
4. Players continue taking turns drawing horizontal or vertical lines at either end of the snake, one line at a time.
5. The last player to add a line to the snake without connecting to any other part of the snake is the winner.

Variation

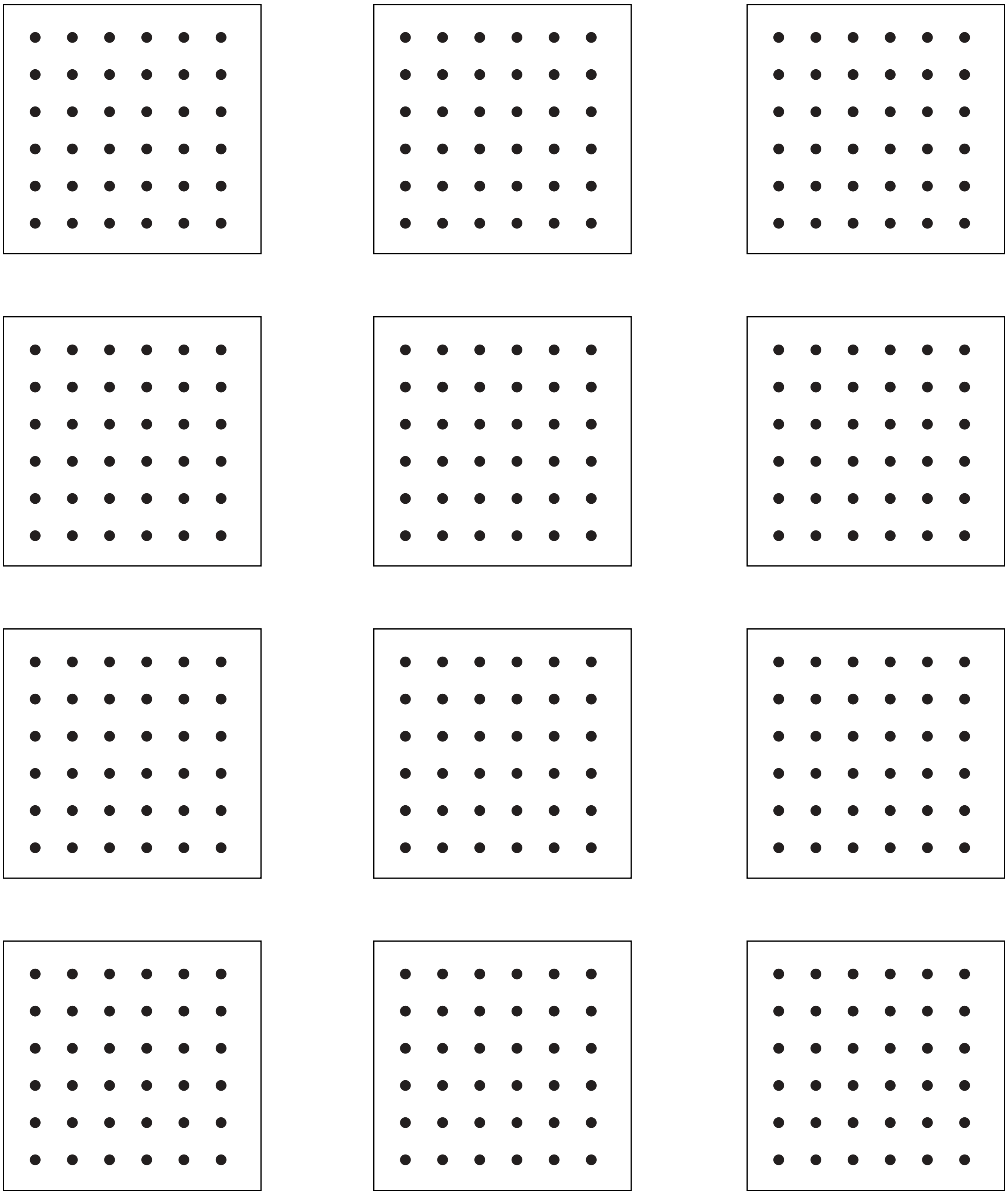
Any size gameboard can be used.

Suggestions

- Start with a small gameboard so the game is not too lengthy.
- Try different-sized gameboards.
- Ask students for strategies for winning.
- Is going first an advantage?

