

Grade 12
Pre-Calculus Mathematics
Achievement Test

Booklet 1

January 2018

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Booklet 1. January 2018

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This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

Grade 12 Pre-Calculus Mathematics Achievement Test

DESCRIPTION

Time: 3 hours

Numbers and Marks by Question Type

	Selected Response	Constructed Response	Marks
Booklet 1*	—	15	32
Booklet 2	9	25	57
Total	9	40	89

* The first 5 questions in *Booklet 1* require a calculator.  You will have access to your calculator for the first 45 minutes of the test.

Note that diagrams and graphs provided in the test booklets may not be drawn to scale.

DIRECTIONS

- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions must include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized, and clear.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to the nearest thousandth (3 decimal places) unless instructed otherwise.

Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited.

No marks will be awarded for work done on this page.

Formula Sheet

$$s = \theta r$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 1 - 2 \sin^2 \alpha$$

$$\cos 2\alpha = 2 \cos^2 \alpha - 1$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\log_a (MN) = \log_a M + \log_a N$$

$$\log_a \left(\frac{M}{N} \right) = \log_a M - \log_a N$$

$$\log_a (M^n) = n \log_a M$$

$$P(n, r) \text{ or } {}_n P_r = \frac{n!}{(n-r)!}$$

$$C(n, r) \text{ or } {}_n C_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_n C_k a^{n-k} b^k$$

For $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Terminology Sheet

Some questions may contain directing words such as *explain*, *identify*, and *justify*. These words are defined below.

Describe: Use words to provide the process or to report details of the response.

Determine: Use a mathematical formula, an algebraic equation, or a numerical calculation to solve a problem.

Evaluate: Find the numerical value.

Explain: Use words to provide the cause of or reason for the response, or to render the response more clear and understandable.

Identify/Indicate: Recognize and select the answer by stating or circling it.

Justify: Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

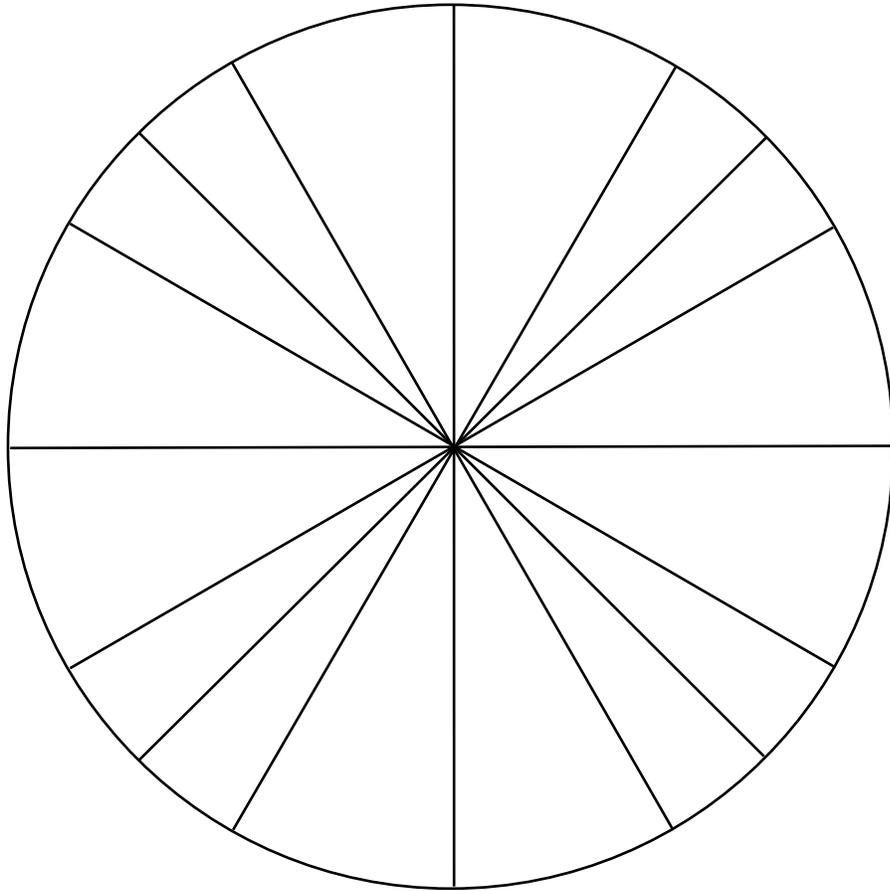
Sketch the graph: Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

