Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

January 2024

Manitoba

Grade 12 pre-calculus mathematics achievement test.
Marking guide. January 2024

This resource is available in print and electronic formats.
ISBN: 978-0-7711-6367-8 (print)
ISBN: 978-0-7711-6369-2 (pdf)

Manitoba Education and Early Childhood Learning
Winnipeg, Manitoba, Canada

This resource will be available on the Manitoba Education and Early Childhood Learning website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

Table of Contents

General Marking Instructions 1

Scoring Guidelines for Booklet 1 Questions 5

Scoring Guidelines for Booklet 2 Questions 47

Answer Key for Selected Response Questions 48

Appendices 115

Appendix A: Marking Guidelines 117

Appendix B: Irregularities in Provincial Tests 118

Irregular Test Booklet Report...119

Appendix C: Table of Questions by Unit and Learning Outcome 121

General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of constructed response questions and selected response questions. Constructed response questions are worth 1 to 5 marks each, and selected response questions are worth 1 mark each. An answer key for the selected response questions can be found at the beginning of the section "Booklet 2 Questions."

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "0" only (e.g., student was present but did not attempt any questions), please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

Provincial Assessment Program Unit
Telephone: 204-945-5011
Toll-Free: 1-800-282-8069, ext. 5011 (8:30 a.m. to 4:30 p.m.)
Email: assesseval@gov.mb.ca

Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called “Communication Errors” (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student’s mark), with a maximum deduction of 5 marks from the total test mark.

When a given response includes multiple types of communication errors, deductions are indicated in the order in which the errors occur in the response. No communication errors are recorded for work that has not been awarded marks. The total deduction may not exceed the marks awarded.

The student’s final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

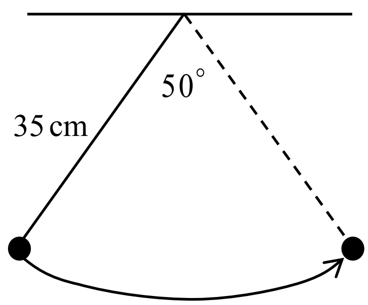
COMMUNICATION ERRORS / ERREURS DE COMMUNICATION				
Shade in the circles below for a maximum total deduction of 5 marks (½ mark deduction per error). **Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).**				
E1 ●	**E2** ○	**E3** ○	**E4** ○	**E5** ○
E6 ○	**E7** ●	**E8** ●	**E9** ○	**E10** ○

Example: Marks assigned to the student

Marks Awarded	Booklet 1 25	Selected Response 7	Booklet 2 40	Communication Errors (Deduct) 1½	Total 70½
Total Marks	**36**	**9**	**45**	**maximum deduction of 5 marks**	**90**

Scoring Guidelines for Booklet 1 Questions

A pendulum that is 35 cm long swings through an angle of 50°. Determine the length of the arc through which the pendulum swings.



Solution

$\theta = 50\left(\frac{\pi}{180}\right)$ 1 mark for conversion

$= \frac{5\pi}{18}$

or

$= 0.872\ 664...$

$s = \theta r$

$= \left(\frac{5\pi}{18}\right)(35)$ 1 mark for substitution

$= \frac{175\pi}{18}$ cm **2 marks**

or

$= 30.543$ cm

Exemplar 1

$$S = \theta r$$

$$\frac{50^\circ \pi}{180} = 0.873$$

$$S = (17.5)(0.873)$$

$$S = 15.278 \text{ cm}$$

1½ out of 2

award full marks
– ½ mark for procedural error in line 3
E6 (rounding too early)

Exemplar 2

$$S = \theta r$$

$$S = \left(\frac{5\pi}{18}\right)(35)$$

$$\boxed{S = 30.543}$$

$$50^\circ\left(\frac{\pi}{180^\circ}\right)$$

$$= \frac{5\pi}{18}$$

2 out of 2

award full marks
E5 (units of measure omitted in final answer)

Exemplar 3

$$S = 50^\circ \cdot 35 \text{ cm}$$

$$S = 1750\text{cm}$$

1 out of 2

+ 1 mark for substitution

Question 2 T5

Solve algebraically, where $0 \le \theta \le 2\pi$.

$$2\cos^2\theta = \sin^2\theta - 2\cos\theta$$

Solution

$2\cos^2\theta = 1 - \cos^2\theta - 2\cos\theta$ — 1 mark for substitution of appropriate identity

$3\cos^2\theta + 2\cos\theta - 1 = 0$

$(\cos\theta + 1)(3\cos\theta - 1) = 0$

$\cos\theta = -1$ $\quad$ $\cos\theta = \frac{1}{3}$ — 1 mark for solving for $\cos\theta$

$\theta = \pi$ $\quad$ $\theta_r = 1.230\,959...$

$\theta = 1.231, 5.052$ — 2 marks for solving for θ (1 mark for each branch)

4 marks

Exemplar 1

$$2\cos^2\theta = 1 - \cos^2\theta - 2\cos\theta$$

$$\frac{0}{-1} = \frac{-3\cos^2\theta - 2\cos\theta + 1}{-1}$$

$$0 = 3\cos^2\theta + 2\cos\theta - 1$$

$$0 = (3\cos\theta + 1)(\cos\theta - 1)$$

$$\cos\theta = \frac{-1}{3} \qquad \cos\theta = 1$$

$$\theta = 0, 2\pi$$

S | A
T | C

$$\theta_r = \cos^{-1}\left(\frac{1}{3}\right)$$

$$\theta_r = 1.230959\ldots$$

Quad II

$$\theta = \pi - 1.230959\ldots$$

$$\theta = 1.911$$

Quad III

$$\theta = \pi + 1.230959\ldots$$

$$\theta = 4.372$$

$$\boxed{\theta = 0, 1.911, 4.372, 2\pi}$$

3½ out of 4

award full marks
– ½ mark for arithmetic error in line 4

Exemplar 2

$$2\cos^2\theta = 1 - \cos^2\theta - 2\cos\theta$$
$$+\cos^2\theta \qquad +\cos^2\theta$$

$$3\cos^2\theta = 1 - 2\cos\theta$$
$$+2\cos\theta \qquad +2\cos\theta$$

$$3\cos^2\theta + 2\cos\theta = 1$$
$$-1 \quad -1$$

$$3\cos^2\theta + 2\cos\theta - 1 = 0$$

$$(3\cos\theta - 1)(\cos\theta + 1)$$

$$\cos\theta = \frac{1}{3} \qquad \cos\theta = -1$$

$$\theta = 1.23 \qquad \theta = 0$$
$$\theta = 4.37 \qquad \theta = \pi$$
$$\theta = 2\pi$$

2½ out of 4

+ 1 mark for substitution of appropriate identity
+ 1 mark for solving for $\cos\theta$
+ ½ mark for one correct value of θ on the left branch
E2 (changing an equation to an expression in line 5)
E6 (rounding error)

Exemplar 3

S | A
T | C

$$2\cos^2\theta = (1-\cos^2\theta) - 2\cos\theta$$

$$3\cos^2\theta + 2\cos\theta - 1 = 0$$

$a = \cos\theta$

$$3a^2 + 2a - 1 = 0$$

$$(3a - 1)(a + 1) = 0$$

$$a = \tfrac{1}{3}, -1$$

$$\cos\theta = \tfrac{1}{3}, -1$$

$\cos\theta = \tfrac{1}{3}$

$\theta = \boxed{70.53^\circ}$

$360 - 70.53 = \boxed{289.5^\circ}$

$\cos\theta = -1$

$\boxed{\theta = 180}$

$$\theta = 70.53^\circ, 289.5^\circ, 180^\circ$$

4 out of 4

award full marks
E5 (answer stated in degrees instead of radians)
E6 (rounding error)

Determine the number of arrangements of the letters in the word ATTENTION which begin with the letter A.

