

## GRADE 12 PRE-CALCULUS MATHEMATICS (40S)

Grade 12 Pre-Calculus Mathematics (40S) is designed for students who intend to study calculus and related mathematics as part of post-secondary education. It builds on the topics studied in Grade 11 Pre-Calculus Mathematics and provides background knowledge and skills for the study of calculus in post-secondary institutions.

The course comprises a high-level study of theoretical mathematics with an emphasis on problem solving and mental mathematics. The topics include study of transformations of functions, trigonometric functions, exponential functions, logarithmic functions, polynomial functions, radical functions, rational functions, and the binomial theorem.

Assessment of Grade 12 Pre-Calculus Mathematics should be a balance of assessment *for* learning, assessment *as* learning and assessment *of* learning. Assessment tools used in Grade 12 Pre-Calculus Mathematics should be varied and may include observation, homework, learning conversations or interviews, summative unit essays, demonstrations, presentations, performance tasks, learning logs, projects, investigations, reflective journals, portfolios, quizzes, tests,

and examinations. An appropriately prepared portfolio requires a consistent effort throughout the school term and a commitment to completing quality work on a daily basis.

The learning outcomes are divided into three topics: Trigonometry; Relations and Functions; and Permutations, Combinations, and Binomial Theorem. For instructional purposes, the outcomes could be arranged into units. Outcomes from different topics could be taught in the same unit. Some outcomes may fit into multiple units and parts of the outcome could be taught in one unit while the remaining parts can be taught later. Two possible sequences of the outcomes into units with suggested time allotments follow. The suggested times include time for instruction and assessment. These are not the only possibilities but will provide some direction for teachers for the first time through the course.

Regardless of the organization of the outcomes into units, students should constantly be looking for and be given opportunities to see connections between the various outcomes in Grade 12 Pre-Calculus Mathematics.

| Possibility 1                          |                        |                 | Possibility 2                 |                        |                 |
|--|------------------------|-----------------|-------------------------------|------------------------|-----------------|
| Unit                                   | Outcomes               | Suggested hours | Unit                          | Outcomes               | Suggested hours |
| Transformations of Functions           | R1, R2, R3, R4, R5, R6 | 25              | Functions                     | R11, R12, R13, R14     | 25              |
| Trigonometric Functions                | T1, T2, T3, T4         | 25              | Transformations of Functions  | R1, R2, R3, R4, R5, R6 | 25              |
| Binomial Theorem                       | P1, P2, P3, P4         | 10              | Trigonometric Functions       | T1, T2, T3, T4, T5     | 28              |
| Polynomial Functions                   | R11, R12               | 12              | Permutations and Combinations | P1, P2, P3             | 5               |
| Trigonometric Equations and Identities | T5, T6                 | 15              | Exponents and Logarithms      | R7, R8, R9, R10        | 10              |
| Exponents and Logarithms               | R7, R8, R9, R10        | 10              | Trigonometric Identities      | T6                     | 12              |
| Radicals and Rationals                 | R13, R14               | 13              | Binomial Theorem              | P4                     | 5               |
|  | <b>Total</b>           | <b>110</b>      |                               | <b>Total</b>           | <b>110</b>      |

# General and Specific Learning Outcomes with Achievement Indicators by Course

## Grade 12 Pre-Calculus Mathematics

|  |                      |
|--|----------------------|
| [C] Communication                      | [PS] Problem Solving |
| [CN] Connections                       | [R] Reasoning        |
| [ME] Mental Mathematics and Estimation | [T] Technology       |
|  | [V] Visualization    |

### Trigonometry

**General Outcome:**  
Develop trigonometric reasoning.

#### Specific Outcomes

*It is expected that students will:*

#### Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

12P.T.1. Demonstrate an understanding of angles in standard position, expressed in degrees and radians.