Solve: Give a solution for a problem or determine the value(s) of a variable.

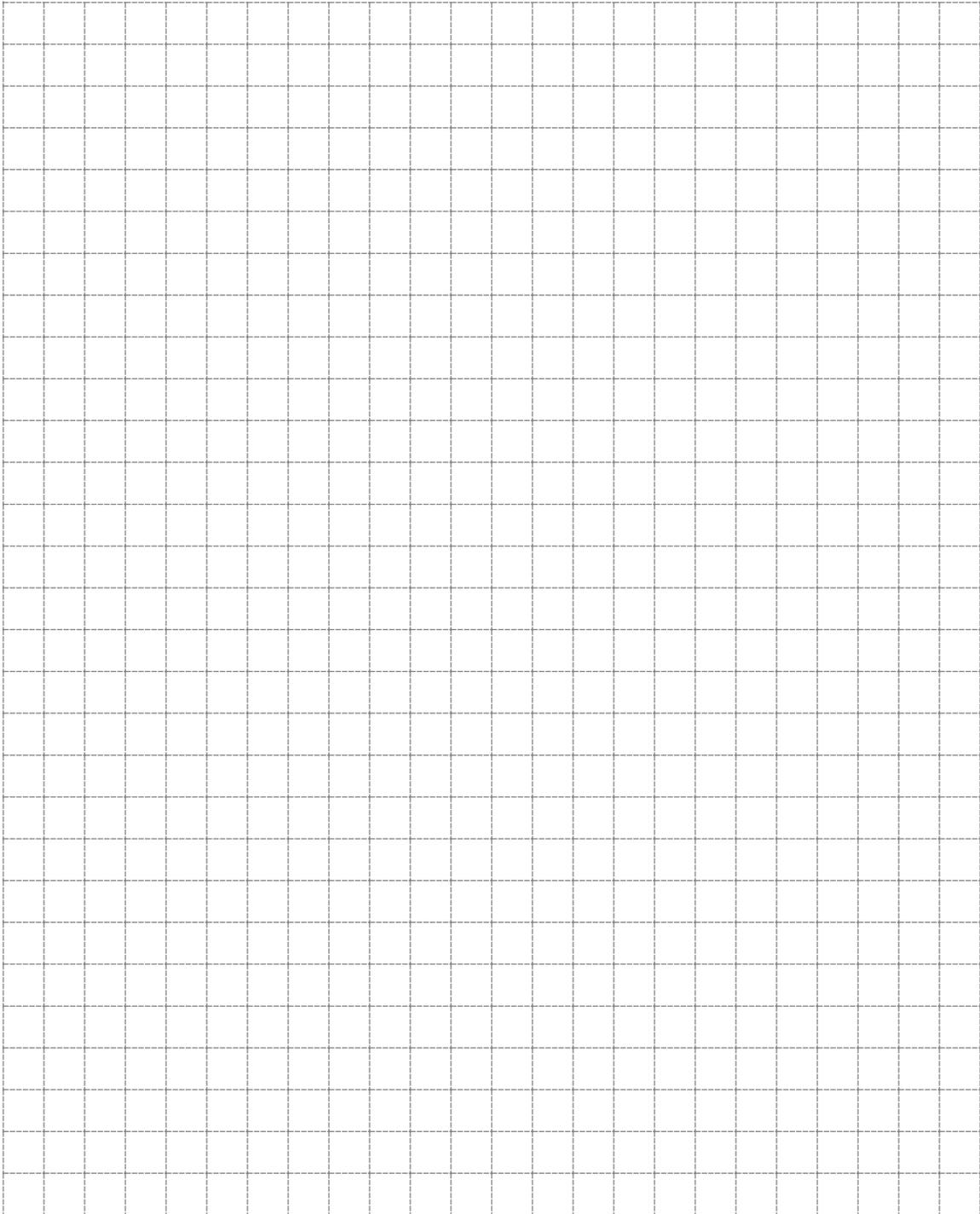
State: Give an answer without an explanation or a justification.

