Grade 12
Pre-Calculus Mathematics
Standards Test

Booklet 1

June 2011

Manitoba

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Grade 12 Pre-Calculus Mathematics
Standards Test

DESCRIPTION

Total Possible Marks: 100 Time: 3 hours

Part	Description	Time	Value
Part 1	**Long Answer:** 11 questions worth a total of 36 marks Calculators (scientific or graphing) are permitted.	60 minutes	36%
Part 2	**Multiple Choice:** 15 questions worth 1 mark each **Short Answer:** 15 questions worth 1 mark each **Long Answer:** 12 questions worth a total of 34 marks Calculators are not permitted.	120 minutes	64%

GENERAL DIRECTIONS

- Read all instructions carefully.
- A page of formulas, helpful hints, and scrap paper are provided on perforated pages at the beginning of this test booklet. Please do not tear out any other pages. No marks will be given for work done on these pages.
- The blank pages at the back of each booklet may also be used as scrap paper, but must **not** be removed from the test booklet. No marks will be given for work done on these pages.
- Note that diagrams and graphs provided in the test booklets may not be drawn to scale.
- When *Booklet 2* is distributed, you may keep *Booklet 1* in case you wish to continue working on it without a calculator.

No marks will be awarded for work done on this page.

Formula Sheet: Pre-Calculus Mathematics

$$s = \theta r$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$
$$\cos 2\alpha = 1 - 2\sin^2\alpha$$
$$\cos 2\alpha = 2\cos^2\alpha - 1$$
$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$P(n,r) \text{ or } {}_nP_r = \frac{n!}{(n-r)!}$$

$$C(n,r) \text{ or } {}_nC_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_nC_k a^{n-k} b^k$$

$$\mathrm{P(A \text{ or } B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A \text{ and } B)}$$
$$\mathrm{P(A \text{ and } B)} = \mathrm{P(A)} \bullet \mathrm{P(B|A)}$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$A = Pe^{rt}$$
$$e \approx 2.718\ 28$$

$$\log_a(MN) = \log_a M + \log_a N$$
$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$
$$\log_a(M^n) = n\log_a M$$
$$\log_a M = \frac{\log_b M}{\log_b a}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \quad a > b$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, \quad a > b$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$(y-k) = a(x-h)^2$$

$$(x-h) = a(y-k)^2$$

$$t_n = t_1 r^{n-1}$$

$$S_n = \frac{t_1(1-r^n)}{1-r} = \frac{t_1(r^n-1)}{r-1}$$

$$S_n = \frac{t_1 - t_n r}{1-r} = \frac{t_n r - t_1}{r-1}$$

$$S_\infty = \frac{t_1}{1-r}, \quad |r| < 1$$

Helpful Hints

Clearly labelled graphs include:

- labels on both axes
- scales on both axes (only one number on each axis is required)
- arrowheads or endpoints to indicate if the graph continues or stops
- accurate shape (whether it is straight or curved and whether the curve opens up, down, left, or right)
- asymptotes (if applicable) drawn as non-solid lines with the graph approaching the asymptote
- intercepts labelled with numeric values (only if they are requested in the question)
- vertices and centre of conic sections accurately located according to the scale shown

Guess and check
Marks may be awarded for this method if explanations are provided and work is backed up with evidence.

Exact value solutions	
Incomplete solution	Exact value solution
$5-3$	2
$4(3)$	12
$\frac{\sqrt{2}\sqrt{3}}{4}$	$\frac{\sqrt{6}}{4}$
$\frac{1}{2}-\frac{2\sqrt{11}}{5}$	$\frac{5-4\sqrt{11}}{10}$
$\frac{\sqrt{7}}{3}-\frac{\sqrt{5}}{3}$	$\frac{\sqrt{7}-\sqrt{5}}{3}$
$\frac{1+\frac{1}{3}}{2}$	$\frac{2}{3}$
$2\left(\frac{2\sqrt{3}}{\sqrt{5}}\right)$	$\frac{4\sqrt{3}}{\sqrt{5}}$
$\frac{0.7}{1.4}$	$\frac{7}{14}$ or $\frac{1}{2}$ or 0.5

Rough work
If you clearly label work as "rough work," it will not be marked.