[C, CN, ME, R, V]

- Sketch, in standard position, an angle (positive or negative) when the measure is given in degrees.
- Describe the relationship among different systems of angle measurement, with emphasis on radians and degrees.
- Sketch, in standard position, an angle with a measure of 1 radian.
- Sketch, in standard position, an angle with a measure expressed in the form  $k\pi$  radians, where  $k \in \mathbb{Q}$ .
- Express the measure of an angle in radians (exact value or decimal approximation), given its measure in degrees.
- Express the measure of an angle in degrees, given its measure in radians (exact value or decimal approximation).
- Determine the measures, in degrees or radians, of all angles in a given domain that are coterminal with an angle in standard position.
- Determine the general form of the measures, in degrees or radians, of all angles that are coterminal with an angle in standard position.
- Explain the relationship between the radian measure of an angle in standard position and the length of the arc cut on a circle of radius  $r$ , and solve a problem based upon that relationship.

## Grade 12 Pre-Calculus Mathematics

|   |                             |
|---|-----------------------------|
| <b>[C]</b> Communication                      | <b>[PS]</b> Problem Solving |
| <b>[CN]</b> Connections                       | <b>[R]</b> Reasoning        |
| <b>[ME]</b> Mental Mathematics and Estimation | <b>[T]</b> Technology       |
|   | <b>[V]</b> Visualization    |

### Trigonometry (*continued*)

**General Outcome:**  
Develop trigonometric reasoning.

#### Specific Outcomes

*It is expected that students will:*

#### Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

12P.T.2. Develop and apply the equation of the unit circle.

[CN, R, V]

- Derive the equation of the unit circle from the Pythagorean theorem.
- Describe the six trigonometric ratios, using a point P ( $x, y$ ) that is the intersection of the terminal arm of an angle and the unit circle.
- Generalize the equation of a circle with centre (0, 0) and radius  $r$ .

12P.T.3. Solve problems, using the six trigonometric ratios for angles expressed in radians and degrees.

[C, ME, PS, R, T, V]

- Determine, with technology, the approximate value of a trigonometric ratio for any angle with a measure expressed in either degrees or radians.
- Determine, using the unit circle or reference triangle, the exact value of a trigonometric ratio for angles expressed in degrees that are multiples of  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ , or  $90^\circ$ , or for angles expressed in radians that are multiples of  $0$ ,  $\frac{\pi}{6}$ ,  $\frac{\pi}{4}$ ,  $\frac{\pi}{3}$ , or  $\frac{\pi}{2}$ , and explain the strategy.
- Determine, with or without technology, the measures, in degrees or radians, of the angles in a specified domain, given the value of a trigonometric ratio.
- Explain how to determine the exact values of the six trigonometric ratios, given the coordinates of a point on the terminal arm of an angle in standard position.
- Determine the measures of the angles in a specified domain in degrees or radians, given a point on the terminal arm of an angle in standard position.
- Determine the exact values of the other trigonometric ratios, given the value of one trigonometric ratio in a specified domain.
- Sketch a diagram to represent a problem that involves trigonometric ratios.
- Solve a problem, using trigonometric ratios.

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|  |                             |
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| <b>[ME]</b> Mental Mathematics<br>and Estimation | <b>[T]</b> Technology       |
|  | <b>[V]</b> Visualization    |

### Trigonometry (*continued*)

**General Outcome:**  
Develop trigonometric reasoning.

#### Specific Outcomes

*It is expected that students will:*

#### Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

12P.T.4. Graph and analyze the trigonometric functions sine, cosine, and tangent to solve problems.

