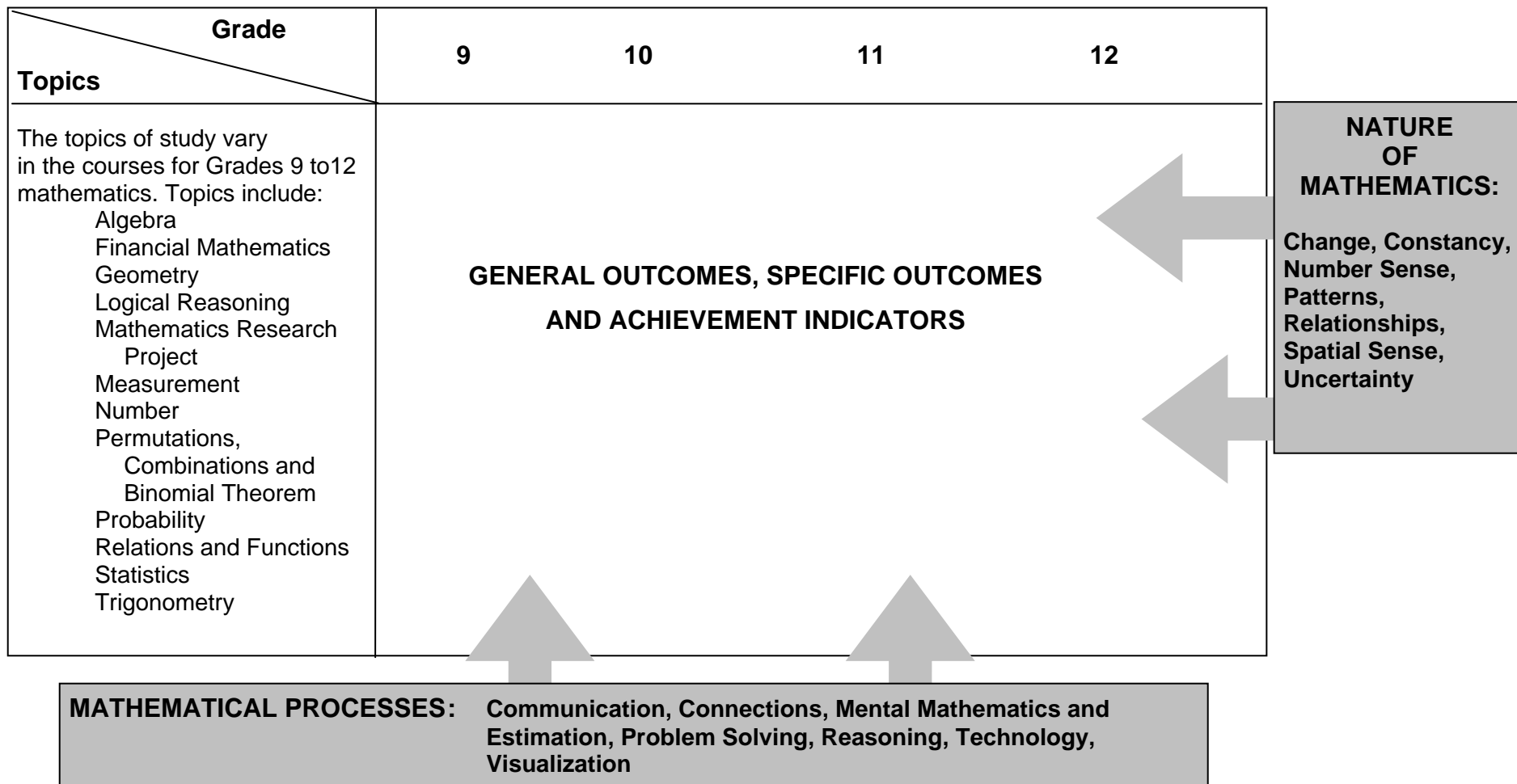


# CONCEPTUAL FRAMEWORK FOR GRADES 9 TO 12 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



## MATHEMATICAL PROCESSES

The seven mathematical processes are critical aspects of learning, doing and understanding mathematics. Students must encounter these processes regularly in a mathematics program in order to achieve the goals of mathematics education.

The common curriculum framework incorporates the following interrelated mathematical processes. It is intended that they permeate the teaching and learning of mathematics.