Expressing the answer correct to 3 decimal places means that you should not round before the final answer.

Explanations
It is not always necessary to explain in words; simply indicate or label any restrictions or chosen elements.

Scrap Paper

No marks will be awarded for work done on this page.

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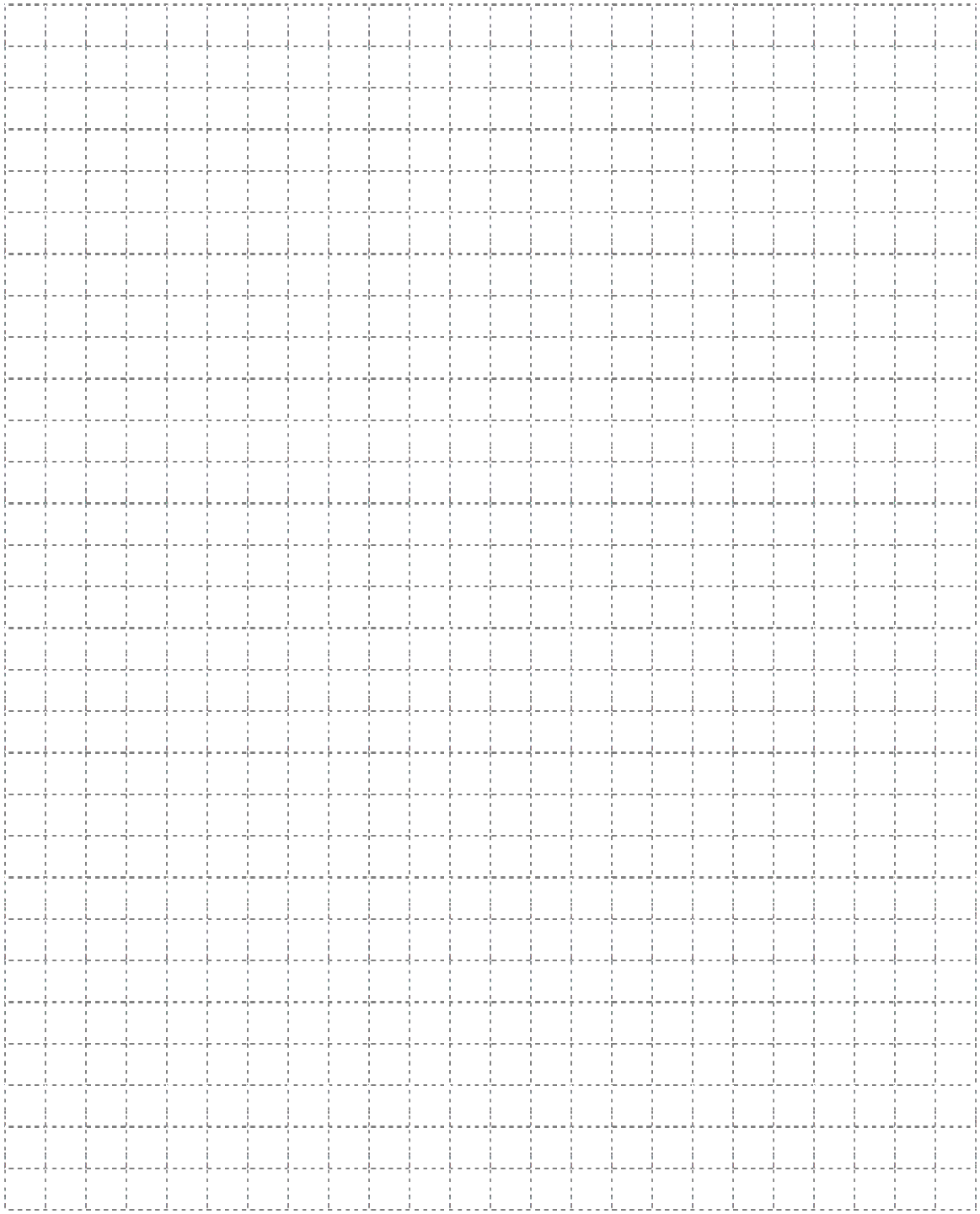
No marks will be awarded for work done on this page.