[C, CN, PS, T, V]

- Sketch, with or without technology, the graph of  $y = \sin x$ ,  $y = \cos x$  or  $y = \tan x$ .
- Determine the characteristics (amplitude, asymptotes, domain, period, range and zeros) of the graph of  $y = \sin x$ ,  $y = \cos x$  or  $y = \tan x$ .
- Determine how varying the value of  $a$  affects the graphs of  $y = a \sin x$  or  $y = a \cos x$ .
- Determine how varying the value of  $d$  affects the graphs of  $y = \sin x + d$  or  $y = \cos x + d$ .
- Determine how varying the value of  $c$  affects the graphs of  $y = \sin(x - c)$  or  $y = \cos(x - c)$ .
- Determine how varying the value of  $b$  affects the graphs of  $y = \sin bx$  or  $y = \cos bx$ .
- Sketch, without technology, graphs of the form  $y = a \sin b(x - c) + d$  or  $y = a \cos b(x - c) + d$ , using transformations, and explain the strategies.
- Determine the characteristics (amplitude, asymptotes, domain, period, phase shift, range, and zeros) of the graph of a trigonometric function of the form  $y = a \sin b(x - c) + d$  or  $y = a \cos b(x - c) + d$ .
- Determine the values of  $a$ ,  $b$ ,  $c$  and  $d$  for functions of the form  $y = a \sin b(x - c) + d$  or  $y = a \cos b(x - c) + d$  that correspond to a graph, and write the equation of the function.
- Determine a trigonometric function that models a context to solve a problem.
- Explain how the characteristics of the graph of a trigonometric function relate to the conditions in a problem context.
- Solve a problem by analyzing the graph of a trigonometric function.

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|   |                             |
|---|-----------------------------|
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| <b>[CN]</b> Connections                       | <b>[R]</b> Reasoning        |
| <b>[ME]</b> Mental Mathematics and Estimation | <b>[T]</b> Technology       |
|   | <b>[V]</b> Visualization    |

|                                   |   |
|-----------------------------------|---|
| Trigonometry ( <i>continued</i> ) | <b>General Outcome:</b><br>Develop trigonometric reasoning. |
|-----------------------------------|---|

### Specific Outcomes

*It is expected that students will:*

### Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

12P.T.5. Solve, algebraically and graphically, first and second degree trigonometric equations with the domain expressed in degrees and radians.

[C, CN, PS, R, T, V]

- Verify, with or without technology, that a value is a solution to a trigonometric equation.
- Determine, algebraically, the solution of a trigonometric equation, stating the solution in exact form when possible.
- Determine, using technology, the approximate solution of a trigonometric equation in a restricted domain.
- Relate the general solution of a trigonometric equation to the zeros of the corresponding trigonometric function.
- Determine, using technology, the general solution of a trigonometric equation.
- Identify and correct errors in a solution for a trigonometric equation.
- Use identities to simplify and solve a trigonometric equation.

12P.T.6. Prove trigonometric identities, using

- reciprocal identities
- quotient identities
- Pythagorean identities
- sum or difference identities (restricted to sine, cosine, and tangent)
- double-angle identities (restricted to sine, cosine, and tangent)

[C, R, T, V]

- Explain the difference between a trigonometric identity and a trigonometric equation.
- Verify a trigonometric identity numerically for a given value in either degrees or radians.
- Explain why verifying that the two sides of a trigonometric identity are equal for given values is insufficient to conclude that the identity is valid.
- Determine, graphically, the potential validity of a trigonometric identity, using technology.
- Determine the non-permissible values of a trigonometric identity.
- Prove a trigonometric identity algebraically.
- Determine, using the sum, difference or double-angle identities, the exact value of a trigonometric ratio.

## Grade 12 Pre-Calculus Mathematics

|  |                             |
|--|-----------------------------|
| <b>[C]</b> Communication                         | <b>[PS]</b> Problem Solving |
| <b>[CN]</b> Connections                          | <b>[R]</b> Reasoning        |
| <b>[ME]</b> Mental Mathematics<br>and Estimation | <b>[T]</b> Technology       |
|  | <b>[V]</b> Visualization    |

### Relations and Functions

#### General Outcome:

Develop algebraic and graphical reasoning through the study of relations.

#### Specific Outcomes

*It is expected that students will:*

#### Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

12P.R.1. Demonstrate an understanding of operations on, and compositions of, functions.

