Grade 12
Pre-Calculus Mathematics
Achievement Test

Booklet 1

January 2015
Grade 12 pre-calculus mathematics achievement test.
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Websites are subject to change without notice.

Disponible en français.

Available in alternate formats upon request.
DESCRIPTION

Time: 3 hours

Numbers and Marks by Question Type

<table>
<thead>
<tr>
<th></th>
<th>Multiple Choice</th>
<th>Constructed Response</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Booklet 1*</td>
<td>–</td>
<td>15</td>
<td>33</td>
</tr>
<tr>
<td>Booklet 2</td>
<td>10</td>
<td>20</td>
<td>57</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td>35</td>
<td>90</td>
</tr>
</tbody>
</table>

* The first 6 questions in Booklet 1 require a calculator. You will have access to your calculator for the first 45 minutes of the test.

GENERAL DIRECTIONS

- Read all instructions carefully.
- The perforated Formula Sheet and the Terminology Sheet can be removed from the test booklet. No marks will be given for work done on these pages.
- The blank pages at the back of each booklet may be used as scrap paper, but must not be removed from the test booklet. No marks will be given for work done on these pages.
- Note that diagrams and graphs provided in the test booklets may not be drawn to scale.
- After 45 minutes, put away your calculator. Even though you may not have finished Booklet 1, Booklet 2 will be distributed at this time. You may choose to continue working on Booklet 1 or start working on Booklet 2, but you will no longer have access to your calculator.
Instructions

- There are 15 questions for a total of 33 marks.
- Calculators (scientific or graphing) are allowed for the first 45 minutes of the test.
- A calculator icon appears next to the questions that require a calculator.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions must include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized, and clear.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to 3 decimal places unless instructed otherwise.
Formula Sheet

\[ s = \theta r \]

\[ \sin^2 \theta + \cos^2 \theta = 1 \]

\[ \tan^2 \theta + 1 = \sec^2 \theta \]

\[ 1 + \cot^2 \theta = \csc^2 \theta \]

\[ \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \]

\[ \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \]

\[ \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \]

\[ \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \]

\[ \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \]

\[ \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \]

\[ \sin 2\alpha = 2 \sin \alpha \cos \alpha \]

\[ \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha \]

\[ \cos 2\alpha = 1 - 2 \sin^2 \alpha \]

\[ \cos 2\alpha = 2 \cos^2 \alpha - 1 \]

\[ \tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \]

\[ \log_a (MN) = \log_a M + \log_a N \]

\[ \log_a \left( \frac{M}{N} \right) = \log_a M - \log_a N \]

\[ \log_a (M^n) = n \log_a M \]

\[ P(n, r) \text{ or } \binom{n}{r} = \frac{n!}{(n-r)!} \]

\[ C(n, r) \text{ or } \binom{n}{r} = \frac{n!}{r!(n-r)!} \]

\[ \binom{n}{r} \]

For \( ax^2 + bx + c = 0 \),

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Some questions may contain directing words such as explain, identify, and justify. These words are defined below.

**Evaluate:** Find the numerical value.

**Explain:** Use words to provide the cause of or reason for the response, or to render the response more clear and understandable.

**Sketch the graph:** Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

**Identify/Indicate:** Recognize and select the answer by stating or circling it.

**Justify:** Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

**Solve:** Give a solution for a problem or determine the value(s) of a variable.

**Verify:** Establish the truth of a statement by substitution or comparison.

**Determine:** Use a mathematical formula, an algebraic equation, or a numerical calculation to solve a problem.

**State:** Give an answer without an explanation or justification.

Unit Circle (can be used if needed)
Question 1

Convert $\frac{13\pi}{5}$ to degrees.
Question 2

a) 1 mark  

b) 1 mark  

c) 1 mark

a)  From a group of 9 people, in how many ways can you select a committee of 4 members?

b)  From a group of 9 people, in how many ways can you select a president, a vice president, a secretary, and a treasurer?

c)  Explain why the answers in a) and b) are different.
A population of 500 bacteria will triple in 20 hours.

Using the formula given below,

\[ A = Pe^{rt} \]

\( A = \) population after \( t \) hours  
\( P = \) initial population  
\( r = \) rate of growth  
\( t = \) time in hours

a) Determine the rate of growth, \( r \).

b) Determine how many hours it will take for the initial population to double with the same rate of growth.
Talla incorrectly solved the following trigonometric equation:

Solve: $2 \sec x - 5 = 0; \ 0^\circ \leq x \leq 360^\circ$.

Talla’s work:

$$2 \sec x - 5 = 0$$

$$\sec x = \frac{5}{2}$$

No solution, $\sec x$ cannot be greater than 1.

a) Explain her error.

b) Determine the correct solution.
Simplify the 6th term in the expansion of:

\[ \left( 2x - \frac{3}{x^2} \right)^{10} \]
Determine the arc length subtended by a central angle if the diameter is 19 cm and the central angle is 1.6 radians.
Question 7

Solve the following equation algebraically for $x$, where $0 \leq x \leq 2\pi$.

$$2\cos^2 x = -3\sin x$$
In how many different ways can you arrange the letters in the word VOLLEYBALL?
State your answer as a factorial.
Question 9

Is \((x - 2)\) a factor of the polynomial \(p(x) = -x^4 - 3x^3 + 11x^2 + 3x - 10\)?

Justify your response.
Question 10

Determine the period of the sinusoidal function \( y = \frac{1}{2} \sin \left( \frac{1}{3} x \right) \).

State your answer in radians.
Question 11 2 marks

The domain of $f(x)$ is $x \leq 2$. The domain of $g(x)$ is $x \geq -7$.

State the domain of $f(x) + g(x)$.

Justify your answer.
Prove the identity below for all permissible values of $\theta$.

$$\frac{1}{1 + \cos \theta} = \csc^2 \theta - \frac{\cot \theta}{\sin \theta}$$

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<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
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</table>
Question 13

Explain how the end behaviours of the graphs of polynomial functions with an even degree and with an odd degree are different.
Given the graphs of \( f(x) \) and \( g(x) \), sketch the graph of \( g(x) - f(x) \).
Given \( f(x) = -3x + 7 \), evaluate \( f^{-1}(-2) \).
No marks will be awarded for work done on this page.
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