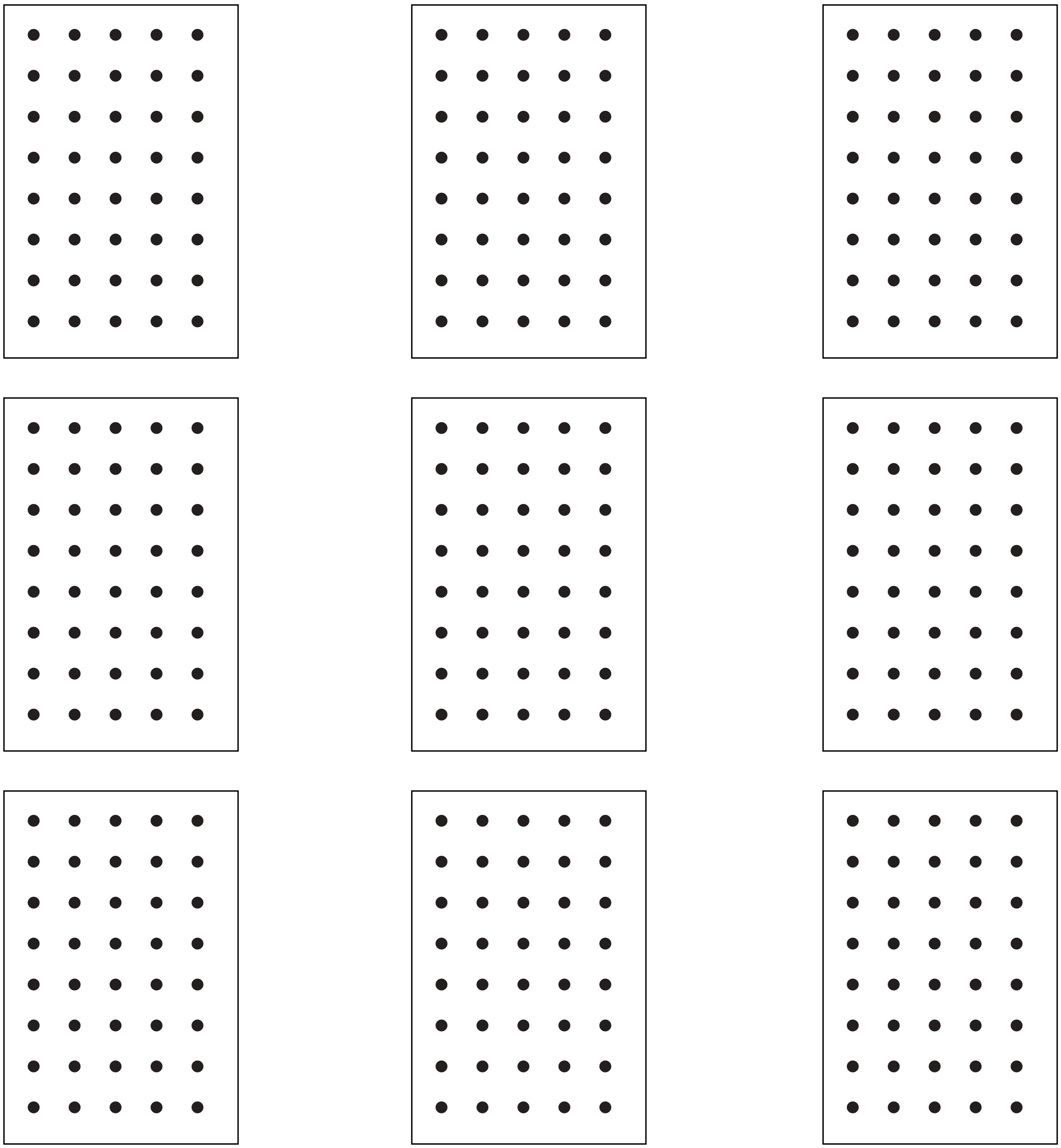
Blackline Master: Snake

Gameboard



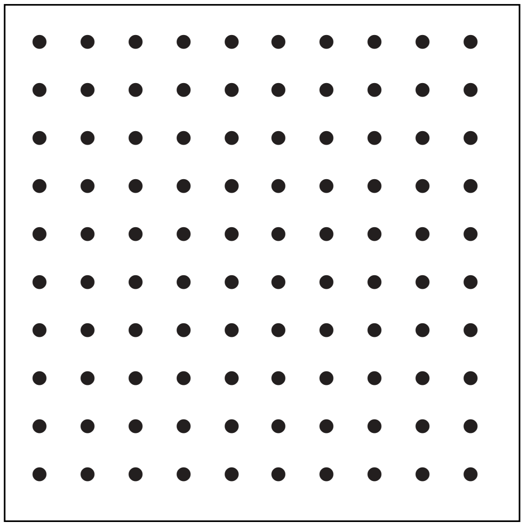
Blackline Master: Snake (continued)

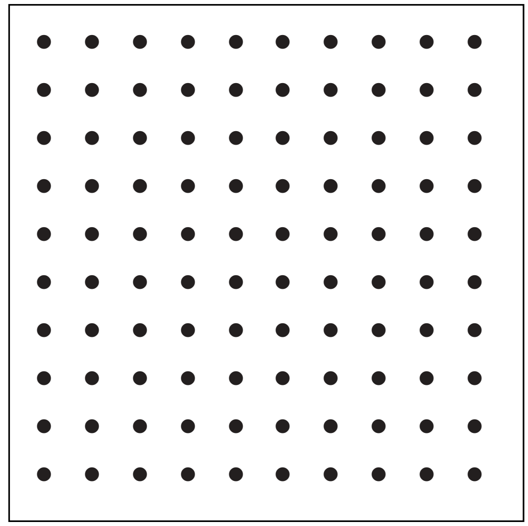
Gameboard

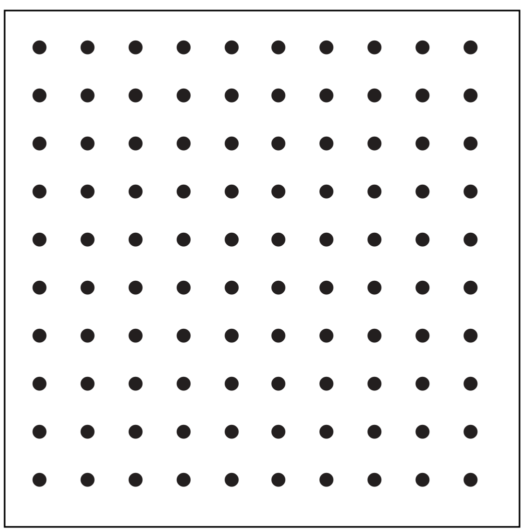


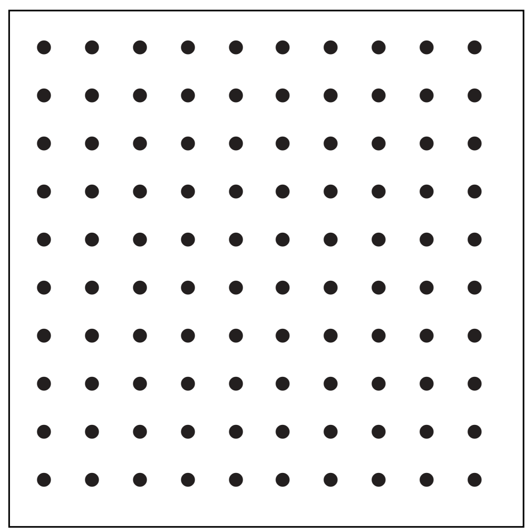
Blackline Master: Snake (continued)

Gameboard









Teacher Information: Poplar Lane Mystery

Skills Required

- logic
- deductive reasoning

When To Do

Any time.

Teaching Suggestions

1. Enlarge the grid and put on an overhead sheet. Work through the introduction and the first clue with the class. Have students work through the other clues.
2. Logic problems such as this can be found in *Dell Puzzle Magazine* and *Penny Press Magazine*. Some of the puzzles in these magazines are quite difficult, so choose carefully.