Students are expected to:

- *Communication [C]*
  - *Connections [CN]*
  - *Mental Mathematics and Estimation [ME]*
  - *Problem Solving [PS]*
  - *Reasoning [R]*
  - *Technology [T]*
  - *Visualization [V]*
- use *communication* in order to learn and express their understanding
  - make *connections* among mathematical ideas, other concepts in mathematics, everyday experiences and other disciplines
  - demonstrate fluency with *mental mathematics and estimation*
  - develop and apply new mathematical knowledge through *problem solving*
  - develop mathematical *reasoning*
  - select and use *technology* as a tool for learning and solving problems
  - develop *visualization* skills to assist in processing information, making connections and solving problems.

All seven processes should be used in the teaching and learning of mathematics. Each specific outcome includes a list of relevant mathematical processes. The identified processes are to be used as a primary focus of instruction and assessment.

### Communication [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links among their own language and ideas, the language and ideas of others, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can play a significant role in helping students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

Emerging technologies enable students to engage in communication beyond the traditional classroom to gather data and share mathematical ideas.

*Students must be able to communicate mathematical ideas in a variety of ways and contexts.*

## Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. “Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding... Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching” (Caine and Caine, 1991, p. 5).

*Through connections, students begin to view mathematics as useful and relevant.*

## Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves using strategies to perform mental calculations.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility in reasoning and calculating.

“Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math” (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein, 2001, p. 442).

Mental mathematics “provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and non-standard techniques for finding answers” (Hope, 1988, p. v).

Estimation is used for determining approximate values or quantities, usually by referring to benchmarks or referents, or for determining the reasonableness of calculated values. Estimation is also used to make mathematical judgements and to develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to learn which strategy to use and how to use it.

*Mental mathematics and estimation are fundamental components of number sense.*

## Problem Solving [PS]

Problem solving is one of the key processes and foundations within the field of mathematics. Learning through problem solving should be the focus of mathematics at all grade levels. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts. Problem solving is to be employed throughout all of mathematics and should be embedded throughout all the topics.

*Learning through problem solving should be the focus of mathematics at all grade levels.*

When students encounter new situations and respond to questions of the type, *How would you ...?* or *How could you ...?*, the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. Students should not know the answer immediately. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement. Students will be engaged if the problems relate to their lives, cultures, interests, families or current events.

Both conceptual understanding and student engagement are fundamental in moulding students' willingness to persevere in future problem-solving tasks.

Problems are not just simple computations embedded in a story, nor are they contrived. They are tasks that are rich and open-ended, so there may be more than one way of arriving at a solution or there may be multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill or understanding. Problem solving can vary from being an individual activity to a class (or beyond) undertaking.

In a mathematics class, there are two distinct types of problem solving: solving contextual problems outside of mathematics and solving mathematical problems. Finding the maximum profit given manufacturing constraints is an example of a contextual problem, while seeking and developing a general formula to solve a quadratic equation is an example of a mathematical problem.

Problem solving can also be considered in terms of engaging students in both inductive and deductive reasoning strategies. As students make sense of the problem, they will be creating conjectures and looking for patterns that they may be able to generalize. This part of the problem-solving process often involves inductive reasoning. As students use approaches to solving the problem they often move into mathematical reasoning that is deductive in nature. It is crucial that students be encouraged to engage in both types of reasoning and be given the opportunity to consider the approaches and strategies used by others in solving similar problems.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk-takers.

## Reasoning [R]

*Mathematical reasoning helps students think logically and make sense of mathematics.*

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. Questions that challenge students to think, analyze and synthesize help them develop an understanding of mathematics. All students need to be challenged to answer questions such as, *Why do you believe that's true/correct?* or *What would happen if ....*

Mathematical experiences provide opportunities for students to engage in inductive and deductive reasoning. Students use inductive reasoning when they explore and record results, analyze observations, make generalizations from patterns and test these generalizations. Students use deductive reasoning when they reach new conclusions based upon the application of what is already known or assumed to be true. The thinking skills developed by focusing on reasoning can be used in daily life in a wide variety of contexts and disciplines.

## Technology [T]

Technology can be used effectively to contribute to and support the learning of a wide range of mathematical outcomes. Technology enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- generate and test inductive conjectures
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- increase the focus on conceptual understanding by decreasing the time spent on repetitive procedures
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- model situations
- develop number and spatial sense.

*The use of technology should not replace mathematical understanding.*

Technology contributes to a learning environment in which the curiosity of students can lead to rich mathematical discoveries at all grade levels. The use of technology should not replace mathematical understanding. Instead, technology should be used as one of a variety of approaches and tools for creating mathematical understanding.