Long-Answer Questions

Instructions

- There are 11 long-answer questions for a total of 36 marks.
- Calculators (scientific or graphing) are allowed for this part of the test.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions should include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized, and clear.
- If any curve contains asymptotes, the asymptotes should be included in the graph.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to 3 decimal places unless instructed otherwise.

No marks will be awarded for work done on this page.

Long-Answer Questions

Do Not Use

(3 marks) 1. Find and simplify the 7th term in the binomial expansion of $\left(3a - 2b^2\right)^{10}$.

101

Do Not Use

(2 marks) 2. The population of Boomsville in 1990 was 20 000. Exactly 13 years later, the population had reached 32 000. The population continues to increase according to the formula:

102

$$P = P_o e^{rt}$$

where P = the final population

P_o = the initial population

r = the rate of growth

t = the time in years

Find the value of r.
Express your answer correct to 3 decimal places.

Do Not Use

(3 marks) 3. Solve the following equation where $\theta \in \mathbb{R}$.

103

$$\cos 2\theta = -\frac{3}{4}$$

Give the general solution in radian measure correct to 3 decimal places.

Do Not Use

104

(4 marks) 4. Solve algebraically:

$$7^{2x+5} = \frac{\left(5^{x}\right)}{7}$$

Express your answer correct to 3 decimal places.

Do Not Use

(1 mark) 5. a) There are 4 boys and 3 girls to be seated in a row. 105

How many arrangements are possible if Alexandra, one of the 7 people, must sit in the middle?

Express your answer as a whole number.

(2 marks) b) There are 4 boys and 3 girls to be seated in a row. 106

How many arrangements are possible if Shawn and Dave, 2 of the boys, **cannot** sit beside each other?

Express your answer as a whole number.

Briefly explain your calculations.

Do Not Use

(1 mark) 6. a) You want to select a committee of 3 girls and 2 boys from a group of 8 girls and 6 boys. 107

How many committees are possible?
Express your answer as a whole number.
Briefly explain your calculations.

(3 marks) b) You want to select a committee of 3 girls and 2 boys from a group of 8 girls and 6 boys. 108

John is one of the boys and Marie is one of the girls.
Either John or Marie must be on the committee, but not both together.

How many committees are possible?
Express your answer as a whole number.
Briefly explain your calculations.

Do Not Use

7. The probability of a sunny day is 70%.
When it is sunny, the probability of going to the beach is 90%.
When it is not sunny, the probability of going to the beach is 25%.

(2 marks) a) What is the probability of **not** going to the beach? 109
Express your answer correct to 3 decimal places.

(2 marks) b) Given that a person did **not** go to the beach, what is the probability that it was sunny? 110
Express your answer correct to 3 decimal places.

Do Not Use

8. Willy B. Rich receives a salary that increases every month according to a geometric sequence. He received \$1200 for month 2 and \$4050 for month 5 during his first year of work.

(2 marks) a) Calculate the value of r. 111

(2 marks) b) Calculate the total amount of money he earned in his first year of work. 112

Express your answer correct to 2 decimal places.