[CN, R, T, V]

- Sketch the graph of a function that is the sum, difference, product or quotient of two functions, given their graphs.
- Write the equation of a function that is the sum, difference, product or quotient of two or more functions, given their equations.
- Determine the domain and range of a function that is the sum, difference, product or quotient of two functions.
- Write a function  $f(x)$  as the sum, difference, product or quotient of two or more functions.
- Determine the value of the composition of functions when evaluated at a point using the forms  $f(f(a))$ ,  $f(g(a))$ , or  $g(f(a))$ .
- Determine, given the equations of two functions  $f(x)$  and  $g(x)$ , the equation of the composite function of the forms  $f(f(x))$ ,  $f(g(x))$ , or  $g(f(x))$  and explain any restrictions.
- Sketch, given the equations of two functions  $f(x)$  and  $g(x)$ , the graph of the composite function in the forms  $f(f(x))$ ,  $f(g(x))$ , or  $g(f(x))$ .
- Sketch the graph of the function  $y = |f(x)|$  or  $y = \frac{1}{f(x)}$  given the graph of  $y = f(x)$ , and explain the strategies used.
- Write a function  $f(x)$  as the composition of two or more functions.
- Write a function  $f(x)$  by combining two or more functions through operations on, or compositions of, functions.

|   |                             |
|---|-----------------------------|
| <b>[C]</b> Communication                      | <b>[PS]</b> Problem Solving |
| <b>[CN]</b> Connections                       | <b>[R]</b> Reasoning        |
| <b>[ME]</b> Mental Mathematics and Estimation | <b>[T]</b> Technology       |
|   | <b>[V]</b> Visualization    |

Relations and Functions (*continued*) **General Outcome:**  
Develop algebraic and graphical reasoning through the study of relations.

**Specific Outcomes**  
*It is expected that students will:*

**Achievement Indicators**  
*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

12P.R.2. Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations.  
[C, CN, R, V]

- Compare the graphs of a set of functions of the form  $y - k = f(x)$  to the graph of  $y = f(x)$ , and generalize, using inductive reasoning, a rule about the effect of  $k$ .
- Compare the graphs of a set of functions of the form  $y = f(x - h)$  to the graph of  $y = f(x)$ , and generalize, using inductive reasoning, a rule about the effect of  $h$ .
- Compare the graphs of a set of functions of the form  $y - k = f(x - h)$  to the graph of  $y = f(x)$ , and generalize, using inductive reasoning, a rule about the effects of  $h$  and  $k$ .
- Sketch the graph of  $y - k = f(x)$ ,  $y = f(x - h)$  or  $y - k = f(x - h)$  for values of  $h$  and  $k$ , given a sketch of the function  $y = f(x)$ , where the equation of  $y = f(x)$  is not given.
- Write the equation of a function whose graph is a vertical or horizontal translation of the graph of the function.

## Grade 12 Pre-Calculus Mathematics

|  |                             |
|--|-----------------------------|
| <b>[C]</b> Communication                         | <b>[PS]</b> Problem Solving |
| <b>[CN]</b> Connections                          | <b>[R]</b> Reasoning        |
| <b>[ME]</b> Mental Mathematics<br>and Estimation | <b>[T]</b> Technology       |
|  | <b>[V]</b> Visualization    |

### Relations and Functions (*continued*)

#### General Outcome:

Develop algebraic and graphical reasoning through the study of relations.

#### Specific Outcomes

*It is expected that students will:*

#### Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

12P.R.3. Demonstrate an understanding of the effects of horizontal and vertical compressions and stretches on the graphs of functions and their related equations.