## **Visualization [V]**

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world” (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and spatial reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate and involves knowledge of several estimation strategies (Shaw and Cliatt, 1989, p. 150).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations. It is through visualization that abstract concepts can be understood concretely by the student. Visualization is a foundation to the development of abstract understanding, confidence and fluency.

*Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.*

## NATURE OF MATHEMATICS

- *Change*
- *Constancy*
- *Number Sense*
- *Patterns*
- *Relationships*
- *Spatial Sense*
- *Uncertainty*

Mathematics is one way of understanding, interpreting and describing our world. There are a number of characteristics that define the nature of mathematics, including change, constancy, number sense, patterns, relationships, spatial sense and uncertainty.

### Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

*Change is an integral part of mathematics and the learning of mathematics.*

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, ... can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain (Steen, 1990, p. 184).

Students need to learn that new concepts of mathematics as well as changes to already learned concepts arise from a need to describe and understand something new. Integers, decimals, fractions, irrational numbers and complex numbers emerge as students engage in exploring new situations that cannot be effectively described or analyzed using whole numbers.

Students best experience change to their understanding of mathematical concepts as a result of mathematical play.

### Constancy

Many important properties in mathematics do not change when conditions change. Examples of constancy include:

- the conservation of equality in solving equations
- the sum of the interior angles of any triangle
- the theoretical probability of an event.

*Some problems in mathematics require students to focus on properties that remain constant.*

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems such as those involving constant rates of change, lines with constant slope, or direct variation situations.

## Number Sense

Number sense, which can be thought of as deep understanding and flexibility with numbers, is the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p. 146). Continuing to foster number sense is fundamental to growth of mathematical understanding.

*Number sense can be developed by providing mathematically rich tasks that allow students to make connections.*

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Students with strong number sense are able to judge the reasonableness of a solution, describe relationships between different types of numbers, compare quantities and work with different representations of the same number to develop a deeper conceptual understanding of mathematics.

Number sense develops when students connect numbers to real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about numbers. Evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing mathematically rich tasks that allow students to make connections.

## Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all of the mathematical topics, and it is through the study of patterns that students can make strong connections between concepts in the same and different topics. Working with patterns also enables students to make connections beyond mathematics. The ability to analyze patterns contributes to how students understand their environment.

Patterns may be represented in concrete, visual, auditory or symbolic form. Students should develop fluency in moving from one representation to another.

Students need to learn to recognize, extend, create and apply mathematical patterns. This understanding of patterns allows students to make predictions and justify their reasoning when solving problems.

Learning to work with patterns helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics.

*Mathematics is about recognizing, describing and working with numerical and non-numerical patterns.*

*Mathematics is used to describe and explain relationships.*

## **Relationships**

Mathematics is used to describe and explain relationships. Within the study of mathematics, students look for relationships among numbers, sets, shapes, objects, variables and concepts. The search for possible relationships involves collecting and analyzing data, analyzing patterns and describing possible relationships visually, symbolically, orally or in written form.

## **Spatial Sense**

Spatial sense involves the representation and manipulation of 3-D objects and 2-D shapes. It enables students to reason and interpret among 3-D and 2-D representations.

*Spatial sense offers a way to interpret and reflect on the physical environment.*

Spatial sense is developed through a variety of experiences with visual and concrete models. It offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions.

Spatial sense is also critical in students' understanding of the relationship between the equations and graphs of functions and, ultimately, in understanding how both equations and graphs can be used to represent physical situations.

## **Uncertainty**

In mathematics, interpretations of data and the predictions made from data inherently lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important that students recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of an interpretation or conclusion is directly related to the quality of the data it is based upon. An awareness of uncertainty provides students with an understanding of why and how to assess the reliability of data and data interpretation.