Do Not Use

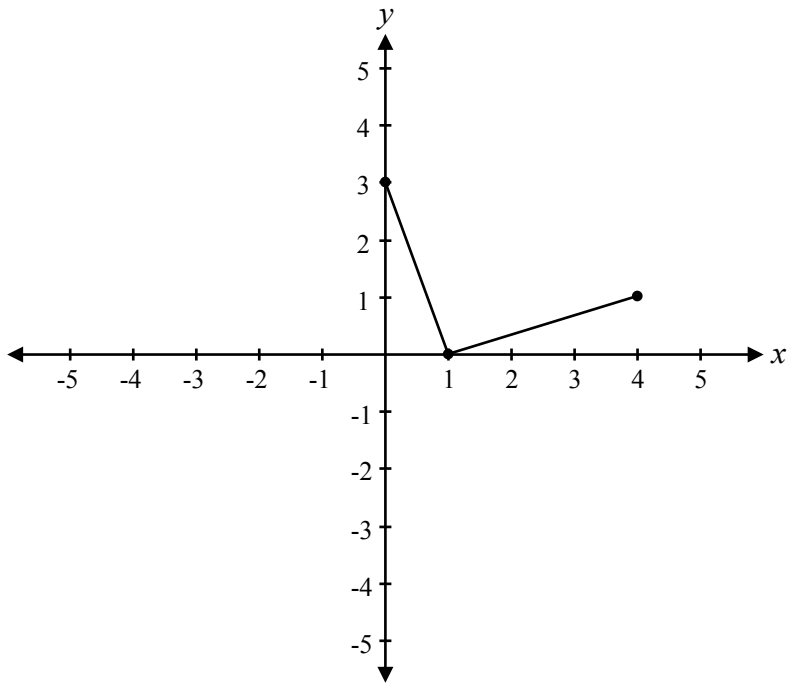
(2 marks) 9. Find the length, in centimetres, of an arc intercepted by a central angle of 50° on a circle with a radius of 36 cm.

Express your answer as an exact value or correct to 3 decimal places.

113

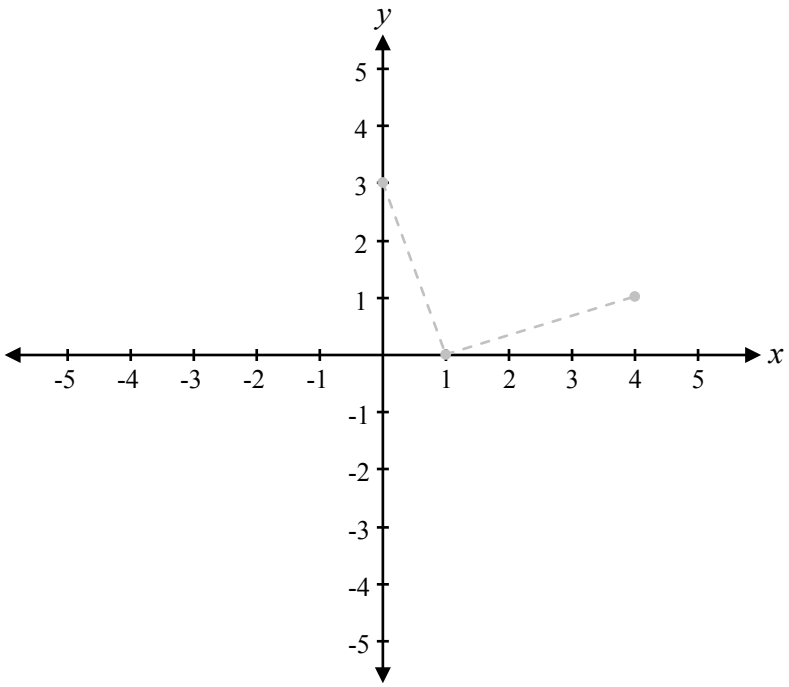
Do Not Use

10. The graph of the function $y = f(x)$ is shown below.



Sketch a clearly labelled graph of:

