

Grade 12  
Pre-Calculus Mathematics  
Achievement Test

# Booklet 1

June 2014

Manitoba Education and Advanced Learning Cataloguing in Publication Data

Grade 12 pre-calculus mathematics achievement test.  
Booklet 1. June 2014 [electronic resource]

ISBN: 978-0-7711-5585-7

1. Mathematics—Examinations, questions, etc.
  2. Educational tests and measurements—Manitoba.
  3. Mathematics—Study and teaching (Secondary)—Manitoba.
  4. Calculus—Study and teaching (Secondary)—Manitoba.
  5. Mathematical ability—Testing.
- I. Manitoba. Manitoba Education and Advanced Learning.  
515.076

Manitoba Education and Advanced Learning  
School Programs Division  
Winnipeg, Manitoba, Canada

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*Disponible en français.*


Available in alternate formats upon request.

# Grade 12 Pre-Calculus Mathematics Achievement Test

## DESCRIPTION

Time: 3 hours

	Questions	Marks	Total Marks
<b>Booklet 1*</b>	16 Short-Answer	25	36
	3 Long-Answer	11	
<b>Booklet 2</b>	10 Multiple-Choice	10	54
	12 Short-Answer	18	
	8 Long-Answer	26	
<b>Total</b>			<b>90</b>


\* The first 5 questions in *Booklet 1* require a calculator.   
You will have access to your calculator for the first 45 minutes of the test.

## GENERAL DIRECTIONS

- § Read all instructions carefully.
- § The blank pages at the back of each booklet may be used as scrap paper, but must **not** be removed from the test booklet. No marks will be given for work done on these pages.
- § Note that diagrams and graphs provided in the test booklets may not be drawn to scale.
- § After 45 minutes, put away your calculator. Even though you may not have finished *Booklet 1*, *Booklet 2* will be distributed at this time. You may choose to continue working on *Booklet 1* or start working on *Booklet 2*, but you will no longer have access to your calculator.

# Instructions

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- § There are 19 questions for a total of 36 marks.
- § Calculators (scientific or graphing) are allowed for the first 45 minutes of the test.
- § A calculator icon  appears next to the questions that require a calculator.
- § Write each solution in the space provided.
- § For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- § Graphing calculator solutions must include an explanation of how your final answer is obtained.
- § Your solutions should be neat, organized, and clear.
- § Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- § Express your answers as exact values or correct to 3 decimal places unless instructed otherwise.

# Formula Sheet

$$s = \theta r$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 1 - 2 \sin^2 \alpha$$

$$\cos 2\alpha = 2 \cos^2 \alpha - 1$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\log_a (MN) = \log_a M + \log_a N$$

$$\log_a \left( \frac{M}{N} \right) = \log_a M - \log_a N$$

$$\log_a (M^n) = n \log_a M$$

$$P(n, r) \text{ or } {}_n P_r = \frac{n!}{(n-r)!}$$

$$C(n, r) \text{ or } {}_n C_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_n C_k a^{n-k} b^k$$

# Terminology Sheet

Some questions may contain directing words such as *explain*, *identify*, and *justify*. These words are explained below.

**Evaluate:** Find the numerical value.

**Explain:** Use words to provide the cause or reason for the response, or to render the response more clear and understandable.