Verify: Establish the truth of a statement by substitution or comparison.

No marks will be awarded for work done on this page.



No marks will be awarded for work done on this page.



Question 1 

2 marks 101

A group of 7 friends decide to go to a movie.

Determine how many ways the friends can sit in a row if two of the friends refuse to sit next to each other.

Gabrielle listens to her radio at a sound level of 80 dB. She attended a music concert that had a sound level of 115 dB. Determine how many times more intense the music concert was than the radio.

You may use the formula:

$$\beta = 10 \log \left(\frac{I}{I_0} \right)$$

where β is the intensity level of sound, measured in dB

I is the intensity of sound

I_0 is the standard minimum intensity that a person can hear

Question 3 

4 marks 103

Solve, algebraically.

$$2(7)^x = 3^{2x-3}$$

Question 4 

4 marks 104

Solve for θ , algebraically, over the interval $[0, 2\pi]$.

$$\csc^2\theta + 2 \csc\theta - 8 = 0$$

Question 5 

1 mark 105

You have forgotten the code to unlock your cell phone. You know the code is made up of four numbers from 0 to 9.

Determine the number of possible codes, if repetition is allowed.

Note: A calculator is not required for the remaining test questions.

In the binomial expansion of $\left(\frac{7}{x^3} - 3x^7\right)^n$, the 5th term contains x^7 .

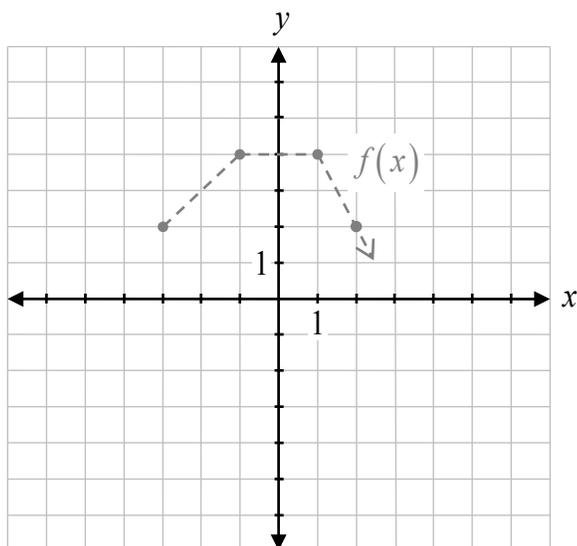
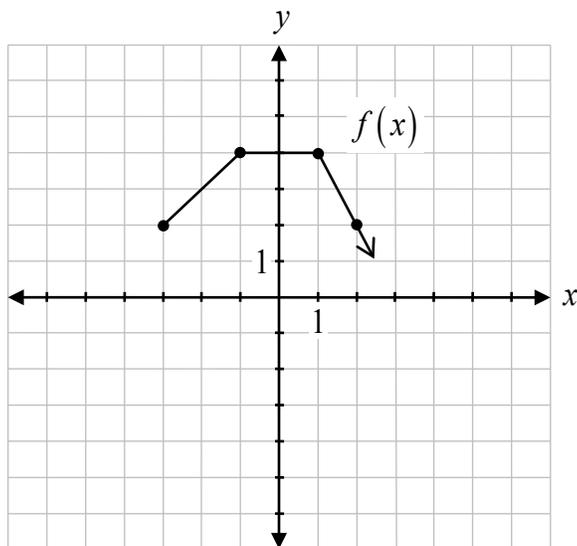
Determine the value of n .