(1 mark) a) $y = f(x + 1)$ 114

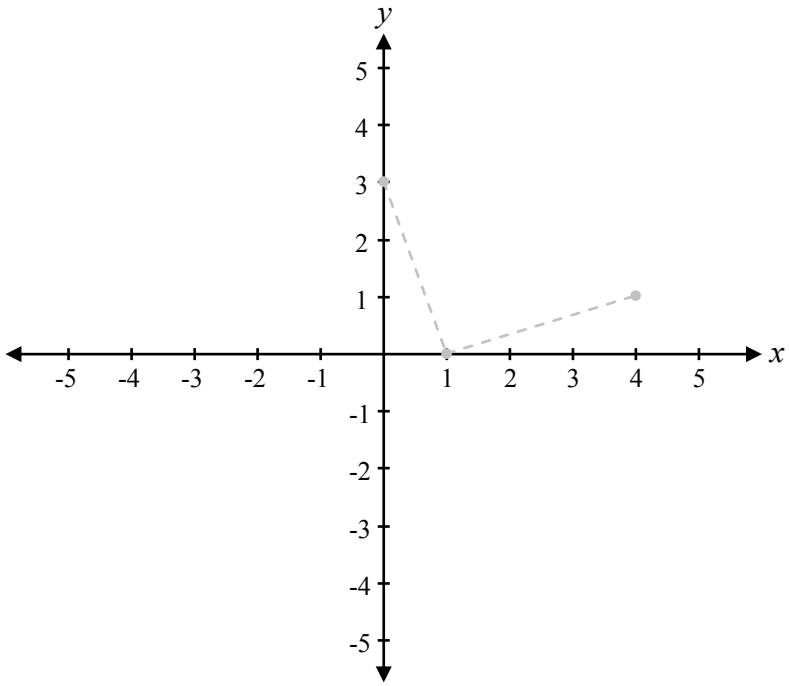


The graph of $y = f(x)$ has already been drawn for your reference.

No marks will be awarded for this graph.

Do Not Use

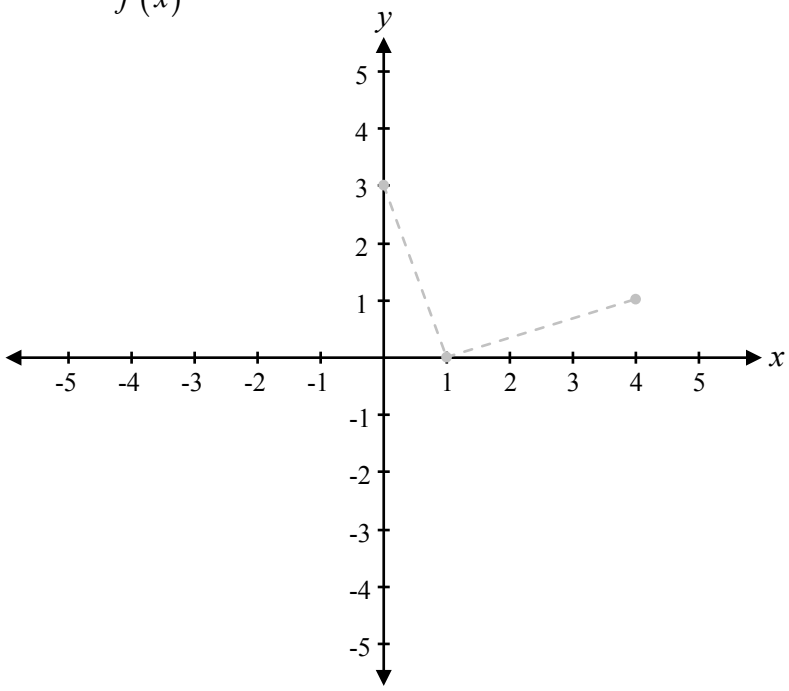
(1 mark) b) $y = f(-x)$ 115



The graph of $y = f(x)$ has already been drawn for your reference.

No marks will be awarded for this graph.

(2 marks) c) $y = \dfrac{1}{f(x)}$ 116



The graph of $y = f(x)$ has already been drawn for your reference.

No marks will be awarded for this graph.

Do Not Use

(2 marks) 11. a) Express $\frac{\csc^2\theta - 1}{1 - \sin^2\theta}$ in terms of $\csc\theta$ only. 117

Completely simplify your answer.