[C, CN, R, V]

*Note: Resources will use the vocabulary of "stretches" and "compressions" differently. A stretch of factor  $\frac{1}{a}$  is the same as a compression of factor,  $a$ .*

- Compare the graphs of a set of functions of the form  $y = af(x)$  to the graph of  $y = f(x)$ , and generalize, using inductive reasoning, a rule about the effect of  $a$ .
- Compare the graphs of a set of functions of the form  $y = f(bx)$  to the graph of  $y = f(x)$ , and generalize, using inductive reasoning, a rule about the effect of  $b$ .
- Compare the graphs of a set of functions of the form  $y = af(bx)$  to the graph of  $y = f(x)$ , and generalize, using inductive reasoning, a rule about the effects of  $a$  and  $b$ .
- Sketch the graph of  $y = af(x)$ ,  $y = f(bx)$  or  $y = af(bx)$  for values of  $a$  and  $b$ , given a sketch of the function  $y = f(x)$ , where the equation of  $y = f(x)$  is not given.
- Write the equation of a function, given its graph which is a vertical or horizontal compression or stretch of the graph of the function  $y = f(x)$ .

12P.R.4. Apply translations, compressions and stretches to the graphs and equations of functions.

[C, CN, R, V]

- Sketch the graph of the function  $y - k = af(b(x - h))$  for values of  $a$ ,  $b$ ,  $h$ , and  $k$ , given the graph of the function  $y = f(x)$ , where the equation of  $y = f(x)$  is not given.
- Write the equation of a function, given its graph which is a translation, compression or stretch of the graph of the function  $y = f(x)$ .

## Grade 12 Pre-Calculus Mathematics

|  |                             |
|--|-----------------------------|
| <b>[C]</b> Communication                         | <b>[PS]</b> Problem Solving |
| <b>[CN]</b> Connections                          | <b>[R]</b> Reasoning        |
| <b>[ME]</b> Mental Mathematics<br>and Estimation | <b>[T]</b> Technology       |
|  | <b>[V]</b> Visualization    |

### Relations and Functions (*continued*)

#### General Outcome:

Develop algebraic and graphical reasoning through the study of relations.

#### Specific Outcomes

*It is expected that students will:*

#### Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

12P.R.5. Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections through the

- x-axis
- y-axis
- line  $y = x$

[C, CN, R, V]

- Generalize the relationship between the coordinates of an ordered pair and the coordinates of the corresponding ordered pair that results from a reflection through the  $x$ -axis, the  $y$ -axis or the line  $y = x$ .
- Sketch the reflection of the graph of a function  $y = f(x)$  through the  $x$ -axis, the  $y$ -axis or the line  $y = x$ , given the graph of the function  $y = f(x)$ , where the equation of  $y = f(x)$  is not given.
- Generalize, using inductive reasoning, and explain rules for the reflection of the graph of the function  $y = f(x)$  through the  $x$ -axis, the  $y$ -axis or the line  $y = x$ .
- Sketch the graphs of the functions  $y = -f(x)$ ,  $y = f(-x)$  and  $y = f^{-1}(x)$  given the graph of the function  $y = f(x)$ , where the equation of  $y = f(x)$  is not given.
- Write the equation of a function, given its graph which is a reflection of the graph of the function through the  $x$ -axis, the  $y$ -axis or the line  $y = x$ .

12P.R.6. Demonstrate an understanding of inverses of relations.

[C, CN, R, V]

- Explain how the graph of the line can be used to sketch the inverse of a relation.
- Explain how the transformation  $(x, y) \Rightarrow (y, x)$  can be used to sketch the inverse of a relation.
- Sketch the graph of the inverse relation, given the graph of a relation.
- Determine if a relation and its inverse are functions.
- Determine restrictions on the domain of a function in order for its inverse to be a function.
- Determine the equation and sketch the graph of the inverse relation, given the equation of a linear or quadratic relation.
- Explain the relationship between the domains and ranges of a relation and its inverse.
- Determine, algebraically or graphically, if two functions are inverses of each other.

## Grade 12 Pre-Calculus Mathematics

|  |                             |
|--|-----------------------------|
| <b>[C]</b> Communication                         | <b>[PS]</b> Problem Solving |
| <b>[CN]</b> Connections                          | <b>[R]</b> Reasoning        |
| <b>[ME]</b> Mental Mathematics<br>and Estimation | <b>[T]</b> Technology       |
|  | <b>[V]</b> Visualization    |

### Relations and Functions (*continued*)

#### **General Outcome:**

Develop algebraic and graphical reasoning through the study of relations.