**Sketch the graph:** Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

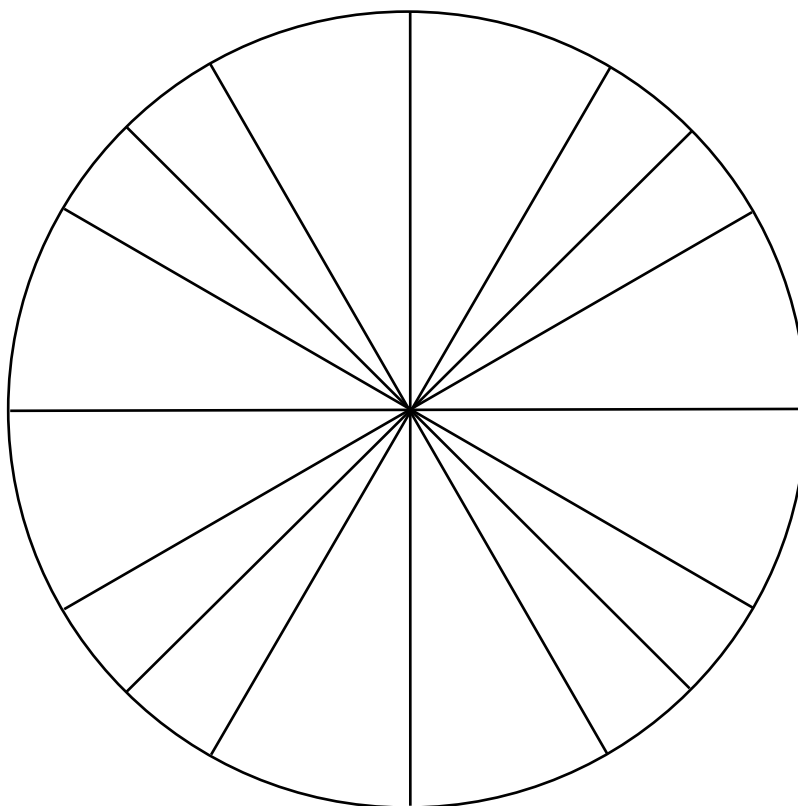
**Identify/Indicate:** Recognize and select the answer by stating or circling it.

**Justify:** Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

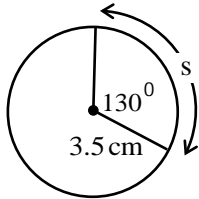
**Solve:** Give a solution for a problem or determine the value(s) of a variable.

**Verify:** Establish the truth of a statement by substitution or comparison.

Unit Circle (can be used if needed)



Use the information in the diagram to determine the value of the arc length “s”.



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Solve the following equation over the interval  $[0, 2\pi)$ .

$$\tan^2 \theta + 2.8 \tan \theta + 1.96 = 0$$

Use the quadratic formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  for  $ax^2 + bx + c = 0$ .



Determine how many monthly investments of \$50 would have to be deposited into a savings account that pays 3% annual interest, compounded monthly, for the account's future value to be \$50,000.

Use the formula:

$$FV = \frac{R \left[ (1 + i)^n - 1 \right]}{i}$$


where FV = the future value

R = the investment amount

$i = \frac{\text{the annual interest rate}}{\text{the number of compounding periods per year}}$

$n = \text{the number of investments}$

Express your answer as a whole number.

Question 4 

2 marks

104

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There are 5 men and 4 women to be seated in a row.

How many arrangements are possible if two men must sit at the beginning of the row and two men must sit at the end of the row?

a) In the binomial expansion of  $\left(\frac{3}{x^2} - 4x^5\right)^8$ , determine the 3rd term.

b) In the binomial expansion of  $\left(\frac{3}{x^2} - 4x^5\right)^n$ , the 6th term contains  $x^{25}$ .  
Solve for  $n$ .

**Note: A calculator is not required for the remaining test questions.**

Question 6

1 mark

107

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Given the following two functions,  $f(x) = \sqrt{x-1}$  and  $g(x) = x^2 + 1$ , evaluate  $g(f(3))$ .

