
Grade 12
Pre-Calculus Mathematics
Standards Test

Booklet 1

January 2012

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Manitoba Education
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Grade 12 Pre-Calculus Mathematics Standards Test

DESCRIPTION

Total Possible Marks: 100

Time: 3 hours

Part	Description	Time	Value
Part 1	Long Answer: 10 questions worth a total of 34 marks Calculators (scientific or graphing) are permitted.	60 minutes	34%
Part 2	Multiple Choice: 15 questions worth 1 mark each Short Answer: 15 questions worth 1 mark each Long Answer: 11 questions worth a total of 36 marks Calculators are not permitted.	120 minutes	66%

GENERAL DIRECTIONS

- Read all instructions carefully.
- A page of formulas, helpful hints, and scrap paper are provided on perforated pages at the beginning of this test booklet. Please do not tear out any other pages. No marks will be given for work done on these pages.
- The blank pages at the back of each booklet may also be used as scrap paper, but must **not** be removed from the test booklet. No marks will be given for work done on these pages.
- Note that diagrams and graphs provided in the test booklets may not be drawn to scale.
- When *Booklet 2* is distributed, you may keep *Booklet 1* in case you wish to continue working on it without a calculator.

No marks will be awarded for work done on this page.

Formula Sheet: Pre-Calculus Mathematics

$$s = \theta r$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 1 - 2 \sin^2 \alpha$$

$$\cos 2\alpha = 2 \cos^2 \alpha - 1$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$P(n, r) \text{ or } {}_n P_r = \frac{n!}{(n-r)!}$$

$$C(n, r) \text{ or } {}_n C_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_n C_k a^{n-k} b^k$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A = Pe^{rt}$$

$$e \approx 2.71828$$

$$\log_a(MN) = \log_a M + \log_a N$$

$$\log_a \left(\frac{M}{N} \right) = \log_a M - \log_a N$$

$$\log_a(M^n) = n \log_a M$$

$$\log_a M = \frac{\log_b M}{\log_b a}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \quad a > b$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, \quad a > b$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$(y-k) = a(x-h)^2$$

$$(x-h) = a(y-k)^2$$

$$t_n = t_1 r^{n-1}$$

$$S_n = \frac{t_1(1-r^n)}{1-r} = \frac{t_1(r^n-1)}{r-1}$$

$$S_n = \frac{t_1 - t_n r}{1-r} = \frac{t_n r - t_1}{r-1}$$

$$S_\infty = \frac{t_1}{1-r}, \quad |r| < 1$$

Helpful Hints

Clearly labelled graphs include:

- labels on both axes
- scales on both axes (only one number on each axis is required)
- arrowheads or endpoints to indicate if the graph continues or stops
- accurate shape (whether it is straight or curved and whether the curve opens up, down, left, or right)
- asymptotes (if applicable) drawn as non-solid lines with the graph approaching the asymptote
- intercepts labelled with numeric values (only if they are requested in the question)
- vertices and centre of conic sections accurately located according to the scale shown

Guess and check

Marks may be awarded for this method if explanations are provided and work is backed up with evidence.

Exact value solutions	
Incomplete solution	Exact value solution
$5 - 3$	2
$4(3)$	12
$\frac{\sqrt{2}\sqrt{3}}{4}$	$\frac{\sqrt{6}}{4}$
$\frac{1}{2} - \frac{2\sqrt{11}}{5}$	$\frac{5 - 4\sqrt{11}}{10}$
$\frac{\sqrt{7}}{3} - \frac{\sqrt{5}}{3}$	$\frac{\sqrt{7} - \sqrt{5}}{3}$
$1 + \frac{1}{\frac{3}{2}}$	$\frac{2}{3}$
$2\left(\frac{2\sqrt{3}}{\sqrt{5}}\right)$	$\frac{4\sqrt{3}}{\sqrt{5}}$
$\frac{0.7}{1.4}$	$\frac{7}{14}$ or $\frac{1}{2}$ or 0.5

Rough work

If you clearly label work as "rough work," it will not be marked.

Expressing the answer correct to 3 decimal places

means that you should not round before the final answer.