Question 7

1 mark 107

Given the domain of $f(x)$ is $\{-6, 1, 3, 4\}$ and the range of $f(x)$ is $\{-4, 7, 10, 15\}$, state the domain of $f^{-1}(x)$.

Given the graph of $y = f(x)$, sketch the graph of its inverse.



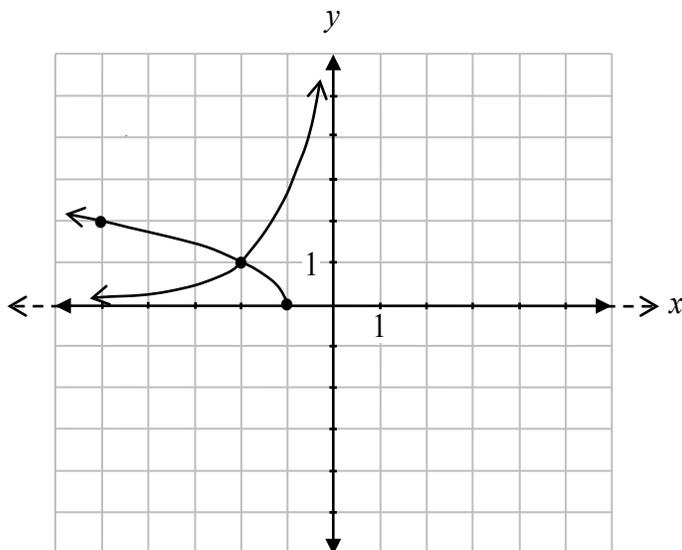
The graph of $f(x)$ has already been drawn for your reference. No marks will be awarded for the graph of $f(x)$.

Prove the following identity for all permissible values of θ .

$$\frac{1 + \cos \theta}{1 - \sin^2 \theta} = \sec \theta + \tan^2 \theta + 1$$

Left-Hand Side	Right-Hand Side

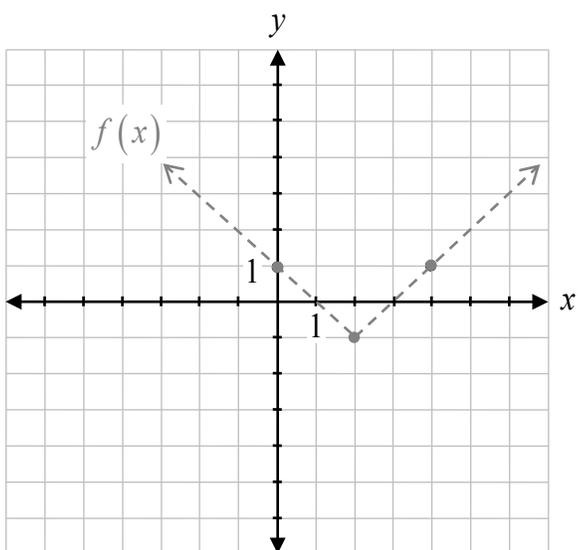
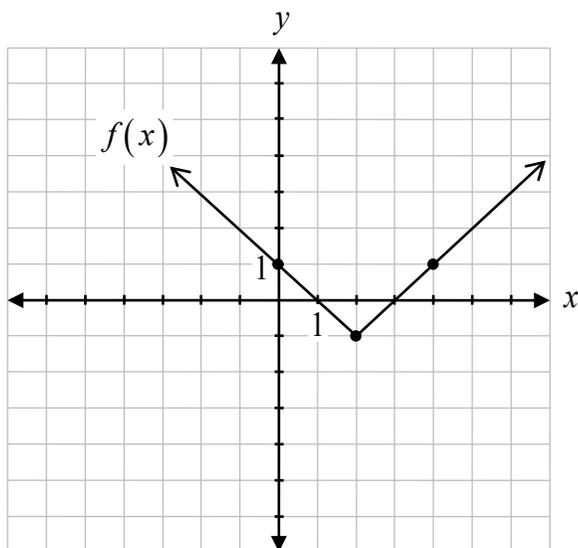
Thomas used graphs to solve the equation $e^{x+2} = \sqrt{-(x+1)}$.