Question 7

2 marks

108

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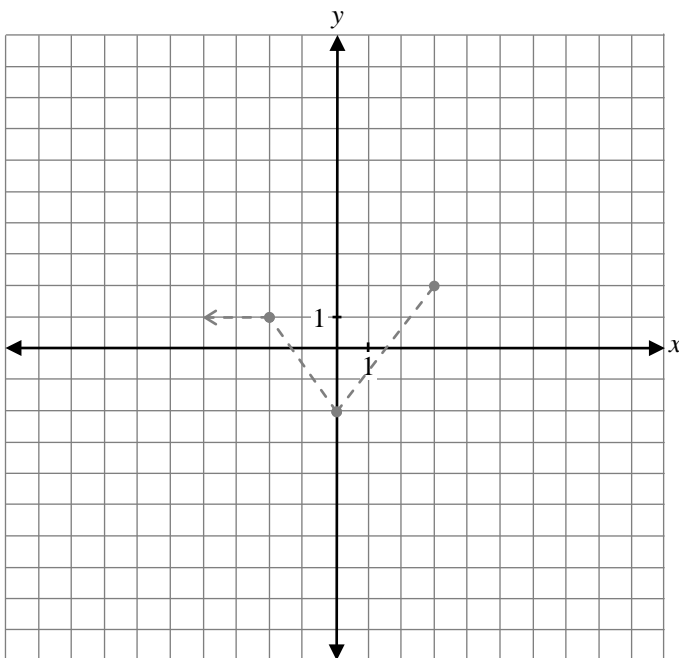
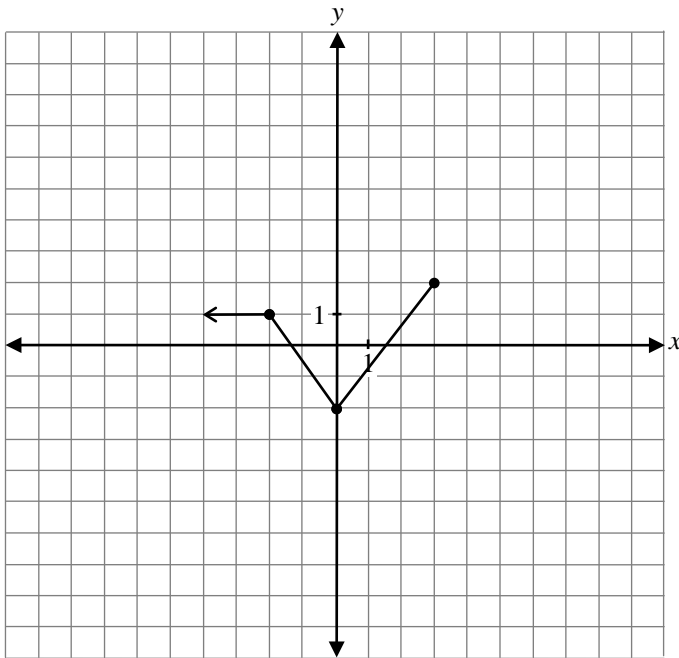
If  $\theta$  terminates in quadrant II and  $\csc \theta = \frac{3}{2}$ , determine the exact value of  $\tan \theta$ .

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a) Determine the remainder when  $x^4 - 3x^2 + 1$  is divided by  $x + 2$ .

b) Is  $x + 2$  a factor of  $x^4 - 3x^2 + 1$ ?  
Explain your reasoning.

Given the graph of  $y = f(x)$  below, sketch the graph of  $y = 2f(x) - 3$ .



The graph of  $f(x)$  has already been drawn for your reference. No marks will be awarded for the graph of  $f(x)$ .

