

GENERAL AND SPECIFIC LEARNING OUTCOMES WITH ACHIEVEMENT INDICATORS BY COURSE

This section presents specific learning outcomes with corresponding achievement indicators for each of the nine courses in Grades 9 to 12 Mathematics. Achievement indicators describe the depth and scope of each specific learning outcome. The list of indicators contained in this document is not intended to be exhaustive but rather to provide teachers with examples of evidence of understanding that may be used in determining whether or not students understand a given outcome. They are not presented in any particular order and need not be explicitly addressed in the classroom. Teachers may use any number of these indicators, or they may choose to use other indicators as evidence that the desired learning has been achieved. However, students need to understand the outcomes at least to the depth indicated by the indicators. Therefore the achievement indicators are sufficient as a basis for instructional design and assessment, and will form the basis for provincial assessment as appropriate.

Grade 9 Mathematics (10F)

Grade 9 Mathematics (10F) is a foundation course to prepare students for multiple possible pathways in Grades 10 to 12. The course builds on the understandings from Kindergarten to Grade 8 Mathematics (for details, please see *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes*).

The activities that take place in the Grade 9 mathematics classroom should stem from a problem-solving approach and be based on the seven mathematical processes. Students should develop an understanding of the nature of mathematics through specific knowledge, skills, and connections among and between strands.

The general focus in most units should be to allow time for hands-on activities that promote concrete understanding of concepts.

A focus on developing problem-solving skills will enable students to move on with a deeper understanding of mathematics. The emphasis should be on “why” and not just “how”.

The learning outcomes are divided into four strands of Number; Patterns and Relations; Shape and Space; Statistics and Probability. For instructional purposes, the outcomes could be arranged into units. Outcomes from different strands could be taught in the same unit. Some outcomes may fit into multiple units and parts of the outcome could be taught in one unit while the remaining parts can be taught later. Two possible sequences of the outcomes into units with suggested time allotments follow. These are not the only possibilities but will provide some direction for teachers for the first time through the course. The time for each unit includes instructional and assessment time.

Regardless of the organization of the outcomes into units, students should constantly be looking for and be given opportunities to see connections between the various outcomes in Grade 9 Mathematics.

Assessment of Grade 9 Mathematics should be a balance of assessment *for* learning, assessment *as* learning, and assessment *of* learning. Assessment tools used in Grade 9 Mathematics should be varied and may include observation,

homework, learning conversations or interviews, summative unit essays, demonstrations, presentations, performance tasks, learning logs, projects, investigations, reflective journals, portfolios, quizzes, tests, and examinations. An appropriately prepared portfolio requires a consistent effort throughout the school term and a commitment to completing quality work on a daily basis.

Possibility 1			Possibility 2		
Unit	Outcomes	Suggested hours	Unit	Outcomes	Suggested hours
Number Sense	N3, PR3, PR4, SP4, N5, N6	25	Circle Geometry	SS1	10
Statistics	SP1, SP2, SP3	15	Number Sense	N3, N5, N6	15
Powers	N1, N2, N4	15	Polynomials	PR5, PR6, PR7	15
Linear Relations	PR1, PR2, SP4	10	Powers	N1, N2, N4	10
Circle Geometry	SS1	10	Linear Relations	PR1, PR2, PR3, PR4	20
Polynomials	PR5, PR6, PR7	15	Statistics and Probability	SP1, SP2, SP3, SP4	20
Similarity	SS2, SS3, SS4	15	Symmetry	SS5	5
Symmetry	SS5	5	Geometry	SS2, SS3, SS4	15
Total		110	Total		110

General and Specific Learning Outcomes with Achievement Indicators by Course

Grade 9

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Number

General Outcome:
Develop number sense.

Specific Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

- 9.N.1. Demonstrate an understanding of powers with integral bases (excluding base 0) and whole-number exponents by
- representing repeated multiplication using powers
 - using patterns to show that a power with an exponent of zero is equal to one
 - solving problems involving powers

[C, CN, ME, PS, R]

- Demonstrate the differences between the exponent and the base by building models of a power, such as 2^3 and 3^2 .
- Explain, using repeated multiplication, the difference between two powers in which the exponent and base are interchanged such as 10^3 and 3^{10} .
- Express a power as a repeated multiplication.
- Express a repeated multiplication as a power.
- Explain the role of parentheses in powers by evaluating a set of powers such as $[-2]^4$, $[-2^4]$, and -2^4 .
- Demonstrate in a variety of ways, that a^0 is equal to 1 for any value of a ($a \neq 0$).
- Evaluate powers with integral bases (excluding base 0) and whole-number exponents.
- Determine the sum of two powers such as $5^2 + 5^3$ or $3^2 + 2^3$ and record the process.
- Determine the difference of two powers such as $4^3 - 4^2$, and record the process.