(1 mark) b) Determine the exact value of the expression above when $\theta = \frac{\pi}{6}$. 118

No marks will be awarded for work done on this page.

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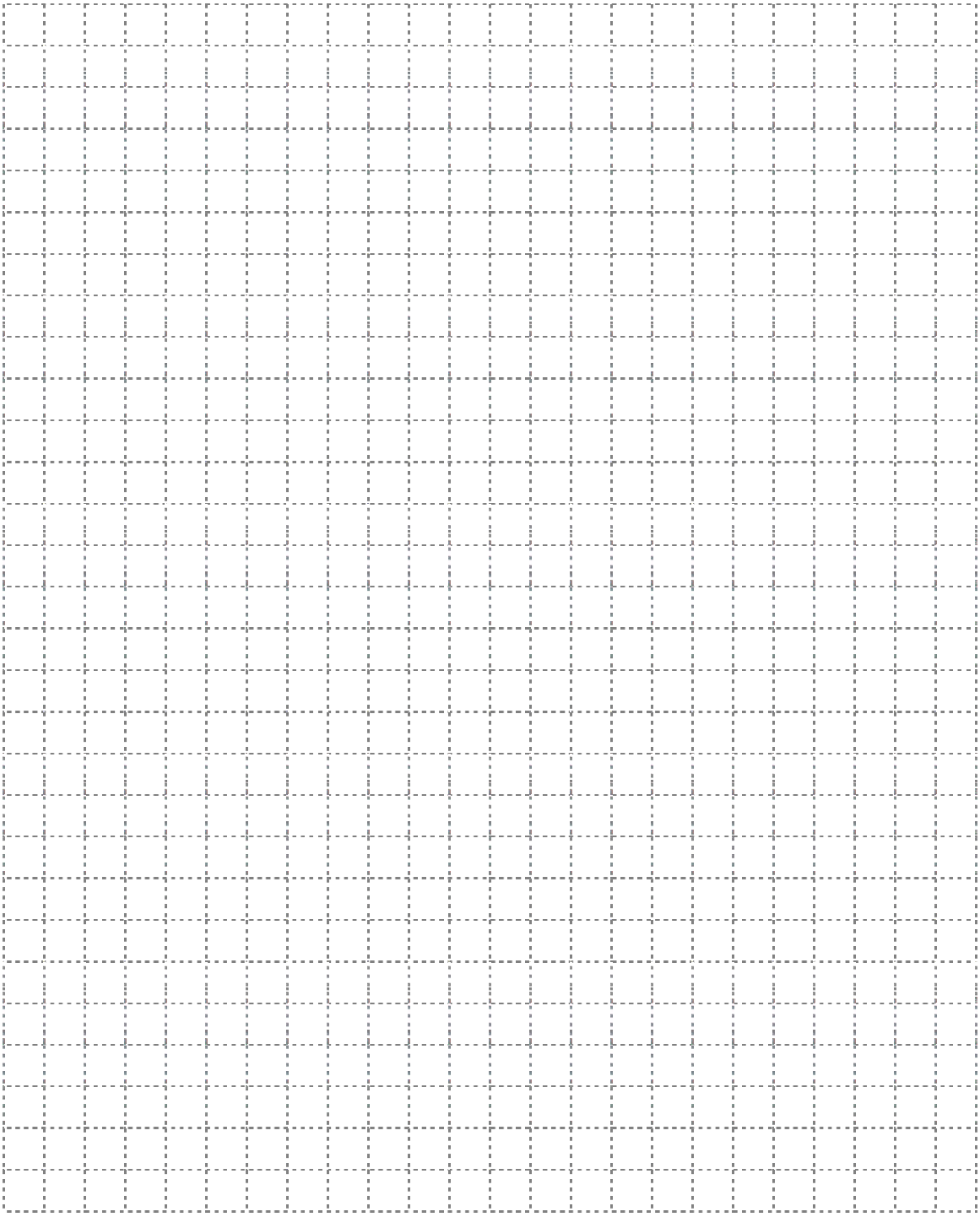

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