Solution

$\frac{8!}{3!2!} = 3360$

1 mark for 8!
1 mark for division by 3!2! (½ mark for 3!; ½ mark for 2!)

2 marks

Exemplar 1

1 8 7 6 5 4 3 2 1

$$\frac{8!}{3!} = \boxed{6720 \text{ arrangements}}$$

1½ out of 2

+ 1 mark for 8!
+ ½ mark for division by 3!

Exemplar 2

$$\frac{9!}{3!\,2!} = 30\,240$$

1 out of 2

+ 1 mark for division by 3!2!

Exemplar 3

$$\frac{8!}{3!\,2!} = 13\,440$$

1½ out of 2

award full marks
– ½ mark for arithmetic error

Question 4 R10

Solve for x, algebraically.

$$e^{2x+1} = 5^x$$

Solution

$\ln\left(e^{2x+1}\right) = \ln\left(5^x\right)$	½ mark for applying logarithms
$(2x+1)\ln e = x\ln 5$	1 mark for power law (½ mark for each)
$2x+1 = x\ln 5$	
$2x - x\ln 5 = -1$	½ mark for collecting terms with x
$x(2-\ln 5) = -1$	
$x = \dfrac{-1}{2-\ln 5}$	½ mark for isolating x
$x = -2.560\ 412...$	
$x = -2.560$	½ mark for evaluating quotient of logarithms

3 marks

Exemplar 1

$$\ln e^{2x+1} = \ln 5^{x}$$

$$2x+1 = x\ln 5$$

$$\frac{1}{\ln 5} = \frac{x\ln 5 - 2x}{\ln 5}$$

$$\boxed{x = -0.621}$$

2 out of 3

+ ½ mark for applying logarithms
+ 1 mark for power law
+ ½ mark for collecting terms with *x*

Exemplar 2

$$\mathrm{Ln}(e^{2x+1}) = \mathrm{Ln}(5^{x})$$

$$(2x+1)\cancel{\mathrm{Ln}e} = (x)\mathrm{Ln}5$$

$$2x\underset{-1}{+1} = (x)\mathrm{Ln}5^{-1}$$

$$\frac{2x}{x} = \frac{(x)\mathrm{Ln}\ 5 - 1}{x}$$

$$x = \mathrm{Ln}5 - 1$$

$$\boxed{x = 0.609}$$

1½ out of 3

+ ½ mark for applying logarithms
+ 1 mark for power law

Exemplar 3

$$2n+1 \log e = n \log 5$$

$$2n \log e + \log e = n \log 5$$

$$2n \log e - n \log 5 = - \log e$$

$$n(2 \log e - \log 5) = - \log e$$

$$n = \frac{- \log e}{2 \log e - \log 5}$$

2½ out of 3

+ ½ mark for applying logarithms
+ 1 mark for power law
+ ½ mark for collecting terms with *x*
+ ½ mark for isolating *x*
E4 (missing brackets but still implied in line 1)

This page was intentionally left blank.

Question 5 P3

There are 10 teachers and 17 students who would like to attend a field trip.

Determine the number of ways that 3 teachers and 9 students can be selected given that Mr. Jones and Mrs. Carol, two of the teachers, must be selected to attend the field trip.

Solution

$_2C_2 \cdot {}_8C_1 \cdot {}_{17}C_9$

194 480

1 mark for $_8C_1$

½ mark for $_{17}C_9$

½ mark for product of combinations

2 marks

Note:

- $_2C_2$ does not need to be shown.

Exemplar 1

$\underset{\text{teachers}}{\underline{_{10}C_3}} \cdot \underset{\text{students}}{\underline{_nC_9}} = 2,917,200$ ways

1 out of 2

+ ½ mark for $_{17}C_9$

+ ½ mark for product of combinations

Exemplar 2

10 teachers   17 students

8 teachers $_{17}C_9$

$_8C_1 = 8$ +   24310

$= 24318$

1½ out of 2

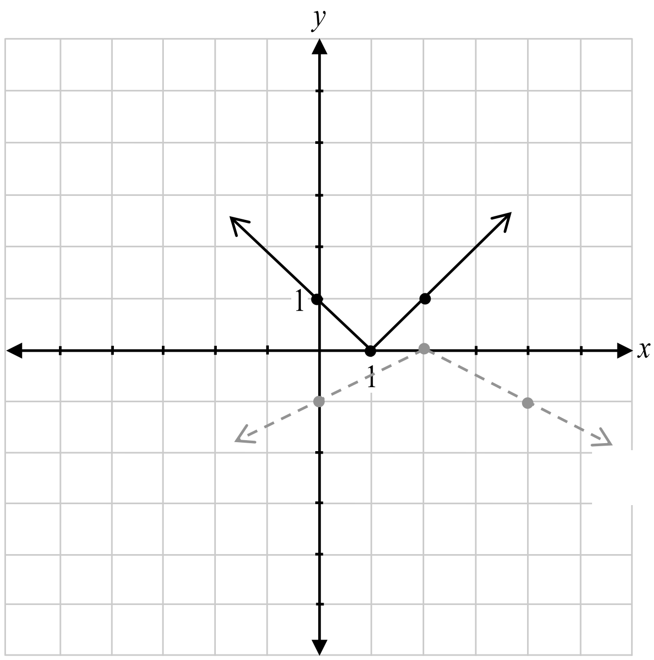
+ 1 mark for $_8C_1$

+ ½ mark for $_{17}C_9$

Question 6 R4, R5

Given the graph of $f(x)$, sketch the graph of $y = -f(2x)$.