## Question 10

1 mark

112

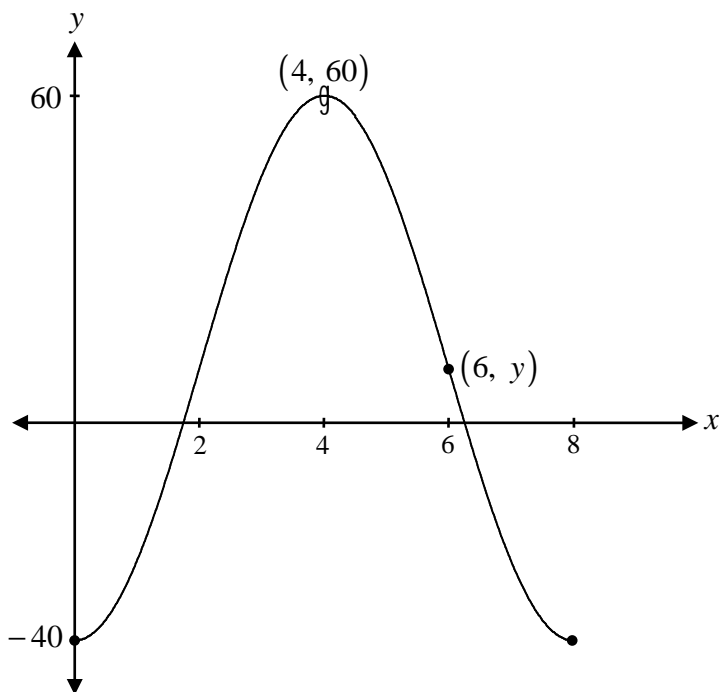
Determine one possible restriction for the domain of  $f(x) = (x - 1)^2$  so that the inverse of  $f(x)$  is a function.

## Question 11

1 mark

113

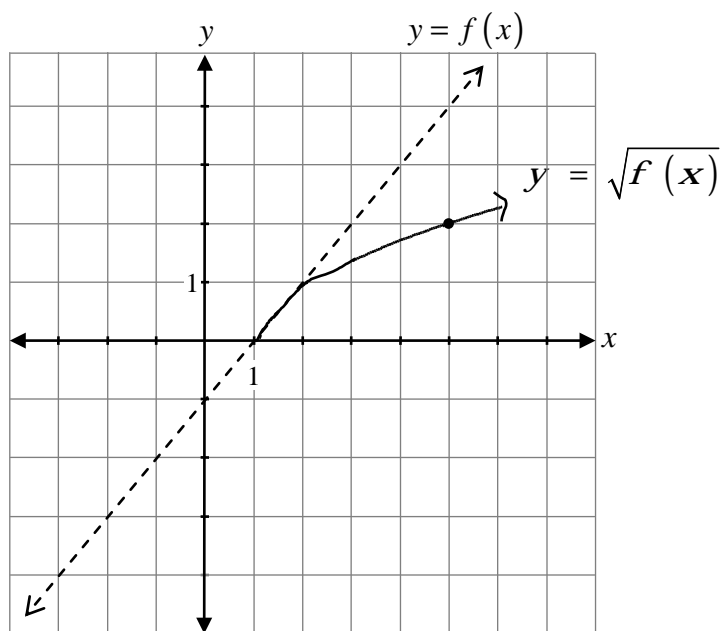
Using the graph of the sinusoidal function below, find the value of  $y$  in the point  $(6, y)$ .



Billy was given the graph of  $y = f(x)$ .

He was asked to sketch the graph of  $y = \sqrt{f(x)}$ .

His answer is given on the graph below.



Explain the error Billy made when sketching the graph of  $y = \sqrt{f(x)}$ .

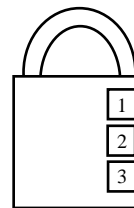


Question 13

1 mark

115

Explain why a locker combination should really be called a locker permutation.



Question 14

1 mark

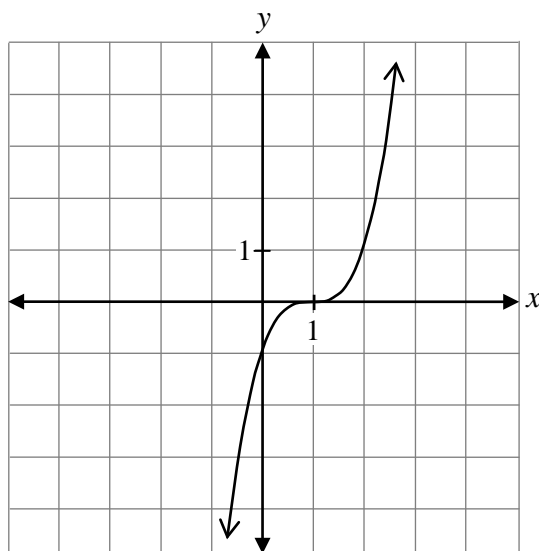
116

The graph of  $f(x) = x^2 + 4$  is reflected over the  $x$ -axis.