Solution

	Hospitality Room	Basement	Kitchen	Patio	Exercise Room	Rope	Silver Tray	Knife	Brick	Poison
Waiter	0	0	0	1	0	1	0	0	0	0
Cook	0	0	1	0	0	0	1	0	0	0
Manager	1	0	0	0	0	0	0	0	0	1
Security Officer	0	1	0	0	0	0	0	0	1	0
Bellhop	0	0	0	0	1	0	0	1	0	0
Rope	0	0	0	1	0					
Silver Tray	0	0	1	0	0					
Knife	0	0	0	0	1					
Brick	0	1	0	0	0					
Poison	1	0	0	0	0					

Combinations

Waiter with the Rope in the Patio
Cook with the Tray in the Kitchen
Manager with the Poison in the Hospitality Room
Security Officer with the Brick in the Basement
Bellhop with the Knife in the Exercise Room

Blackline Master: Poplar Lane Mystery

One cold and rainy evening, five hotel personnel were murdered in the old hotel on Poplar Lane (a waiter, a cook, a manager, a security officer, and a bellhop). The murders took place in the hospitality room, basement, kitchen, patio, and exercise room of the hotel. No two people were murdered in the same room or with the same weapon. The weapons used were a rope, a silver tray, a knife, a brick, and poison. From the clues given, try to determine the room in which each person was killed and the weapon used to do him/her in.

Clues

1. The murder with the brick was not done in the patio or the exercise room; neither the cook nor the manager was killed with the brick, nor was either killed in the patio or the exercise room.
2. The cook was not murdered in the hospitality room.
3. The rope was not the murder weapon used in the exercise room.
4. Neither the waiter nor the bellhop was murdered with poison, a silver tray, or a brick.
5. The person murdered in the basement had just had dinner with the bellhop, the cook, the person done in with poison, and the victim of the rope.

	Hospitality Room	Basement	Kitchen	Patio	Exercise Room	Rope	Silver Tray	Knife	Brick	Poison
Waiter										
Cook										
Manager										
Security Officer										
Bellhop										
Rope										
Silver Tray										
Knife										
Brick										
Poison										

Teacher Information: Checking Your Vision

Skills Required

- reading comprehension
- visualizing the eye-testing procedure

When to Do

This can be done at any time.

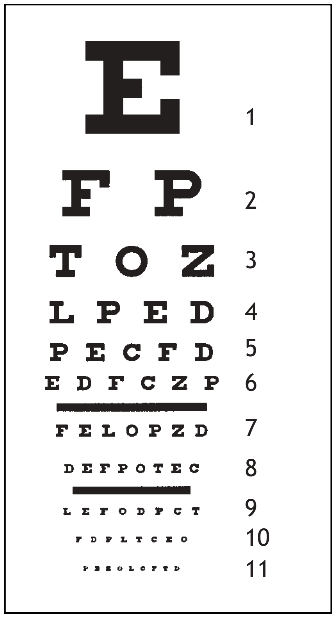
Teaching Suggestions

This is a very practical, consumer-oriented activity. It demonstrates the use of numbers and measurement in everyday life. For students who have never experienced a vision examination, this could be explained as described on the blackline master.

Solutions

1. "Normal" describes the vision of most people who can read a certain line on the chart without corrective lenses.
2. It means that Joan (standing 20 feet away) can read only those lines on the chart that normal eyes can read at 35 feet. Therefore, Joan's vision is not as good as normal vision.
3. Prasad's vision is better than normal, i.e., he can read at 20 feet what normal eyes can only read at 15 feet. Hence, Winnie must stand at 15 feet.
4. Heather's glasses would require no correction for her left eye, but would need some correction (magnification) in the right lens to bring vision in that eye to normal. If her left eye were 20/15, correction would be required to bring it into normal range.

Blackline Master: Checking Your Vision



You have probably heard that good vision is referred to as "20/20." What do these 20s mean? Actually they refer to feet, that is, 12 inches. When you look at the eye chart in a doctor's office, you stand 20 feet away. If you can read the lines of letters on the chart that normal eyes can read at 20 feet, you have 20/20 vision.

1. What do you think "normal" means in this description?

____

____

____

If you can read only those lines which normal people can see at 40 feet, you have 20/40 vision. Your vision can be checked either with or without correction (glasses or contact lenses).

2. Suppose that Joan's uncorrected vision is 20/35. What do the numbers in this expression mean? ______________________________

   ____

   Would you describe Joan's vision as better or worse than normal? ____________

   If your eyesight is better than normal you might be able to read at 20 feet what most people can only read at 12 feet. That is, they must be eight feet closer to read the same thing you can read at 20 feet. This means that you have 20/12 vision. Notice that the first number 20 never changes.