Explanations

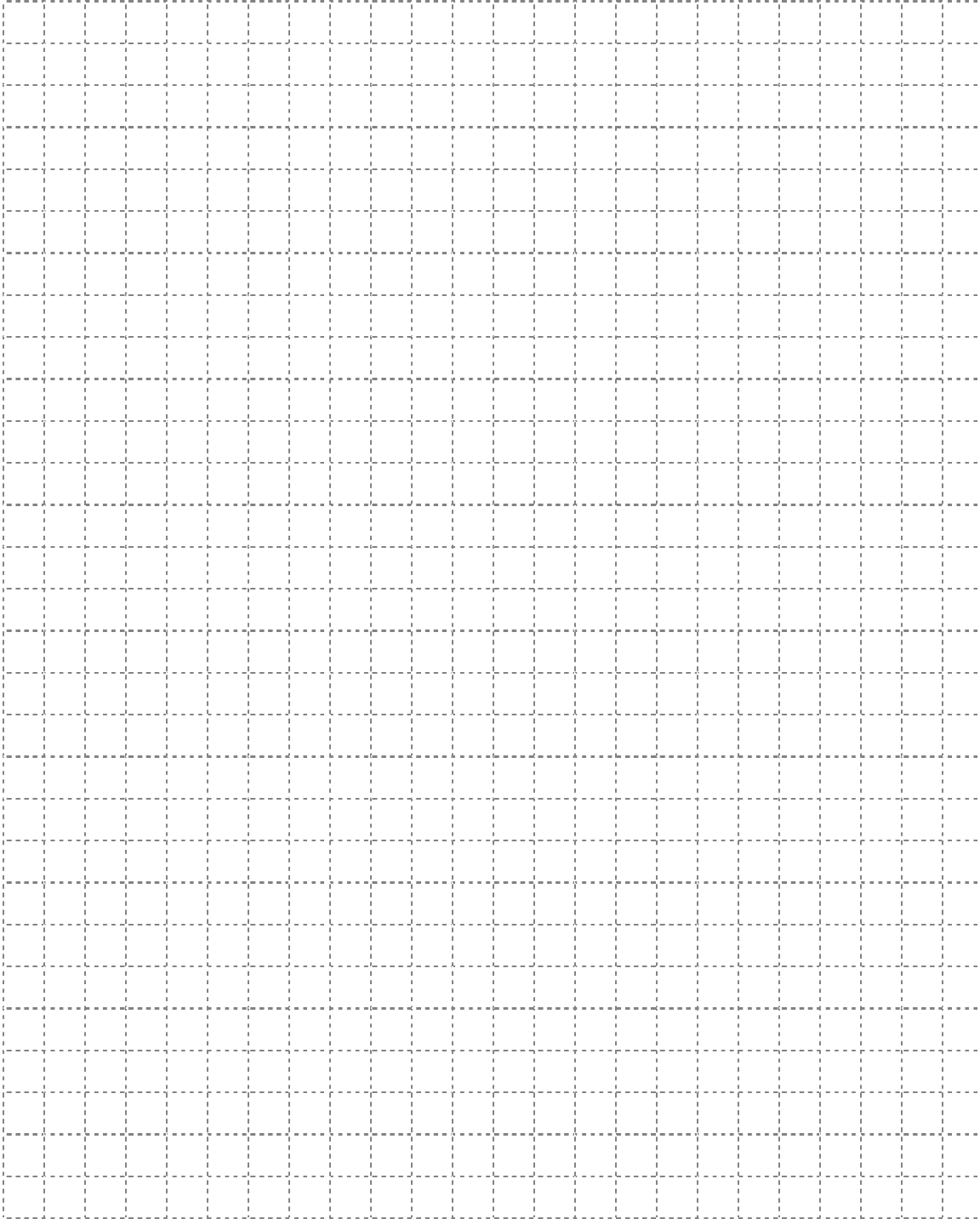
It is not always necessary to explain in words; simply indicate or label any restrictions or chosen elements.

Scrap Paper

No marks will be awarded for work done on this page.

Scrap Paper

No marks will be awarded for work done on this page.



Long-Answer Questions

Instructions

- There are 10 long-answer questions for a total of 34 marks.
- Calculators (scientific or graphing) are allowed for this part of the test.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions should include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized, and clear.
- If any curve contains asymptotes, the asymptotes should be included in the graph.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to 3 decimal places unless instructed otherwise.

No marks will be awarded for work done on this page.

Long-Answer Questions

Do Not
Use

(1 mark) 1. a) Convert $\frac{\pi}{7}$ radians into degrees.

Express your answer correct to 3 decimal places.

101

(1 mark) b) A unit circle is divided into equal sections that each have a central angle of $\frac{\pi}{7}$.
How many sections are there?

102

(1 mark) c) Give a negative co-terminal angle for $\frac{\pi}{7}$ radians.

103

(4 marks) 2. Solve the trigonometric equation below given that $\theta \in R$.

$$12 \cos^2 \theta + \cos \theta - 1 = 0$$

Give the general solution in radian measure correct to 3 decimal places.