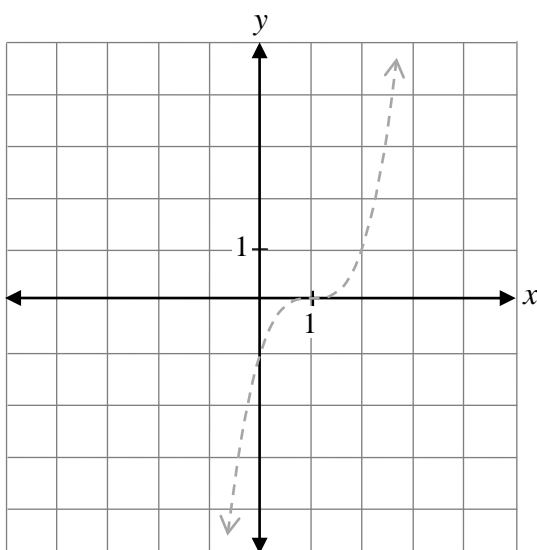
Write the equation of the new function.

$y =$  \_\_\_\_\_

Given the graph of  $y = f(x)$  below,



sketch the graph of  $y = \frac{1}{f(x)}$ .



The graph of  $f(x)$  has already been drawn for your reference.

No marks will be awarded for the graph of  $f(x)$ .

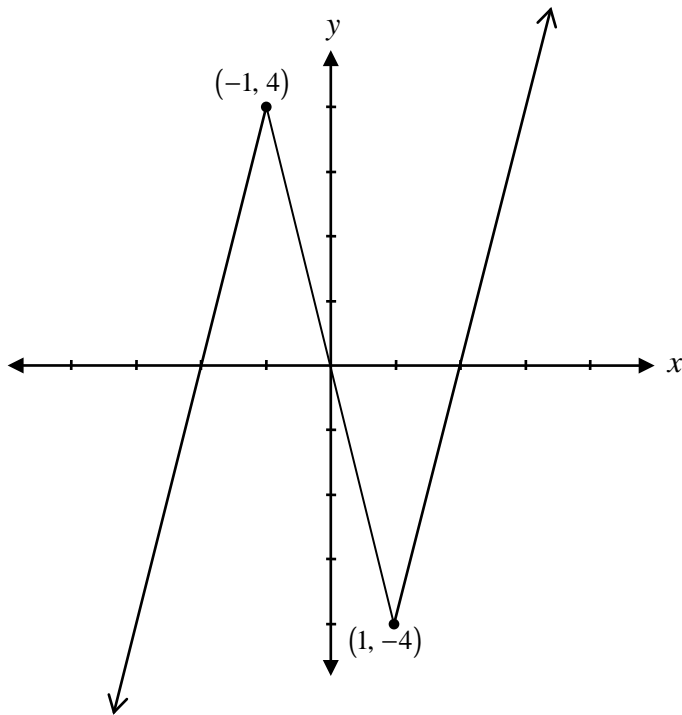
Divide  $(x^3 - 5x - 4)$  by  $(x + 1)$ .

You are given the following row of Pascal's Triangle.

1    7    21    35    35    21    7    1

Determine the values of the next row.

Given the graph of  $y = f(x)$  below, state the domain and range of  $y = \sqrt{f(x)}$ .