- 9.N.2. Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole-number exponents.

[C, CN, ME, PS, R, T]

- Explain, using examples, the exponent laws of powers with integral bases (excluding base 0) and whole-number exponents: NOTE a & b are non zero integers and m & n are whole numbers.
 - $(a^m)(a^n) = a^{m+n}$
 - $a^m \div a^n = a^{m-n}, m \geq n$
 - $(a^m)^n = a^{mn}$
 - $(ab)^m = a^m b^m$
 - $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$.
- Evaluate an expression by applying the exponent laws.
- Evaluate an expression where the exponent laws do not apply.
- Identify the error(s) in a simplification of an expression involving powers.

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Number (*continued*)

General Outcome:
Develop number sense.

Specific Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

<p>9.N.3. Demonstrate an understanding of rational numbers by</p> <ul style="list-style-type: none"> ■ comparing and ordering rational numbers ■ solving problems that involve arithmetic operations on rational numbers <p>[C, CN, ME, PS, R, T, V]</p>	<ul style="list-style-type: none"> ■ Order a set of rational numbers, in fraction or decimal form, by placing them on a vertical or horizontal number line (e.g., $\frac{3}{5}$, $-0.666 \dots$, 0.5, $-\frac{5}{8}$). ■ Identify a rational number that is between two rational numbers. ■ Solve a problem involving operations on rational numbers in fraction form, decimal form, or a combination of rational forms.
<p>9.N.4. Explain and apply the order of operations, including exponents, with and without technology.</p> <p>[ME, PS, T]</p>	<ul style="list-style-type: none"> ■ Demonstrate and explain with examples, the need for a standardized order of operations. ■ Solve a problem by applying the order of operations without the use of technology and record the process. ■ Solve a problem by applying the order of operations with the use of technology and record the process. ■ Identify the error in applying the order of operations in an incorrect solution.
<p>9.N.5. Determine the square root of positive rational numbers that are perfect squares.</p> <p>[C, CN, ME, PS, R, T]</p>	<ul style="list-style-type: none"> ■ Determine whether or not a rational number is a square number and explain the reasoning. ■ Determine the square root of a positive rational number that is a perfect square. ■ Identify the error made in a calculation of a square root. ■ Determine a positive rational number given the square root of that positive rational number. ■ Explain with examples why a positive rational number has both a positive and a negative square root.
<p>9.N.6. Determine an approximate square root of positive rational numbers that are non-perfect squares.</p> <p>[C, CN, ME, PS, R, T]</p>	<ul style="list-style-type: none"> ■ Estimate the square root of a rational number that is not a perfect square using the roots of perfect squares as benchmarks. ■ Determine an approximate square root of a rational number that is not a perfect square, using technology. ■ Explain why the square root of a rational number as shown on a calculator may be an approximation. ■ Identify a number with a square root that is between two numbers.

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Patterns and Relations (Patterns)

General Outcome:
Use patterns to describe the world and solve problems.

Specific Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

9.PR.1. Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.
[C, CN, PS, R, V]

- Write an expression representing a pictorial, oral, or written pattern.
- Write a linear equation to represent a context.
- Create a context for a linear equation.
- Solve, using a linear equation, a problem that involves pictorial, oral, or written linear patterns.
- Write a linear equation representing the pattern in a table of values and verify the equation by substituting values from the table.

9.PR.2. Graph linear relations, analyze the graph, and interpolate or extrapolate to solve problems.
[C, CN, ME, PS, R, T, V]

- Describe a pattern found in a graph.
- Graph a linear relation from a table of values or from a context
- Match a contextual situation to a linear relation or graph.
- Extend a graph (extrapolate) to determine the value of an unknown element.
- Interpolate the approximate value of one variable on a graph given the value of the other variable.
- Extrapolate the approximate value of one variable from a graph given the value of the other variable.
- Solve a problem by graphing a linear relation and analyzing the graph.

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Patterns and Relations (Variables and Equations)

General Outcome:
Represent algebraic expressions in multiple ways.

Specific Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

9.PR 3. Model and solve problems using linear equations of the form

- $ax = b$
- $ax + b = c$
- $ax = b + cx$
- $a(x + b) = c$
- $ax + b = cx + d$
- $a(bx + c) = d(ex + f)$
- $\frac{a}{x} = b, x \neq 0$

where $a, b, c, d, e,$ and f are rational numbers.

[C, CN, ME, PS, V]

(It is intended that this outcome build on the prior work of solving linear equations with integral coefficients.)

- Model the solution of a linear equation using concrete or pictorial representations, and record the process.
- Model the use of preservation of equality when solving linear equations concretely, pictorially, or symbolically.
- Determine, by substitution, whether a rational number is a solution to a linear equation.
- Develop a list of strategies for solving any linear equation.
- Solve a linear equation symbolically.
- Identify and correct an error in an incorrect solution of a linear equation.
- Represent a problem using a linear equation.
- Create a context for a linear equation.
- Solve a problem using a linear equation and record the process.