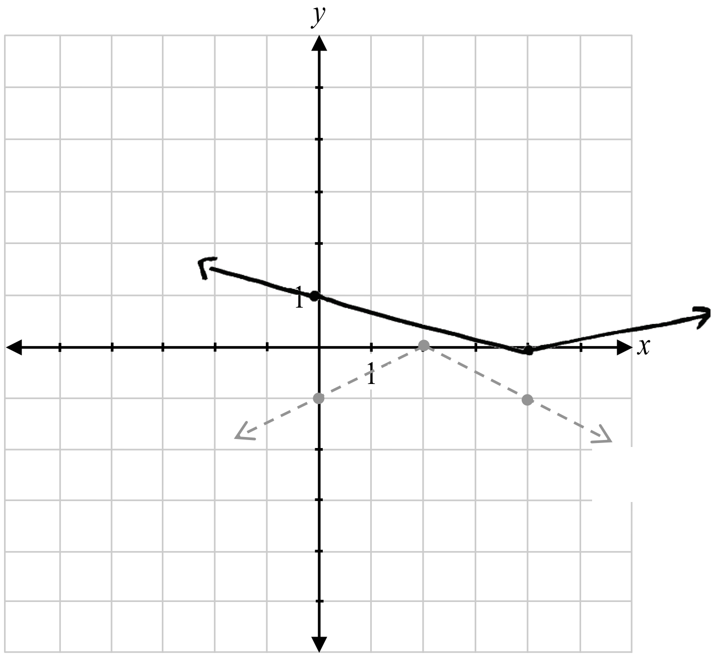
Solution



1 mark for vertical reflection
1 mark for horizontal compression

2 marks

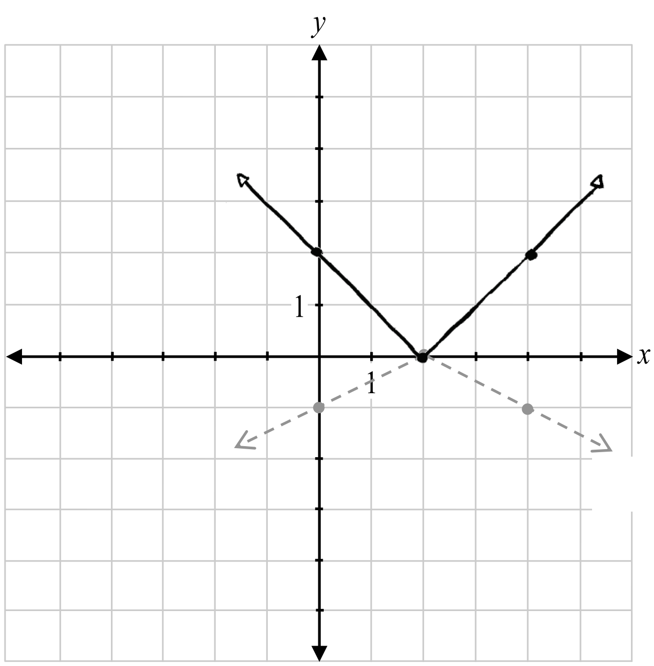
Exemplar 1



1 out of 2

+ 1 mark for vertical reflection

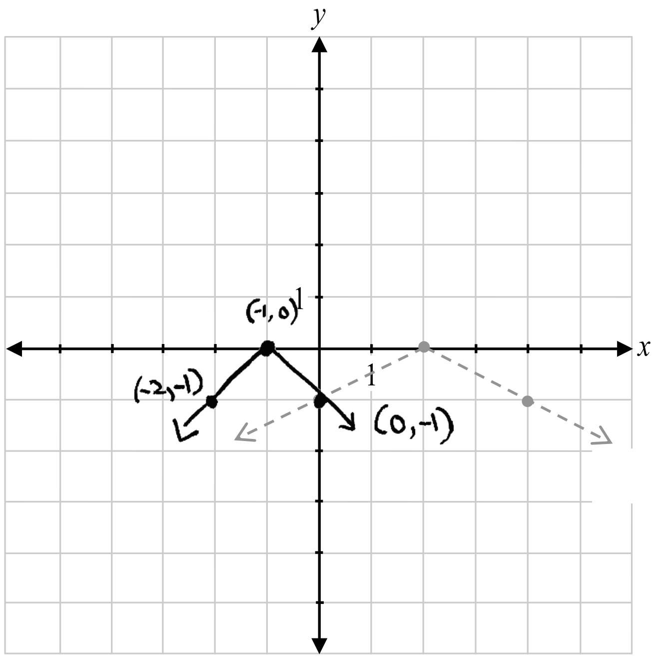
Exemplar 2



1 out of 2

+ 1 mark for vertical reflection

Exemplar 3



1 out of 2

+ 1 mark for horizontal compression

This page was intentionally left blank.

There are 5 roads between Anneville and Berrybourg, and 2 roads between Berrybourg and Carriton.

Determine how many ways Blake can travel from Anneville to Carriton and back to Anneville, given the following conditions:

- he must travel through Berrybourg in both directions
- he cannot use the same road twice

Solution

$$\underset{A \to B}{\underline{\quad 5 \quad}} \bullet \underset{B \to C}{\underline{\quad 2 \quad}} \bullet \underset{C \to B}{\underline{\quad 1 \quad}} \bullet \underset{B \to A}{\underline{\quad 4 \quad}}$$

40 ways

½ mark for correct number of routes between towns
½ mark for product

1 mark

Exemplar 1

case #1 A → C

case #2 C → A

$$({}_5C_1 \cdot {}_2C_1) + ({}_4C_1 \cdot {}_1C_1) = 14 \text{ ways}$$

$$5 \cdot 2 \qquad 4 \cdot 1$$

½ out of 1

+ ½ mark for all correct numbers of routes between towns

Exemplar 2

A B C

$$5! \cdot 2! = 240 \text{ ways.}$$

½ out of 1

+ ½ mark for product

Exemplar 3

A → B 5 roads

B → C 2 roads

$$\underline{\ \ }_5 \ \underline{\ \ }_2 \ \underline{\ \ }_4 = 40 \text{ ways}$$

1 out of 1

Determine which term contains x^0 in the binomial expansion of $\left(x^2+\frac{1}{x}\right)^6$.

Solution

Method 1

$x^0=\left(x^2\right)^{6-k}\left(\frac{1}{x}\right)^k$ ½ mark for substitution

$x^0=x^{12-2k}x^{-k}$

$x^0=x^{12-3k}$

$3k=12$

$k=4$ ½ mark for solving for *k*

$\therefore$ the fifth term 1 mark for the fifth term (or a term consistent with the value of *k*)

2 marks

Method 2

$\left(x^2\right)^6, \left(x^2\right)^5\left(\frac{1}{x}\right), \left(x^2\right)^4\left(\frac{1}{x}\right)^2$ 1 mark for determining the pattern

$x^{12}, x^9, x^6 \ldots$

$\therefore$ the fifth term 1 mark for the fifth term (or a term consistent with the pattern)

2 marks

Exemplar 1

1 3 1 2 -1
1 4 6 3 1
1 1 5 10 10 5 1
1 6 15 20 15 6 1

$$(x^2)^6 + 6(x^2)^5\left(\frac{1}{x}\right) + 15(x^2)^4\left(\frac{1}{x}\right)^2 + 20(x^2)^3\left(\frac{1}{x}\right)^3 + 15(x^2)^2\left(\frac{1}{x}\right)^4 + 6(x^2)\left(\frac{1}{x}\right)^5 + \left(\frac{1}{x}\right)^6$$

↑ term 4

1 out of 2

+ 1 mark for determining the pattern

Exemplar 2

$$\left(x^2+\frac{1}{x}\right)^6 = {}_6C_0(x^2)^6\left(\frac{1}{x}\right)^0 + {}_6C_1(x^2)^5\left(\frac{1}{x}\right)^1 + {}_6C_2(x^2)^4\left(\frac{1}{x}\right)^2 + {}_6C_3(x^2)^3\left(\frac{1}{x}\right)^3 + {}_6C_4(x^2)^2\left(\frac{1}{x}\right)^4 + {}_6C_5(x^2)^1\left(\frac{1}{x}\right)^5 + {}_6C_6(x^2)^0\left(\frac{1}{x}\right)^6$$

$$= x^{12} + 6x^{10}\left(\frac{1}{x}\right) + 15x^8\left(\frac{1}{x^2}\right) + 20x^6\left(\frac{1}{x^3}\right) + 15x^4\left(\frac{1}{x^4}\right) + 6x^2\left(\frac{1}{x^5}\right) + \frac{1}{x^6}$$

$$= x^{12} + 6x^9 + 15x^6 + 20x^3 + 15 + 6x^{-3} + \frac{1}{x^6}$$

0 1 2 3 4 5 6

term 4 = 15

1½ out of 2

award full marks

– ½ mark for procedural error in line 3

Given $f(x) = x^3 + 1$, determine the equation of $f^{-1}(x)$.

Solution

Let $y = f(x)$

$y = x^3 + 1$

To determine the inverse of $f(x)$, switch x and y.

$x = y^3 + 1$

1 mark for switching x and y

$x - 1 = y^3$

$\sqrt[3]{x-1} = y$

½ mark for isolating y

$f^{-1}(x) = \sqrt[3]{x-1}$

½ mark for writing equation in terms of $f^{-1}(x)$

2 marks

Exemplar 1

$x = y^3 + 1$

$-1 \qquad -1$

$x - 1 = \sqrt[3]{y}^{\,3}$

$y = \sqrt[3]{x-1}$

$\boxed{f(x)^{-1} = \sqrt[3]{x-1}}$

1½ out of 2

award full marks
– ½ mark for procedural error in line 2
E7 (notation error in line 4)

Exemplar 2

$y = x^3 + 1$

$\sqrt[3]{}x - 1 = \sqrt[3]{y^3}$

$\sqrt[3]{x-1} = y$

1½ out of 2

+ 1 mark for switching x and y
+ ½ mark for isolating y
E7 (notation error in line 2)

Exemplar 3

$y = x^3 + 1$

$x = y^3 + 1$

$-1 \qquad -1$

$\pm\sqrt[3]{x-1} = \sqrt[3]{y^3}$

$\pm\sqrt[3]{x-1} = y$

$\boxed{\pm\sqrt[3]{x-1} = f(x)^{-1}}$

1½ out of 2

+ 1 mark for switching x and y
+ ½ mark for writing equation in terms of $f^{-1}(x)$
E7 (notation error in line 5)

Question 10 P2

Guillermo was asked to determine the number of ways to select a president, a vice president, and a treasurer from a group of 11 people.