(3 marks) 3. An insect population grows continuously according to the equation:

$$P = P_0 e^{rt}$$

where P = the population after t years

P_0 = the initial population

r = the rate of growth

t = time in years

The initial population is 300 insects. Exactly 4 years later, the population is 560 insects.

If the rate of growth remains the same, how many insects will there be 15 years after the initial insect population?

Express your answer as a whole number.

4. In a particular class, there are 5 girls and 10 boys.

(2 marks)

- a) How many committees of 7 people can be formed if there must be 3 girls and 4 boys on the committee?

Express your answer as a whole number.

106

(2 marks)

- b) How many committees of 7 people can be formed if there is at least one girl on the committee?

Briefly explain your calculations.

Express your answer as a whole number.

107

- (3 marks) 5. Find the 14th term in the binomial expansion of $\left(\frac{3}{2x^2} - x^3\right)^{16}$.
Simplify your answer completely.

108

6. The probability that Sam wears a hat is 0.24.
If Sam wears a hat, the probability he wears jeans that day is 0.71.
If Sam does not wear a hat, the probability he wears jeans is 0.34.

(2 marks)

- a) Find the probability that Sam wears jeans.

109

(1 mark)

- b) Find the probability that Sam does not wear jeans.

110

(2 marks)

- c) Given that Sam wears jeans, find the probability that he wears a hat.
Express your answer correct to 3 decimal places.

111

7. The 1st term in a geometric sequence is 5 and the 6th term is 12.4416.

(2 marks)

a) Find the 30th term of this sequence.

Express your answer correct to 3 decimal places.

112

(1 mark)

b) Find the sum of the first 30 terms.

Express your answer correct to 3 decimal places.

113

(3 marks) 8. Solve algebraically:

$$3^{4x} = 5^{x+2}$$

Express your answer correct to 3 decimal places.

9. The equation of a conic section is $5x^2 + 5y^2 - 30x + 20y - 115 = 0$.

(1 mark)

a) Identify this conic section.

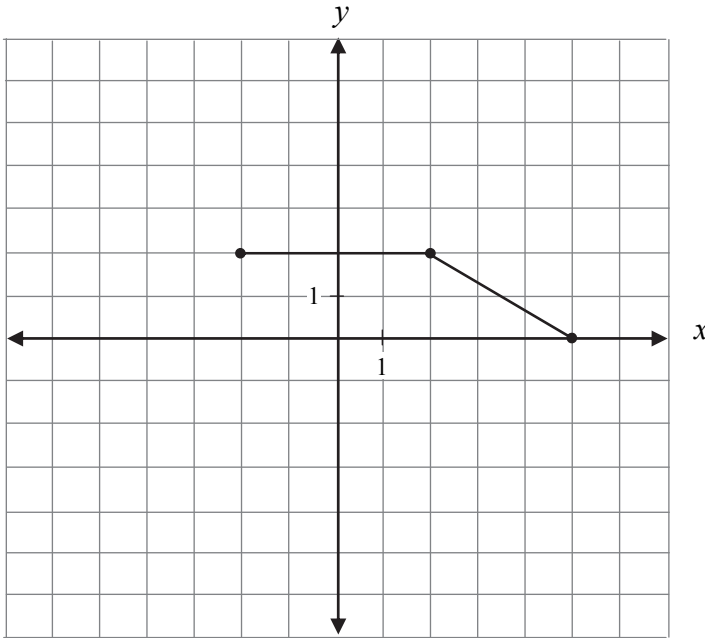
115

(3 marks)

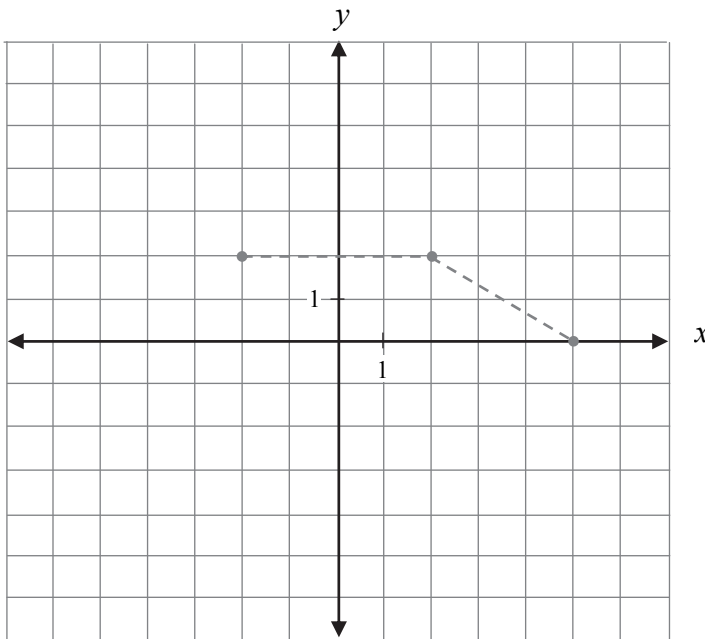
b) Write the above equation in standard form.