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Patterns and Relations
(Variables and Equations) (*continued*)

General Outcome:
Represent algebraic expressions in multiple ways.

Specific Outcomes
It is expected that students will:

9.PR 4. Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.
[C, CN, ME, PS, R, V]

Achievement Indicators
*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

- Translate a problem into a single variable linear inequality using the symbols \geq , $>$, $<$, or \leq .
- Create a context for a linear inequality expressed graphically or symbolically.
- Determine if a rational number is a possible solution of a linear inequality.
- Generalize and apply a rule for adding or subtracting a positive or negative number to determine the solution of an inequality.
- Generalize and apply a rule for multiplying or dividing by a positive or negative number to determine the solution of an inequality.
- Solve a linear inequality algebraically and explain the process orally or in written form.
- Graph the solution of a linear inequality on a vertical or horizontal number line.
- Compare and explain the process for solving a linear equation to the process for solving a linear inequality.
- Compare and explain the solution of a linear equation to the solution of a linear inequality.
- Verify the solution of a linear inequality using substitution for multiple elements in the solution.
- Solve a problem involving a single variable linear inequality and graph the solution.

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Patterns and Relations (Variables and Equations) (*continued*)

General Outcome:
Represent algebraic expressions in multiple ways.

Specific Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

9.PR 5. Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2).
[C, CN, R, V]

- Create a concrete model or a pictorial representation for a polynomial expression.
- Write the expression for a model of a polynomial.
- Identify the variables, exponent, number of terms, and coefficients, including the constant term, of a simplified polynomial expression.
- Create a situation for a first-degree polynomial expression.
- Match equivalent polynomial expressions in simplified form.

9.PR 6. Model, record, and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially, and symbolically (limited to polynomials of degree less than or equal to 2).
[C, CN, ME, PS, R, V]

- Model addition of two polynomial expressions concretely or pictorially, and record the process symbolically.
- Model subtraction of two polynomial expressions concretely or pictorially, and record the process symbolically.
- Apply a personal strategy for addition and subtraction of polynomial expressions, and explain the process.
- Identify equivalent polynomial expressions from a set of polynomial expressions, including pictorial and symbolic representations.
- Identify the error(s) in a simplification of a polynomial expression.

9.PR 7. Model, record, and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially, and symbolically.
[C, CN, R, V]

- Model multiplication of a polynomial expression by a monomial, concretely or pictorially, and record the process symbolically.
- Model division of a polynomial expression by a monomial, concretely or pictorially, and record the process symbolically.
- Apply a personal strategy for multiplication and division of a polynomial expression by a monomial and explain the process.
- Provide examples of equivalent polynomial expressions.
- Identify the error(s) in a simplification of a polynomial expression.

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Shape and Space (Measurement)

General Outcome:
Use direct or indirect measurement to solve problems.

Specific Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

9.SS.1. Solve problems and justify the solution strategy using circle properties including

- the perpendicular from the centre of a circle to a chord bisects the chord
- the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
- the inscribed angles subtended by the same arc are congruent
- a tangent to a circle is perpendicular to the radius at the point of tangency

[C, CN, PS, R, T, V]

- Provide an example that illustrates
 - the perpendicular from the centre of a circle to a chord bisects the chord
 - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
 - the inscribed angles subtended by the same arc are congruent
 - a tangent to a circle is perpendicular to the radius at the point of tangency
- Explore the reverse relationship of the above circle properties.
- Solve a problem involving application of one or more of the circle properties.
- Determine the measure of any inscribed angle subtended by the diameter using the circle properties.
- Solve a problem involving application of one or more of the circle properties.
- Explain the relationship among the centre of a circle, a chord, and the perpendicular bisector of the chord.

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Shape and Space (3-D Objects and 2-D Shapes)

General Outcome:
Describe the characteristics of 3-D objects and 2-D shapes,
and analyze the relationships among them.

Specific Outcomes
It is expected that students will:

Achievement Indicators
*The following set of indicators **may** be used to determine whether students
have met the corresponding specific outcome.*

9.SS.2. Determine the surface area of composite 3-D objects to solve problems.
[C, CN, ME, PS, R, V]

- Determine the surface area of a concrete composite 3-D object (limited to right cylinders, right rectangular prisms, and right triangular prisms).
- Determine the area of overlap in a concrete composite 3-D object, and explain its effect on determining the surface area (limited to right cylinders, right rectangular prisms, and right triangular prisms).
- Solve a problem involving surface area.