His solution: ${}_{11}C_3$.

Explain why he should have used a permutation instead of a combination.

Solution

He should have used a permutation because the people are being selected for specific positions.

1 mark

Exemplar 1

The situation is not a combination because order matters. The correct solution is $_{11}P_3 = 990$ combinations.

1 out of 1

Exemplar 2

Because that is three different categories that the 11 people are going into. It should be like this: $_{11}C_1 \cdot {}_{10}C_1 \cdot {}_9C_1$.

1 out of 1

Question 11 T6

Prove the following identity for all permissible values of x.

$$\frac{\csc^2 x \sec x}{\tan x + \cot x} = \csc x$$

Solution

Left-Hand Side	Right-Hand Side
$\dfrac{\left(\dfrac{1}{\sin^2 x}\right)\left(\dfrac{1}{\cos x}\right)}{\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}}$	$\csc x$
$\dfrac{\dfrac{1}{\sin^2 x \cos x}}{\dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}}$	
$\dfrac{1}{\sin^{\not 2} x \not{\cos} x} \bullet \dfrac{\not{\sin} x \not{\cos} x}{1}$	
$\dfrac{1}{\sin x}$	
$\csc x$	

1 mark for correct substitution of identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Exemplar 1

Left-Hand Side	Right-Hand Side
$\dfrac{\frac{1}{\sin^2\theta}\cdot\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}$	$\dfrac{1}{\sin\theta}$
$\dfrac{\frac{1}{\sin^2\theta\cos\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}}$	
$\dfrac{1}{\sin^{\not 2}\not\theta\not\cos\not\theta}\cdot\dfrac{\not\cos\not\theta\not\sin\not\theta}{1}$	
$\dfrac{1}{\sin\theta}$	

Q.E.D.

3 out of 3

award full marks
E3 (variable introduced without being defined)

Exemplar 2

Left-Hand Side	Right-Hand Side
$\dfrac{\csc^2 x \sec x}{\tan x + \cot x}$	$\csc x$
$\dfrac{1}{\sin^2 x}\left(\dfrac{1}{\cos x}\right)$	$= \dfrac{1}{\sin x}$
$\dfrac{\dfrac{1}{\sin^2 x \cos x}}{\left(\dfrac{\sin x}{\sin x}\right)\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\left(\dfrac{\cos x}{\cos x}\right)}$	
$\dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}$	
$\dfrac{\dfrac{1}{\sin^2 x \cos x}}{\dfrac{1}{\sin x \cos x}}$	
$\dfrac{1}{\sin^2 \cos x} \times \dfrac{\sin x \cos x}{1}$	
$\dfrac{\sin x \cos x}{\sin^2 x \cos x} = \dfrac{1}{\sin x}$	

LHS = RHS

2 out of 3

+ 1 mark for correct substitution of identities
+ 1 mark for algebraic strategies
E3 (variable omitted in an identity in line 6)

This page was intentionally left blank.

Question 12 R8, R10

Determine the value of x, algebraically.

$$5\log_a 2 - \frac{1}{4}\log_a 16 = \log_a x$$

Solution

$\log_a 2^5 - \log_a 16^{\frac{1}{4}} = \log_a x$ 1 mark for power law (½ mark for each)

$\log_a \left(\frac{32}{2}\right) = \log_a x$ 1 mark for quotient law

$\log_a 16 = \log_a x$

$x = 16$ 1 mark for equating arguments

3 marks

Exemplar 1

$$5\log_a 2-\frac{1}{4}\log_a 16-\log_a X=0$$

$$\log_a\left(\frac{2^5}{\sqrt[4]{16}\cdot x}\right)=0$$

$$a^0=\frac{2^5}{2x}$$

$$1=\frac{2^5}{2x}$$

$$2x=32$$

$$x=16$$

3 out of 3

Exemplar 2

$$\log_a(2^5)-\log_a 16^{\left(\frac{1}{4}\right)}=\log_a x$$

$$\log_a\left(\frac{2^5}{16^{\frac{1}{4}}}\right)=\log_a x$$

$$\boxed{\frac{2^5}{16^{\frac{1}{4}}}=x}$$

3 out of 3

award full marks
E1 (final answer not stated)

Exemplar 3

$$\log_a 2^5 - \log_a(\sqrt[4]{16}) = \log a\ x$$

$$\log_a\left(\frac{(2^5)}{\sqrt[4]{16}}\right) = \log a\ x$$

$$\frac{32}{2} = x$$

$$\boxed{16 = x}$$

2½ out of 3

award full marks
– ½ mark for procedural error in line 2

Exemplar 4

Let $a = 2$

$$\log_2 2^5 - \log_2 16^{1/4} = \log_2 x$$

$$\frac{\log_2 32 - \log_2 2}{\log_2} = \frac{\log_2 x}{\log_2}$$

$$32 - 2 = x$$

$$\boxed{30 = x}$$

1 out of 3

+ 1 mark for power law

Exemplar 5

$$\log_a 2^5 - \log_a (16^{1/4}) = \log_a x$$

$$\log_a 32 - \log_a \sqrt[4]{16} = \log_a x$$

$$\frac{\log a\left(\frac{32}{\sqrt[4]{16}}\right)}{\log a} = \frac{\log a\, x}{\log a}$$

$$x = \frac{32}{\sqrt[4]{16}} = \frac{32}{2} = 16$$

$$\boxed{x = 16}$$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$4 \quad 4 \cdot 2$

$16 \cdot 2$

$$\begin{array}{r} 16 \\ \times\ 2 \\ \hline 32 \end{array}$$

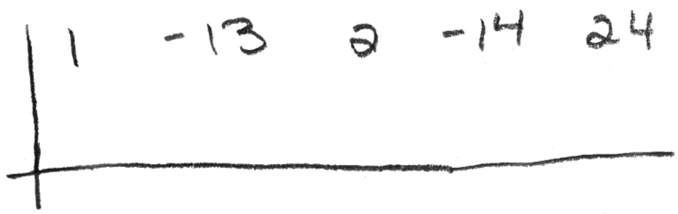
2 out of 3

award full marks
– 1 mark for concept error in line 3

Question 13 R11

Tamara must determine the factors of $x^4 - 13x + 2x^3 - 14x^2 + 24$.

Explain why the coefficients Tamara used to set up her synthetic division are not written correctly.



Solution

Tamara did not arrange the coefficients of the terms in order of descending degree.

1 mark

Exemplar 1

Tamara wrote down here numbers in the order they were given. She should have written down the numbers from highest term to lowest term.

½ out of 1

award full marks
– ½ mark for terminology error in explanation

Exemplar 2

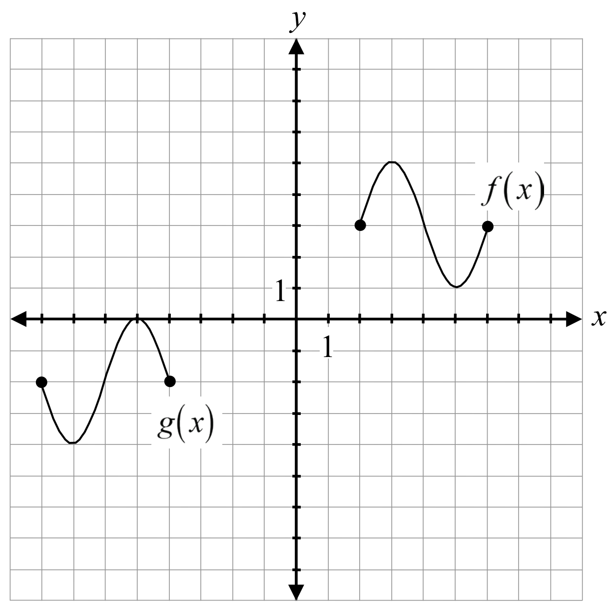
because it needs to be rearranged by descending x

1 2 -14 -13 24

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Determine the equation of the graph of $g(x)$ in terms of $f(x)$.