#### **Specific Outcomes**

*It is expected that students will:*

#### **Achievement Indicators**

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

12P.R.7. Demonstrate an understanding of logarithms.

[C, CN, ME, R]

- Explain the relationship between logarithms and exponents.
- Express a logarithmic expression as an exponential expression and vice versa.
- Determine, without technology, the exact value of a logarithm.
- Estimate the value of a logarithm, using benchmarks, and explain the reasoning.

12P.R.8. Demonstrate an understanding of the product, quotient, and power laws of logarithms.

[C, CN, R, T]

- Develop and generalize the laws for logarithms, using numeric examples and exponent laws.
- Prove each law of logarithms.
- Determine, using the laws of logarithms, an equivalent expression for a logarithmic expression.
- Determine, with technology, the approximate value of a logarithmic expression.

## Grade 12 Pre-Calculus Mathematics

|  |                             |
|--|-----------------------------|
| <b>[C]</b> Communication                         | <b>[PS]</b> Problem Solving |
| <b>[CN]</b> Connections                          | <b>[R]</b> Reasoning        |
| <b>[ME]</b> Mental Mathematics<br>and Estimation | <b>[T]</b> Technology       |
|  | <b>[V]</b> Visualization    |

### Relations and Functions (*continued*)

#### General Outcome:

Develop algebraic and graphical reasoning through the study of relations.

#### Specific Outcomes

*It is expected that students will:*

#### Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

12P.R.9. Graph and analyze exponential and logarithmic functions.

[C, CN, T, V]

- Sketch, with or without technology, a graph of an exponential function of the form  $y = a^x$ ,  $a > 0$ .
- Identify the characteristics of the graph of an exponential function of the form  $y = a^x$ ,  $a > 0$ , including the domain, range, horizontal asymptote and intercepts, and explain the significance of the horizontal asymptote.
- Sketch the graph of an exponential function by applying a set of transformations to the graph of  $y = a^x$ ,  $a > 0$ , and state the characteristics of the graph.
- Sketch, with or without technology, the graph of a logarithmic function of the form  $y = \log_b x$ ,  $b > 1$ .
- Identify the characteristics of the graph of a logarithmic function of the form  $y = \log_b x$ ,  $b > 1$ , including the domain, range, vertical asymptote and intercepts, and explain the significance of the vertical asymptote.
- Sketch the graph of a logarithmic function by applying a set of transformations to the graph of  $y = \log_b x$ ,  $b > 1$ , and state the characteristics of the graph.
- Demonstrate, graphically, that a logarithmic function and an exponential function with the same base are inverses of each other.

12P.R.10. Solve problems that involve exponential and logarithmic equations.

[C, CN, PS, R]

- Determine the solution of an exponential equation in which the bases are powers of one another.
- Determine the solution of an exponential equation in which the bases are not powers of one another, using a variety of strategies.
- Determine the solution of a logarithmic equation, and verify the solution.
- Explain why a value obtained in solving a logarithmic equation may be extraneous.
- Solve a problem that involves exponential growth or decay.
- Solve a problem that involves the application of exponential equations to loans, mortgages, or investments.
- Solve a problem that involves logarithmic scales, such as the Richter scale or the pH scale.
- Solve a problem by modelling a situation with an exponential or a logarithmic equation.

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| <b>[ME]</b> Mental Mathematics<br>and Estimation | <b>[T]</b> Technology       |
|  | <b>[V]</b> Visualization    |

### Relations and Functions (*continued*)

#### General Outcome:

Develop algebraic and graphical reasoning through the study of relations.

#### Specific Outcomes

*It is expected that students will:*

#### Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

12P.R.11. Demonstrate an understanding of factoring polynomials of degree greater than 2 (limited to polynomials of degree  $\leq 5$  with integral coefficients).