He incorrectly states the solution as $(-2, 1)$.

Describe how Thomas should have stated the solution.

Given the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$.



The graph of $f(x)$ has already been drawn for your reference. No marks will be awarded for the graph of $f(x)$.

Question 12

a) 1 mark b) 1 mark

112
113

When a polynomial, $P(x)$, is divided by $(x - 2)$ the resulting equation is

$$\frac{P(x)}{x - 2} = x^2 - x + 1 + \frac{3}{x - 2}.$$

a) Explain why $x - 2$ is not a factor of $P(x)$.

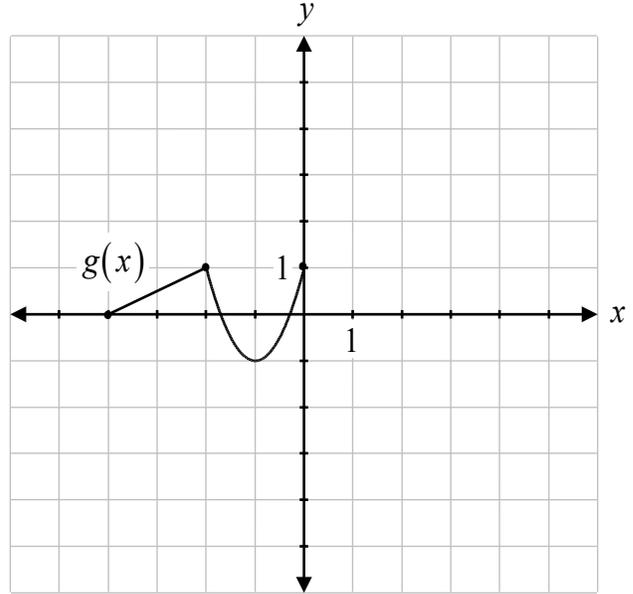
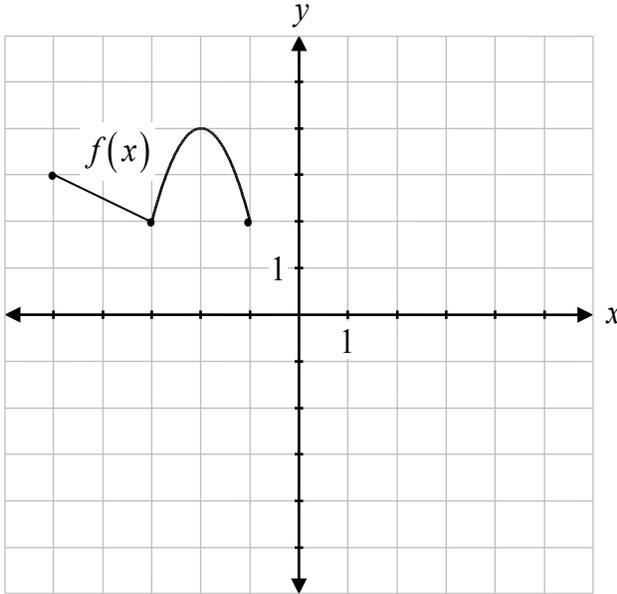
b) Determine the equation for the polynomial function $P(x)$.

$$P(x) = \underline{\hspace{10cm}}$$

Question 13

3 marks 114

Determine the equation for $g(x)$ in terms of $f(x)$.



$g(x) =$ _____

Explain why the binomial expansion of $(2x + y)^9$ does not have a middle term.

Using the laws of logarithms, completely expand the expression $\log\left(\frac{5\sqrt{a}}{b^3}\right)$.

No marks will be awarded for work done on this page.