9.SS.3. Demonstrate an understanding of similarity of polygons.
[C, CN, PS, R, V]

- Determine if the polygons in a pre-sorted set are similar and explain the reasoning.
- Draw a polygon similar to a given polygon and explain why the two are similar.
- Solve a problem using the properties of similar polygons.

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Shape and Space (Transformations)	General Outcome: Describe and analyze position and motion of objects and shapes.
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Specific Outcomes <i>It is expected that students will:</i>	Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific outcome.</i>
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<p>9.SS.4. Draw and interpret scale diagrams of 2-D shapes. [CN, R, T, V]</p>	<ul style="list-style-type: none"> ■ Identify an example in print and electronic media (e.g., newspapers, the Internet) of a scale diagram and interpret the scale factor. ■ Draw a diagram to scale that represents an enlargement or reduction of a 2-D shape. ■ Determine the scale factor for a diagram drawn to scale. ■ Determine if a diagram is proportional to the original 2-D shape and, if it is, state the scale factor. ■ Solve a problem that involves a scale diagram by applying the properties of similar polygons.
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[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Shape and Space (Transformations) <i>(continued)</i>	General Outcome: Describe and analyze position and motion of objects and shapes.
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Specific Outcomes
It is expected that students will:

Achievement Indicators
*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

9.SS.5. Demonstrate an understanding of line and rotation symmetry.
[C, CN, PS, V]

- Classify a set of 2-D shapes or designs according to the number of lines of symmetry.
- Complete a 2-D shape or design given one-half of the shape or design and a line of symmetry.
- Determine if a 2-D shape or design has rotation symmetry about the point at the centre of the shape or design and, if it does, state the order and angle of rotation.
- Rotate a 2-D shape about a vertex and draw the resulting image.
- Identify a line of symmetry or the order and angle of rotation symmetry in a tessellation.
- Identify and describe the types of symmetry created in a piece of artwork.
- Create or provide a piece of artwork that demonstrates line or rotation symmetry, and identify the line(s) of symmetry and the order and angle of rotation.

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Statistics and Probability (Data Analysis)

General Outcome:
Collect, display, and analyze data to solve problems.

Specific Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

9.SP.1. Describe the effect of

- bias
- use of language
- ethics
- cost
- time and timing
- privacy
- cultural sensitivity

on the collection of data.

[C, CN, R, T]

- Analyze a case study of data collection, and identify potential problems related to bias, use of language, ethics, cost, privacy, or cultural sensitivity, time and timing.
- Provide examples to illustrate how bias, use of language, ethics, cost, privacy, or cultural sensitivity, time and timing may influence the data.

9.SP.2. Select and defend the choice of using either a population or a sample of a population to answer a question.

[C, CN, PS, R]

- Identify whether a situation represents the use of a sample or a population.
- Provide an example of a situation in which a population may be used to answer a question and justify the choice.
- Provide an example of a question where a limitation precludes the use of a population and describe the limitation (e.g., too costly, not enough time, limited resources).
- Identify and critique an example in which a generalization from a sample of a population may or may not be valid for the population.

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Statistics and Probability (Data Analysis) (*continued*)

General Outcome:
Collect, display, and analyze data to solve problems.

Specific Outcomes

It is expected that students will:

- 9.SP.3. Develop and implement a project plan for the collection, display, and analysis of data by
- formulating a question for investigation
 - choosing a data collection method that includes social considerations
 - selecting a population or a sample
 - collecting the data
 - displaying the collected data in an appropriate manner
 - drawing conclusions to answer the question

[C, PS, R, T, V]

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

- Create a rubric to assess a project that includes the assessment of
 - a question for investigation
 - the choice of a data collection method that includes social considerations
 - the selection of a population or a sample and justifying the choice
 - the display of the collected data
 - the conclusions to answer the question
- Develop a project plan that describes
 - a question for investigation
 - the method of data collection that includes social considerations
 - the method for selecting a population or a sample
 - the method to be used for collection of the data
 - the methods for analysis and display of the data
- Complete the project according to the plan, draw conclusions, and communicate findings to an audience.
- Self-assess the completed project by applying the rubric.

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

<p>Strand: Statistics and Probability (Chance and Uncertainty)</p>	<p>General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.</p>
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<p>Specific Outcomes <i>It is expected that students will:</i></p>	<p>Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific outcome.</i></p>
<p>9.SP.4. Demonstrate an understanding of the role of probability in society. [C, CN, R, T]</p>	<ul style="list-style-type: none"> ■ Provide an example from print and electronic media (e.g., newspapers, the Internet) where probability is used. ■ Identify the assumptions associated with a given probability and explain the limitations of each assumption. ■ Explain how a single probability can be used to support opposing positions. ■ Explain, using examples, how decisions based on probability may be a combination of theoretical probability, experimental probability, and subjective judgment.