116

(2 marks) 10. The graph of $y = 2f(x - 1)$ is sketched below.

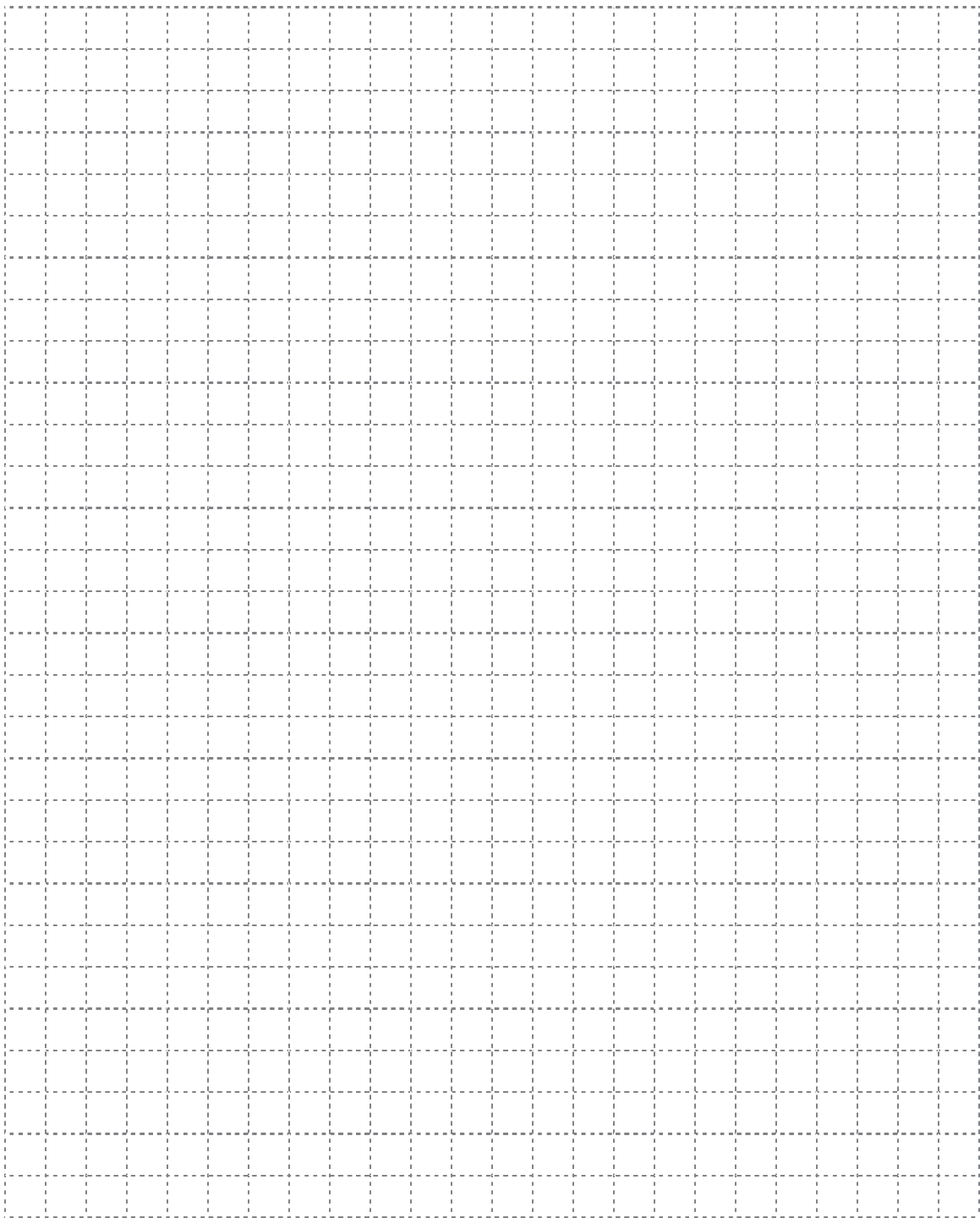


Sketch a clearly labelled graph of $y = f(x)$.



The graph of $y = 2f(x-1)$ has already been drawn for your reference.
No marks will be awarded for this graph.

No marks will be awarded for work done on this page.



No marks will be awarded for work done on this page.