[C, CN, ME]

- Explain how long division of a polynomial expression by a binomial expression of the form  $x - a, a \in I$ , is related to synthetic division.
- Divide a polynomial expression by a binomial expression of the form  $x - a, a \in I$ , using long division or synthetic division.
- Explain the relationship between the linear factors of a polynomial expression and the zeros of the corresponding polynomial function.
- Explain the relationship between the remainder when a polynomial expression is divided by  $x - a, a \in I$ , and the value of the polynomial expression at  $x = a$  (remainder theorem).
- Explain and apply the factor theorem to express a polynomial expression as a product of factors.

12P.R.12. Graph and analyze polynomial functions (limited to polynomial functions of degree  $\leq 5$ ).

[C, CN, PS, T, V]

- Identify the polynomial functions in a set of functions, and explain the reasoning.
- Explain the role of the constant term and leading coefficient in the equation of a polynomial function with respect to the graph of the function.
- Generalize rules for graphing polynomial functions of odd or even degree.
- Explain the relationship between the zeros of a polynomial function, the roots of the corresponding polynomial equation, and the  $x$ -intercepts of the graph of the polynomial function.
- Explain how the multiplicity of a zero of a polynomial function affects the graph.
- Sketch, with or without technology, the graph of a polynomial function.
- Solve a problem by modelling a context with a polynomial function and analyzing the graph of the function.

## Grade 12 Pre-Calculus Mathematics

|  |                             |
|--|-----------------------------|
| <b>[C]</b> Communication                         | <b>[PS]</b> Problem Solving |
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| <b>[ME]</b> Mental Mathematics<br>and Estimation | <b>[T]</b> Technology       |
|  | <b>[V]</b> Visualization    |

### Relations and Functions (*continued*)

#### General Outcome:

Develop algebraic and graphical reasoning through the study of relations.

#### Specific Outcomes

*It is expected that students will:*

#### Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

12P.R.13. Graph and analyze radical functions (limited to functions involving one radical).

[C, CN, R, T, V]

- Sketch the graph of the function  $y = \sqrt{x}$ , using a table of values, and state the domain and range.
- Sketch the graph of the function  $y - k = a\sqrt{b(x - h)}$  by applying transformations to the graph of the function  $y = \sqrt{x}$ , and state the domain and range.
- Sketch the graph of the function  $y = \sqrt{f(x)}$ , given the graph of the function  $y = f(x)$ , and explain the strategies used.
- Compare the domain and range of the function  $y = \sqrt{f(x)}$ , to the domain and range of the function  $y = f(x)$ , and explain why their domains and ranges may differ.
- Describe the relationship between the roots of a radical equation and the  $x$ -intercepts of the graph of the corresponding radical function.
- Determine, graphically, an approximate solution of a radical equation.

12P.R.14. Graph and analyze rational functions (limited to numerators and denominators that are monomials, binomials, or trinomials).

[C, CN, R, T, V]

- Graph, with or without technology, a rational function.
- Analyze the graphs of a set of rational functions to identify common characteristics.
- Explain the behaviour of the graph of a rational function for values of the variable near a non-permissible value.
- Determine if the graph of a rational function will have an asymptote or a hole for a non-permissible value.
- Match a set of rational functions to their graphs, and explain the reasoning.
- Describe the relationship between the roots of a rational equation and the  $x$ -intercepts of the graph of the corresponding rational function.
- Determine, graphically, an approximate solution of a rational equation.

## Grade 12 Pre-Calculus Mathematics

|  |                             |
|--|-----------------------------|
| <b>[C]</b> Communication                         | <b>[PS]</b> Problem Solving |
| <b>[CN]</b> Connections                          | <b>[R]</b> Reasoning        |
| <b>[ME]</b> Mental Mathematics<br>and Estimation | <b>[T]</b> Technology       |
|  | <b>[V]</b> Visualization    |

### Permutations, Combinations and Binomial Theorem

#### General Outcome:

Develop algebraic and numeric reasoning that involves combinatorics.