Solution

$g(x) = \underline{f(-(x+2))-5}$

1 mark for horizontal reflection
1 mark for horizontal translation
1 mark for vertical translation

or

$g(x) = \underline{-f(x+10)+1}$

1 mark for vertical reflection
1 mark for horizontal translation
1 mark for vertical translation

3 marks

Exemplar 1

$g(x) =$ sin(-x +2)-5

1 out of 3

+ 1 mark for horizontal reflection
+ 1 mark for vertical translation
– 1 mark for concept error (incorrect function)

Exemplar 2

$g(x) =$ −g(x +2) −5

1½ out of 3

+ 1 mark for horizontal translation
+ 1 mark for vertical translation
– ½ mark for procedural error (g instead of f)

Exemplar 3

$g(x) =$ −(x+2) −5

2 out of 3

award full marks
– 1 mark for concept error (missing f)

Expand, using the laws of logarithms.

$$\log_2\left[\frac{(x-1)(x-2)}{x}\right]$$

Solution

$\log_2(x-1)+\log_2(x-2)-\log_2 x$

1 mark for product law
1 mark for quotient law

2 marks

Exemplar 1

$\log_2 x - 1 + \log_2 x - 2 - \log_2 x$

2 out of 2

award full marks
E4 (missing brackets but still implied)

Exemplar 2

$= \dfrac{\log_2(x-1) + \log_2(x-2)}{\log_2 x}$

$= \log_2(x-1) + \log_2(x-2) - \log_2 x$

1½ out of 2

award full marks
– ½ mark for procedural error in line 1

Exemplar 3

$(x-1)\log_2 + (x-2)\log_2 - x\log_2$

1 out of 2

award full marks
– 1 mark for concept error (changing argument to coefficient)

This page was intentionally left blank.

Scoring Guidelines for Booklet 2 Questions

Answer Key for Selected Response Questions

Question	Answer	Learning Outcome
16	D	R3
17	A	P4
18	C	T1
19	C	R12
20	B	R14
21	A	R7
22	B	P3
23	B	R1

Question 16 R3

Identify the range of the function $g(x)=\frac{1}{2}f(x+1)$, given that the range of the function $y=f(x)$ is $[-6,4]$.

a) $[-12,8]$

b) $[-7,3]$

c) $[-5,5]$

d) $[-3,2]$

Question 17 P4

Identify the value of a, given that there are 11 terms in the expansion of $(3x^4-y)^{2a}$.

a) 5

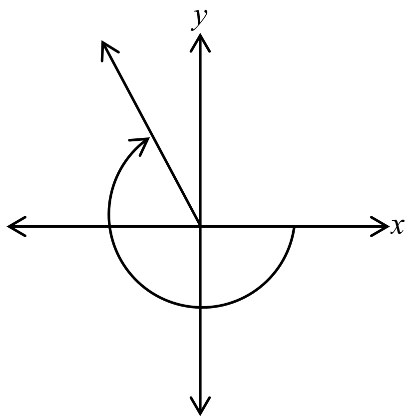
b) 6

c) 10

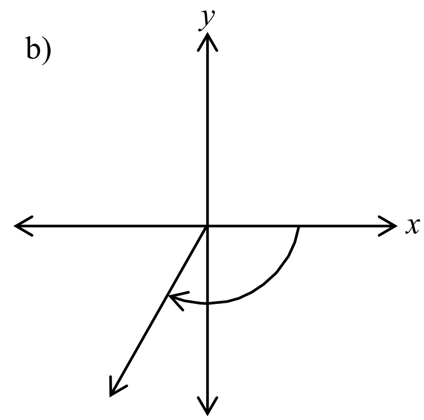
d) 11

Identify the angle that best represents $\theta = -\frac{6\pi}{5}$.

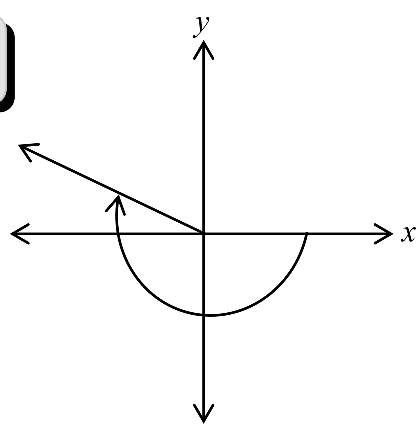
a)



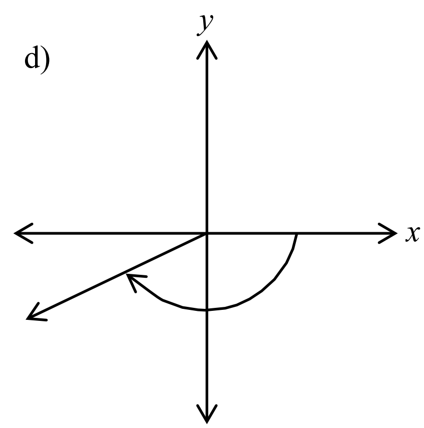
b)



c)



d)



Question 19 R12

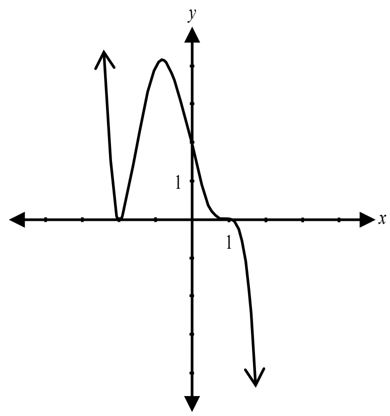
Identify a possible value for *n*, given the graph of $y=-\frac{1}{2}(x+2)^2(x-1)^n$.

a) 1

b) 2

c) 3

d) 4



Question 20 R14

Identify the statement that is false, given $g(x)=\frac{8x^2}{x^2-16}$.

a) the graph of $g(x)$ has one x-intercept.

b) the graph of $g(x)$ has a point of discontinuity (hole) at $x=0$.

c) the graph of $g(x)$ has two vertical asymptotes.

d) the graph of $g(x)$ has a horizontal asymptote at $y=8$.

Question 21 R7

Identify the equivalent form of $\log_a\left(\frac{1}{x^2}\right)$.

a) $-2\log_a x$

b) $1-2\log_a x$

c) $2\log_a x$

d) $-2\log_a\left(\frac{1}{x}\right)$

Question 22 P3

Identify which one of the following expressions is equivalent to ${}_{13}C_6$.

a) ${}_{13}P_6$

b) ${}_{13}C_7$

c) ${}_{12}P_7$

d) ${}_{12}C_6$

Question 23 R1

Identify the equation of $h(x) = f(x) - g(x)$, given $f(x) = x + 5$ and $g(x) = 4x + 1$.

a) $h(x) = -3x + 6$

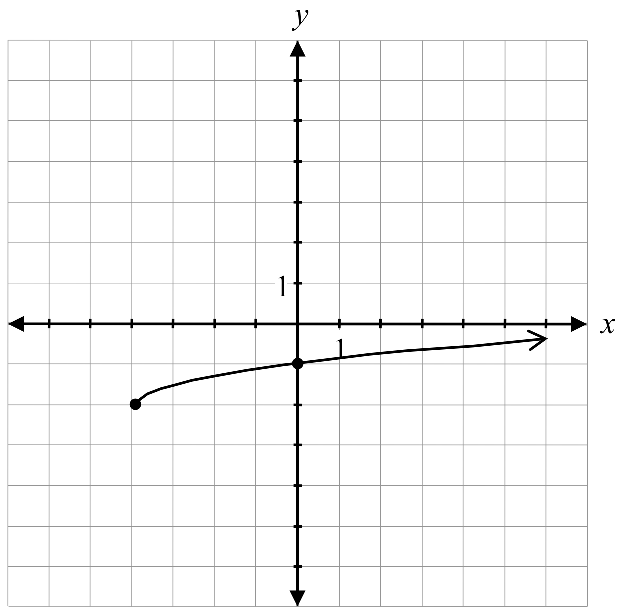
b) $h(x) = -3x + 4$

c) $h(x) = 3x + 6$

d) $h(x) = 3x - 4$

This page was intentionally left blank.