3. Prasad's vision is 20/15 and Winnie's vision is 20/20. Where must Winnie stand to read the same line on the chart that Prasad can read at 20 feet?

   In all the above measurements, only the second number applies to you. As used above, it applies when using both eyes, but the same system can be used to test each eye separately.

Blackline Master: Checking Your Vision (continued)

Suppose that Heather has 20/20 vision in her left eye and 20/30 vision in the right. What do you think Heather's glasses might be like? ______________________________________

____

What if her left eye were 20/15? ____

Since all these measurements contain the number 20, why do they not just drop this first 20? There is, in fact, another test for up-close acuity (acuity refers to your ability to see accurately). In this test, you are shown a card with lines of increasingly larger characters from 14 inches away, the distance at which most people read. Thus, normal close-up vision can be described as 14/14. If you have 14/40 vision, you can read at 14 inches what most people can read at 40 inches! Thus, the first number tells you whether the measurement stands for close-up or distance acuity.

Teacher Information: Cryptograms

Skills Required

- logical thinking
- recognition of patterns

When to Do

Anytime.

Teaching Suggestions

Cryptograms appear in the daily newspaper. To help students, start the cryptograms together using the clue provided.

Other sources for cryptograms are *Penny Press* and *Dell Puzzle* magazines. The *Puzzlemaker* website (<www.puzzlemaker.school.discover.com>) provides an opportunity for teachers (or students) to make their own cryptograms. Because the puzzles are randomly generated, be sure to print both the puzzle and the answer at the same time.

Solutions

1. I like long walks especially when they are taken by people who annoy me.
   — Fred Allen

2. If your parents didn't have any children, there's a good chance that you won't have any.
   — Clarence Day

Extension

Have students create their own cryptograms.

Blackline Master: Cryptograms

Cryptograms are puzzles that appear in many Canadian newspapers. Cryptograms are expressions in which each letter of the expression is replaced by another letter of the alphabet. The original punctuation and spacing between words remain the same. Cryptograms date back to 1841, when Edgar Allan Poe wrote an article titled "Secret Writing" for a magazine of the day.

This activity consists of two cryptograms. Solve them. In each cryptogram, one word is given to help you get started. At the bottom of the cryptogram are the letters of the alphabet. You can use them as you solve the cryptogram. Use pencil when solving the cryptograms in case you have to erase.

Z GZQC GASD RBGQE CEHCWZBGGN RUCS LUCN

BTC LBQCS FN HCAHGC RUA BSSAN YC.

–ITCX BGGCS

One of the words in the cryptogram is "annoy."

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

OP CBHT JUTSVZF YOYV'Z DUMS UVC QDORYTSV,

ZDSTS'F U NBBY QDUVQS ZDUZ CBH EBV'Z DUMS

UVC.

–QRUTSVQS YUC

One of the words in the cryptogram is "didn't."

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Appendix II

Additional Resources

Print

The Diagram Group. *The Little Giant Encyclopedia of Games for 1 or 2*. Sterling Publishing Company Inc. ISBN 0-8069-0981-1.

Fleisher, Paul. *Brain Food: Games That Make Kids Think*. Zephyr Press. ISBN 1-56976-072-1.

Nasht, Helen, and Dorothy Masterson. *Humorous Cryptograms*. Sterling Publishing Company Inc. ISBN 0-8069-3982-6.

Tuller, Dave, and Michael Rios. *Mensa Math and Logic Puzzles*. Sterling Publishing Company Inc. ISBN 0-8069-4199-5.

Internet

There are many sites on the Internet with problems and puzzles. When searching for problems and puzzles, use the words “mathematical games.”

As of February 2002, the following sites were available:

Fun Brain

<http://www.learningnetwork.funbrain.com>

This site offers some interactive games. Some of the games could be adapted to pencil-and-paper games.

Puzzlemaker

<http://www.puzzlemaker.school.discovery.com>

This site allows teachers to create their own puzzles. One of the puzzle types is cryptograms. When using the site, be sure to print the answer with the puzzle as all puzzles are created new with each visit.

This Is Mega Mathematics

<http://www.c3.lanl.gov/mega-math/>

There are a variety of activities here. One of the activities involves map colouring. Another activity involves games with graphs.