Domain: \_\_\_\_\_

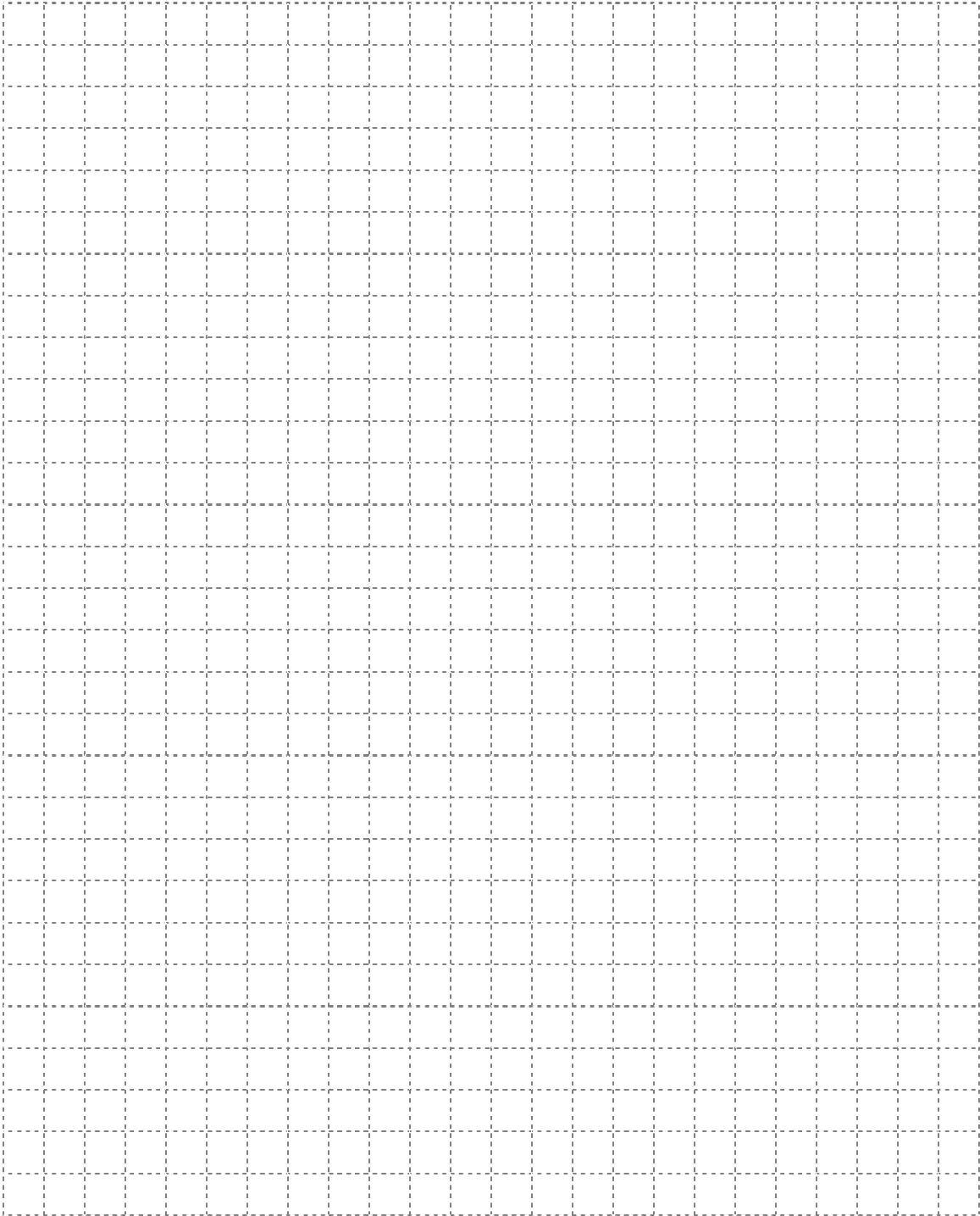
Range: \_\_\_\_\_

Prove the identity below for all permissible values of  $\theta$ :