Determine the equation of the radical function represented by the graph.



Solution

$y = \frac{1}{2}\sqrt{x+4} - 2$

1 mark for vertical compression
1 mark for horizontal translation
1 mark for vertical translation

3 marks

or

$y = \sqrt{\frac{1}{4}(x+4)} - 2$

1 mark for horizontal stretch
1 mark for horizontal translation
1 mark for vertical translation

3 marks

Exemplar 1

$$y = \sqrt{\tfrac{1}{4}x+4} \; - 2$$

2 out of 3

+ 1 mark for horizontal stretch
+ 1 mark for vertical translation

Exemplar 2

$$f(x) = 4\sqrt{(x+4)} \; - 2$$

2 out of 3

+ 1 mark for horizontal translation
+ 1 mark for vertical translation

Exemplar 3

$$\boxed{y = \left(\tfrac{1}{4}(x+4)\right) - 2}$$

2 out of 3

award full marks
– 1 mark for concept error (incorrect function)

Exemplar 4

$$y = f\sqrt{4(x+4)} \; - 2$$

1 out of 3

+ 1 mark for horizontal translation
+ 1 mark for vertical translation
– 1 mark for concept error (introducing f)

Question 25 T3

Determine the exact value of *x*.

$$\sec\left(\frac{2\pi}{3}\right)\left(\sin\left(-\frac{5\pi}{3}\right)\right)(x) = 3$$

Solution

$$(-2)\left(\frac{\sqrt{3}}{2}\right)(x) = 3$$

$$-\sqrt{3}(x) = 3$$

$$x = -\frac{3}{\sqrt{3}}$$

or

$$x = -\sqrt{3}$$

1 mark for $\sec\left(\frac{2\pi}{3}\right)$ (½ mark for value; ½ mark for quadrant)

1 mark for $\sin\left(-\frac{5\pi}{3}\right)$ (½ mark for value; ½ mark for quadrant)

2 marks

Exemplar 1

$$\left(\frac{3}{2\pi}\right)\left(\frac{\sqrt{3}}{2}\right)(x)=3$$

$$\left(\frac{3\sqrt{3}}{4\pi}\right)\cdot 4\pi\,(x)=3\cdot 4\pi$$

$$\frac{(3\sqrt{3})(x)}{(3\sqrt{3})}=\frac{12\pi}{3\sqrt{3}}$$

$$x=\frac{12\pi}{3\sqrt{3}}$$

1 out of 2

+ 1 mark for $\sin\left(-\frac{5\pi}{3}\right)$

Exemplar 2

$$\left(\frac{-2}{1}\right)\left(\frac{\sqrt{3}}{2}\right)(x)=3$$

$$\frac{-\sqrt{6}}{2}(x)=3$$

$$\frac{\sqrt{6}}{2}-\frac{3}{1}=x$$

$$\boxed{\frac{\sqrt{6}-3}{2}}=x$$

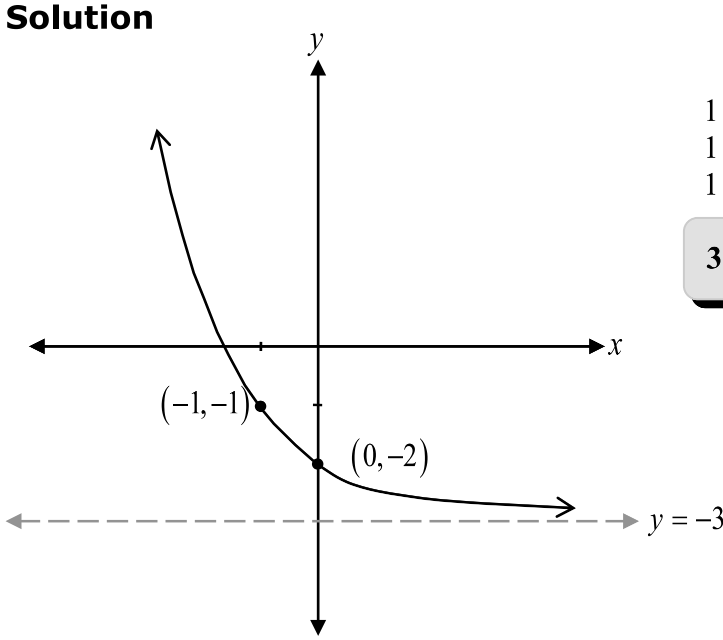
1½ out of 2

award full marks
– ½ mark for arithmetic errors

Question 26 R9

Sketch the graph of $y = 2^{-x} - 3$.

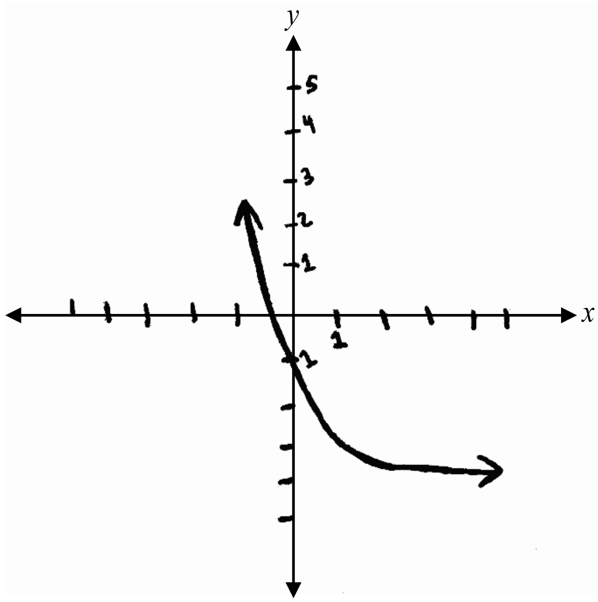
Solution



1 mark for shape of an exponential function
1 mark for horizontal reflection
1 mark for vertical translation

3 marks

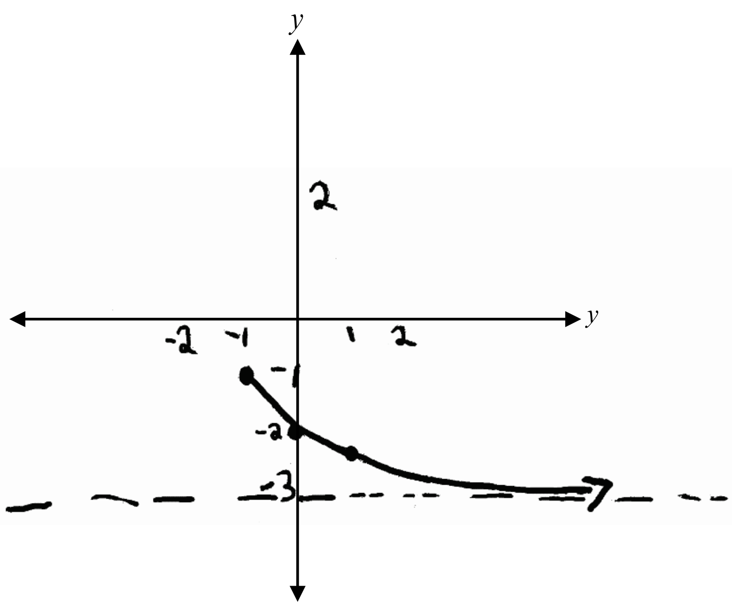
Exemplar 1



1½ out of 3

+ 1 mark for shape of an exponential function
+ 1 mark for horizontal reflection
– ½ mark for procedural error (minimum of 2 points required)
E10 (asymptote omitted but still implied)