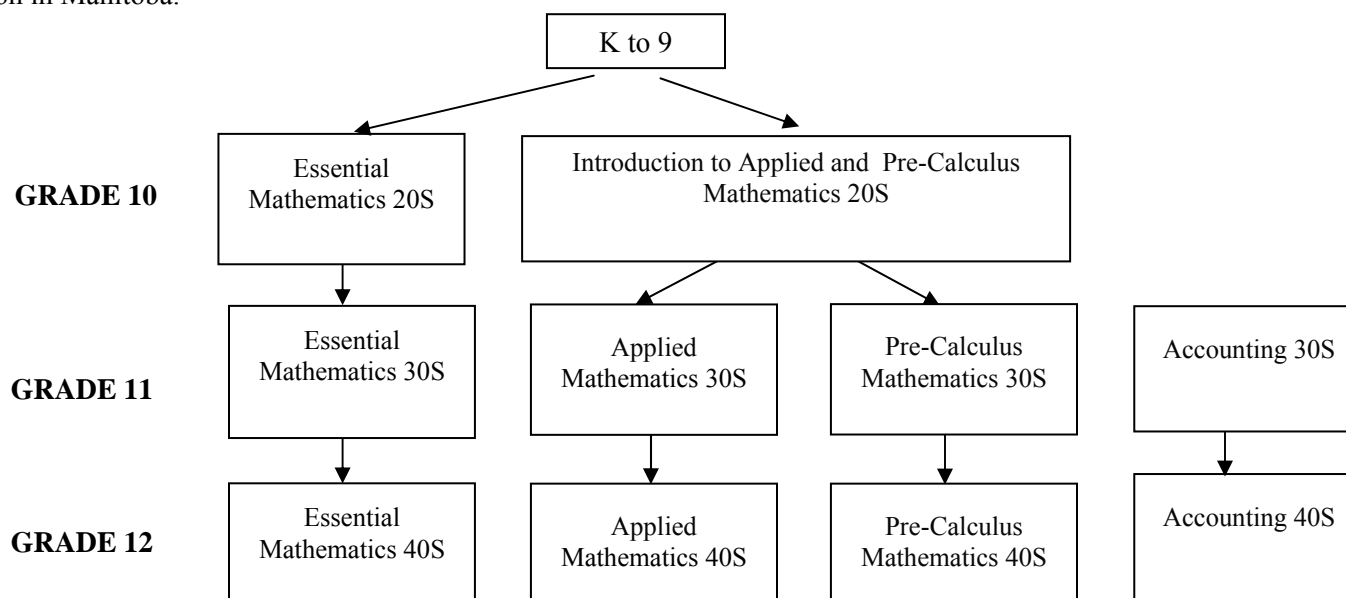
*Uncertainty is an inherent part of making predictions.*

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately. This language must be used effectively and correctly to convey valuable messages.

## PATHWAYS AND TOPICS

*Grades 9 to 12 Mathematics: Manitoba Curriculum Framework for Outcomes: Manitoba Edition—Draft (2008)* includes topics rather than strands as in *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes(2008)*. There is one course available for students in Grade 9. In Grade 10, students may choose between two courses or may choose to take both courses. In Grades 11 and 12, students have four choices for courses and may take one or multiple courses. Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings.

Note: Accounting 30S and Accounting 40S are not referenced in this framework but can be used as a mathematics credit for graduation in Manitoba.



### Goals of Pathways

The goals of all pathways are to provide prerequisite attitudes, knowledge, skills and understandings for specific post-secondary programs or direct entry into the work force. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. When choosing a pathway, students should consider their interests, both current and future. Students, parents and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

### **Design of Pathways**

Each pathway is designed to provide students with the mathematical understandings, rigour and critical-thinking skills that have been identified for specific post-secondary programs of study and for direct entry into the work force.

The content of each pathway has been based on the *Western and Northern Canadian Protocol (WNCP) Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Final Report on Findings* and on consultations with mathematics teachers.

### **Applied Mathematics**

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, logical reasoning, measurement, number, relations and functions, statistics and probability.

### **Essential Mathematics**

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into the majority of trades and for direct entry into the work force. Topics include algebra, geometry, measurement, number, statistics and probability.

### **Pre-Calculus Mathematics**

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, permutations, combinations and binomial theorem, relations and functions, and trigonometry.

## OUTCOMES AND ACHIEVEMENT INDICATORS

The common curriculum framework is stated in terms of general outcomes, specific outcomes and achievement indicators.

**General outcomes** are overarching statements about what students are expected to learn in each course. They remain consistent throughout several years of schooling.

**Specific outcomes** are statements that identify the specific knowledge, skills and understandings that students are required to attain by the end of a given course.

**Achievement indicators** are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome.

In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for clarification and are not requirements that must be addressed to fully meet the learning outcome.

The word *and* used in an outcome indicates that both ideas must be addressed to fully meet the learning outcome, although not necessarily at the same time or in the same question. The word *and* used in an achievement indicator implies that both ideas should be addressed at the same time or in the same question.

## SUMMARY

The Conceptual Framework for Grades 9 to 12 Mathematics describes the nature of mathematics, the mathematical processes, the pathways and topics, and the role of outcomes and achievement indicators in Grades 9 to 12 mathematics. Activities that take place in the mathematics classroom should be based on a problem-solving approach that incorporates the mathematical processes and leads students to an understanding of the nature of mathematics.