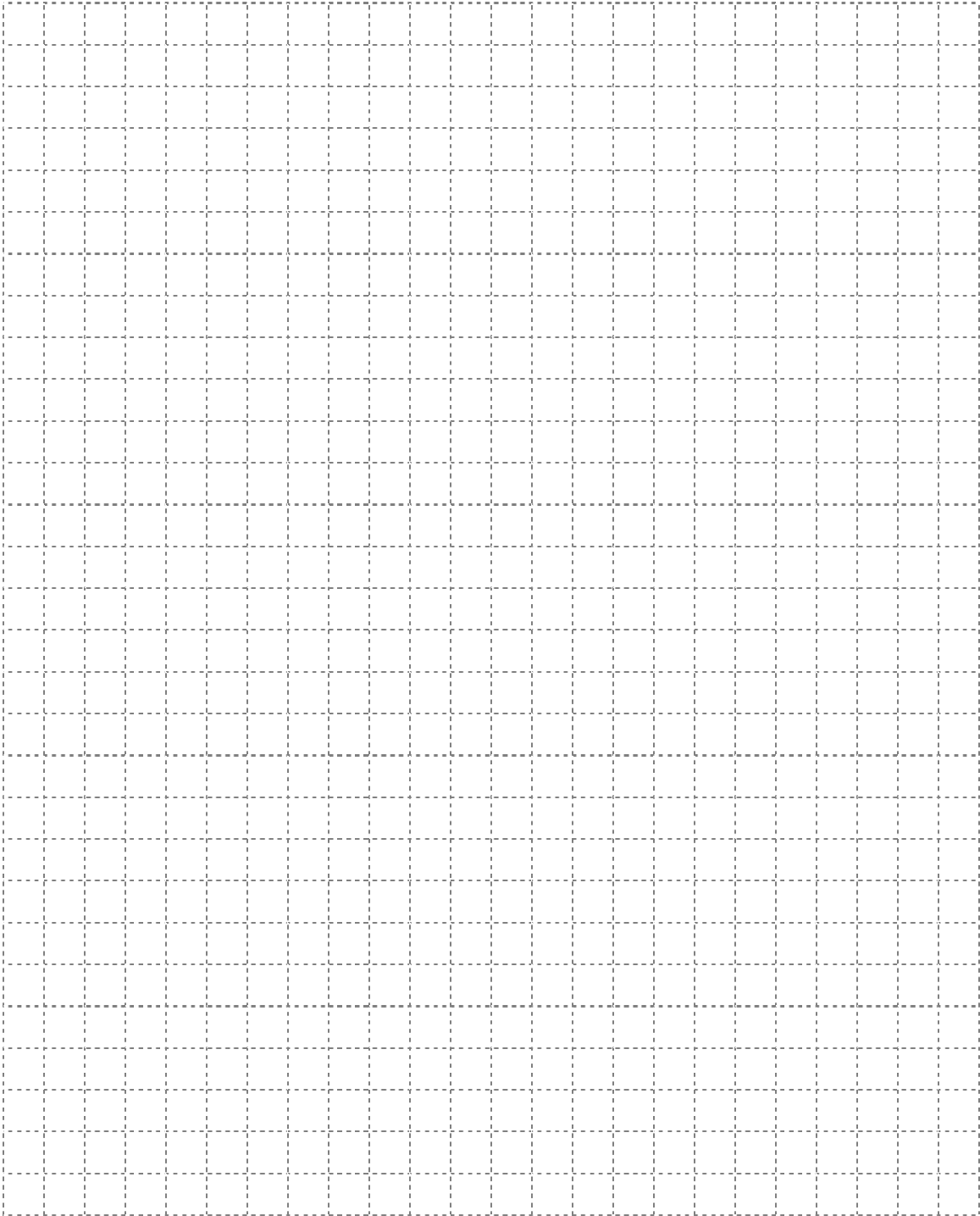
$$\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos 2\theta$$

Left-Hand Side	Right-Hand Side

No marks will be awarded for work done on this page.



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