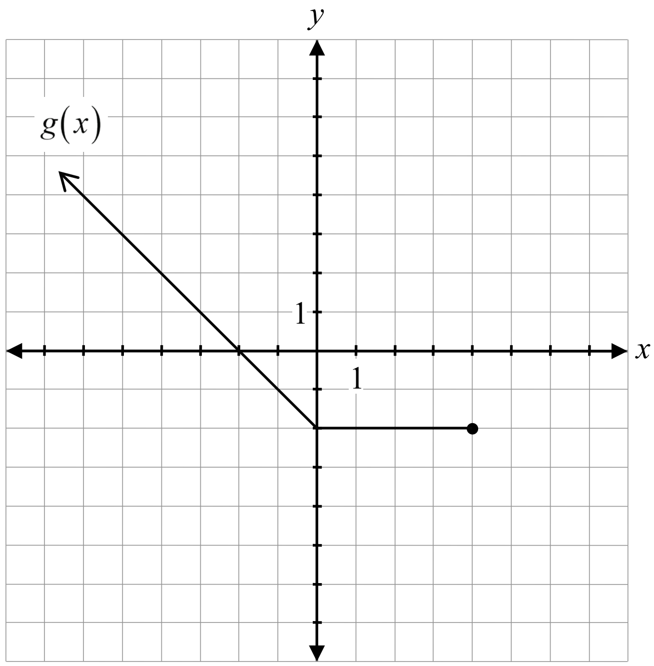
Exemplar 2



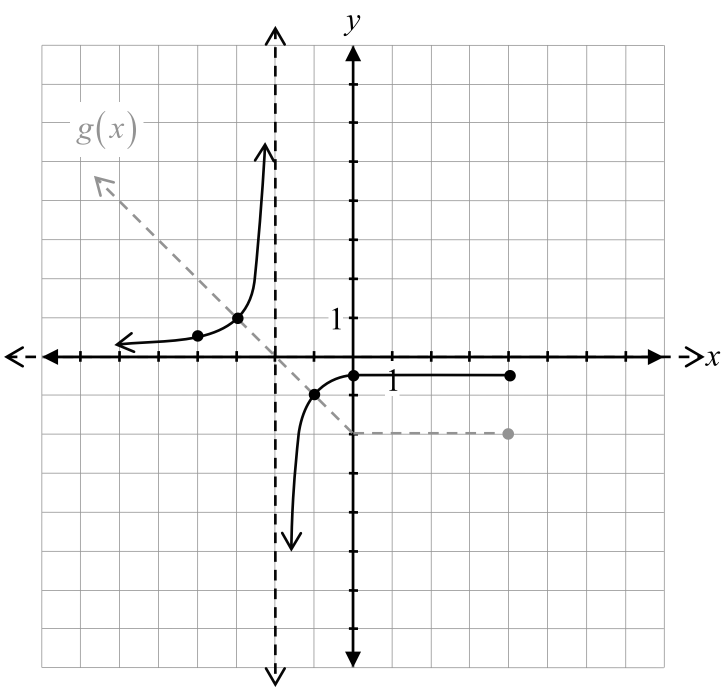
2 out of 3

+ 1 mark for horizontal reflection
+ 1 mark for vertical translation

Given the graph of $y = g(x)$, sketch the graph of $y = \frac{1}{g(x)}$.



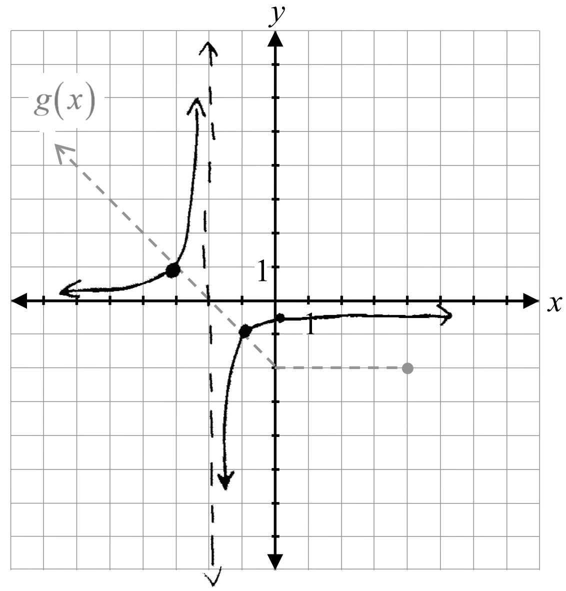
Solution



1 mark for asymptotic behaviour approaching $x = -2$ (½ mark for each side)
½ mark for asymptotic behaviour approaching $y = 0$
½ mark for graph left of $x = -2$
½ mark for graph right of $x = -2$
½ mark for restricting domain

3 marks

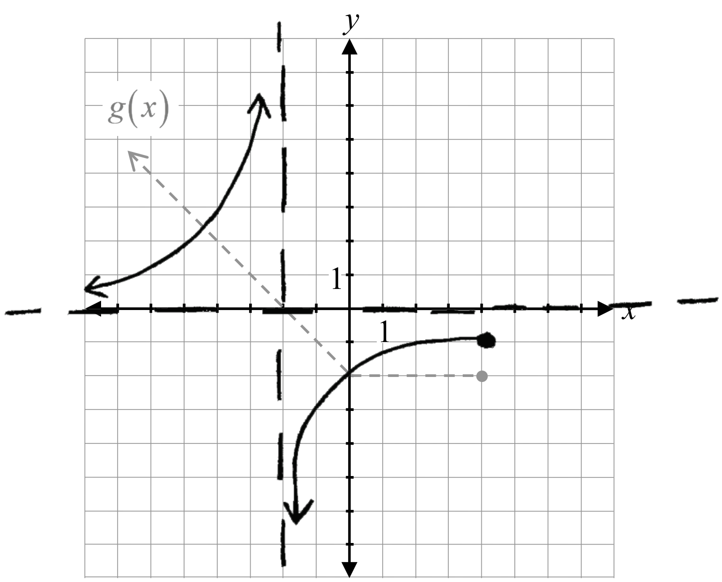
Exemplar 1



2½ out of 3

+ 1 mark for asymptotic behaviour approaching $x = -2$
+ ½ mark for asymptotic behaviour approaching $y = 0$
+ ½ mark for graph left of $x = -2$
+ ½ mark for graph right of $x = -2$
E10 (asymptote omitted but still implied)

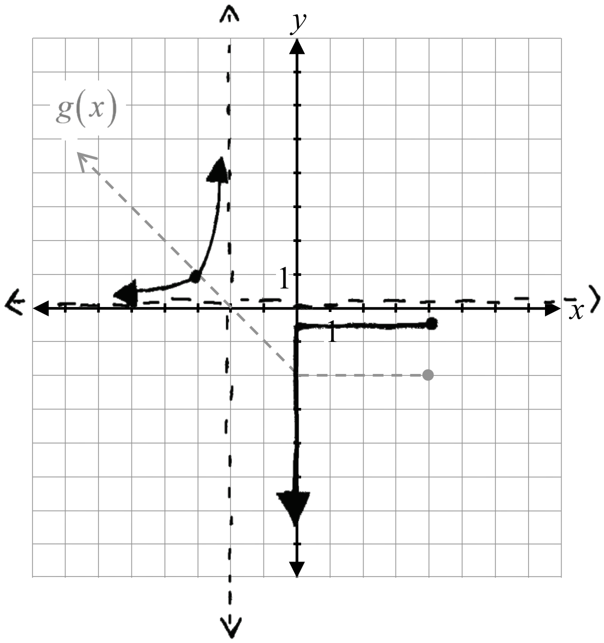
Exemplar 2



2 out of 3

+ 1 mark for asymptotic behaviour approaching $x = -2$
+ ½ mark for asymptotic behaviour approaching $y = 0$
+ ½ mark for restricting domain

Exemplar 3



2 out of 3

+ ½ mark for asymptotic behaviour approaching $x = -2$
+ ½ mark for asymptotic behaviour approaching $y = 0$
+ ½ mark for graph left of $x = -2$
+ ½ mark for restricting domain

This page was intentionally left blank.