#### Specific Outcomes

*It is expected that students will:*

#### Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

12P.P.1 Apply the fundamental counting principle to solve problems.

[C, CN, PS, R, V]

- Count the total number of items in the sample space, using graphic organizers such as lists and tree diagrams.
- Explain, using examples, why the total number of items is found by multiplying rather than adding the number of ways the individual choices can be made.
- Solve a simple counting problem by applying the fundamental counting principle.

12P.P.2. Determine the number of permutations of  $n$  elements taken  $r$  at a time to solve problems.

[C, PS, R, V]

- Count, using graphic organizers such as lists and tree diagrams, the number of ways of arranging the elements of a set in a row.
- Determine, in factorial notation, the number of permutations of  $n$  different elements taken  $n$  at a time to solve a problem.
- Determine, using a variety of strategies, the number of permutations of  $n$  different elements taken  $r$  at a time to solve a problem.
- Explain why  $n$  must be greater than or equal to  $r$  in the notation  ${}_n P_r$ .
- Solve an equation that involves  ${}_n P_r$  notation.
- Explain, using examples, the effect on the total number of permutations when two or more elements are identical.

## Grade 12 Pre-Calculus Mathematics

|  |                             |
|--|-----------------------------|
| <b>[C]</b> Communication                         | <b>[PS]</b> Problem Solving |
| <b>[CN]</b> Connections                          | <b>[R]</b> Reasoning        |
| <b>[ME]</b> Mental Mathematics<br>and Estimation | <b>[T]</b> Technology       |
|  | <b>[V]</b> Visualization    |

### Permutations, Combinations and Binomial Theorem (continued)

#### General Outcome:

Develop algebraic and numeric reasoning that involves combinatorics.

#### Specific Outcomes

*It is expected that students will:*

12P.P.3. Determine the number of combinations of  $n$  different elements taken  $r$  at a time to solve problems.

[C, PS, R, V]

#### Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

- Explain, using examples, the difference between a permutation and a combination.
- Determine the number of combinations of  $n$  different elements taken  $r$  at a time to solve a problem.
- Explain why  $n$  must be greater than or equal to  $r$  in the notation  ${}_n C_r$  or  $\binom{n}{r}$ .
- Explain, using examples, why  ${}_n C_r = {}_n C_{n-r}$  or  $\binom{n}{r} = \binom{n}{n-r}$ .
- Solve an equation that involves  ${}_n C_r$  or  $\binom{n}{r}$  notation.

## Grade 12 Pre-Calculus Mathematics

|  |                             |
|--|-----------------------------|
| <b>[C]</b> Communication                         | <b>[PS]</b> Problem Solving |
| <b>[CN]</b> Connections                          | <b>[R]</b> Reasoning        |
| <b>[ME]</b> Mental Mathematics<br>and Estimation | <b>[T]</b> Technology       |
|  | <b>[V]</b> Visualization    |

### Permutations, Combinations and Binomial Theorem (continued)

#### General Outcome:

Develop algebraic and numeric reasoning that involves combinatorics.

#### Specific Outcomes

*It is expected that students will:*

#### Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*

12P.P.4. Expand powers of a binomial in a variety of ways, including using the binomial theorem (restricted to exponents that are natural numbers).

[C, CN, R, V]

- Explain the patterns found in the expanded form of  $(x + y)^n$ ,  $n \leq 4$ , by multiplying  $n$  factors of  $(x + y)$ .
- Explain how to determine the subsequent row in Pascal's triangle, given any row.
- Relate the coefficients of the terms in the expansion of  $(x + y)^n$  to the  $(n+1)^{\text{st}}$  row in Pascal's triangle.
- Explain, using examples, how the coefficients of the terms in the expansion of  $(x + y)^n$  are determined by combinations.
- Expand, using the binomial theorem,  $(x + y)^n$ .
- Determine a specific term in the expansion of  $(x + y)^n$ .