Question 28 T6

Determine the exact value of $\tan\left(\frac{\pi}{12}\right)$.

Solution

$$\tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$$

$$\tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$$

1 mark for substitution into correct identity

$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$$

1 mark for exact values (½ mark for each value)

2 marks

or

$$= 2 - \sqrt{3}$$

or

$$= \frac{3 - \sqrt{3}}{3 + \sqrt{3}}$$

Note:

- Other combinations are possible.

Exemplar 1

$$\tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= \sqrt{3} - 1$$

1 out of 2

+ 1 mark for exact values

Exemplar 2

$$\tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$$

$$= \frac{\tan(\sqrt{3}) - \tan(1)}{1 + \tan(\sqrt{3})\tan(1)}$$

1 out of 2

award full marks
– 1 mark for concept error in line 2

Exemplar 3

$$\tan\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$

$$= \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{6}}{1 + \left(\tan\frac{\pi}{3}\tan\frac{\pi}{6}\right)}$$

$$= \frac{\left(\frac{\sqrt{3}}{\sqrt{3}}\right)\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \left(\frac{\sqrt{3}}{1} \cdot \frac{1}{\sqrt{3}}\right)}$$

$$= \frac{\frac{3}{\sqrt{3}} - \frac{1}{\sqrt{3}}}{1 + 1}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{1}}$$

$$= \frac{1}{\sqrt{3}}$$

1½ out of 2

award full marks
– ½ mark for procedural error (incorrect combination)

Exemplar 4

$\tan 15^\circ$

$\tan(45^\circ - 30^\circ)$

$\tan\left(1 - \frac{1}{\sqrt{3}}\right)$

$$\frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)}$$

$$\frac{\cancel{1} - \cancel{\frac{1}{\sqrt{3}}}}{\cancel{1} + \cancel{\frac{1}{\sqrt{3}}}} = 1 - 1 = \boxed{0}$$

---

1 out of 2

award full marks
– ½ mark for procedural error in line 3
– ½ mark for arithmetic error in line 6

Explain why the graph of $g(x) = \dfrac{3}{x^2 + 4}$ does not have a vertical asymptote.

Solution

The graph of a rational function has a vertical asymptote when the denominator is equal to zero. The denominator of $g(x)$ will never be equal to zero.

1 mark

Exemplar 1

g(x) does not have a vertical asymptote because it is found in the denominator, however $(x^2 + 4)$ does not factor ∴ having no asymptotes.

0 out of 1

Exemplar 2

Because there is no 'x' in the numerator ∴ no vertical asymptote.

0 out of 1

Exemplar 3

$g(x) = \frac{3}{x^2+4}$ has no non-permissible value.

1 out of 1

Exemplar 4

Because no matter what x is, it will always be positive when you square it.

0 out of 1

Question 30 R10

Solve, algebraically.

$$\log_3 x + \log_3 (x+8) = 2$$

Solution

Method 1

$\log_3 [x(x+8)] = 2$	1 mark for product law
$x^2 + 8x = 3^2$	1 mark for exponential form
$x^2 + 8x - 9 = 0$	
$(x+9)(x-1) = 0$	
~~$x = -9$~~ $x = 1$	½ mark for the permissible value of x ½ mark for showing the rejection of the extraneous root

3 marks

Method 2

$\log_3 x + \log_3 (x+8) = \log_3 3^2$	1 mark for logarithmic form
$\log_3 [x(x+8)] = \log_3 9$	1 mark for product law
$x^2 + 8x = 9$	
$x^2 + 8x - 9 = 0$	
$(x+9)(x-1) = 0$	
~~$x = -9$~~ $x = 1$	½ mark for the permissible value of x ½ mark for showing the rejection of the extraneous root

3 marks

Exemplar 1

$$\log_3(x(x+8)) = 2$$

$$\log_3(x^2+8x) = \log_3 9$$

$$x^2+8x = 9$$

$$-9 \quad -9$$

$$x^2+8x-9 = 0$$

$$(x-9)(x+1)$$

$$x = 9$$

~~$x = -1$~~

$$\boxed{x = 9}$$

2½ out of 3

award full marks
– ½ mark for arithmetic error in line 6
E2 (changing an equation to an expression in line 6)

Exemplar 2

$$\log_3 x + \log_3(x+8) = \log_3 9$$

$$x + x + 8 = 9$$

$$2x = 1$$

$$x = \frac{1}{2}$$

1 out of 3

+ 1 mark for logarithmic form

Exemplar 3

$$\log_3((x)(x+8)) = \log_3 3^2$$

$$\log_3(x^2+8x) = \log_3 3^2$$

$$x^2 + 8x = 2$$

$$-2$$

$$x^2+8x-2=0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-8 \pm \sqrt{8^2-4(1)(-2)}}{2(1)}$$

$$= \frac{-8 \pm \sqrt{64+8}}{2}$$

$$\boxed{= \frac{-8 \pm \sqrt{72}}{2}}$$

2 out of 3

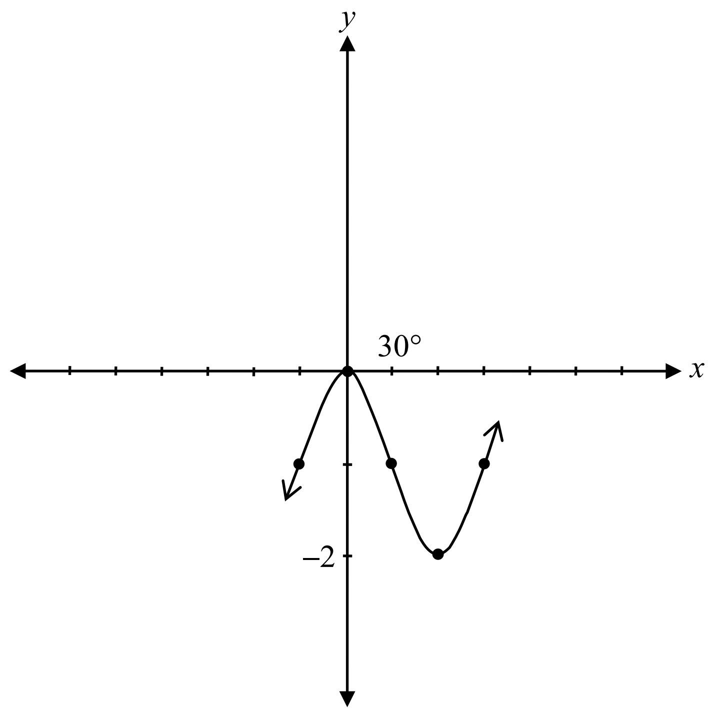
+ 1 mark for logarithmic form
+ 1 mark for product law
+ ½ mark for the permissible value of *x*
– ½ mark for procedural error in line 3

This page was intentionally left blank.

Question 31 T4

Sketch at least one period of the graph of the function $y = \sin\left(3\left(x + 30°\right)\right) - 1$.

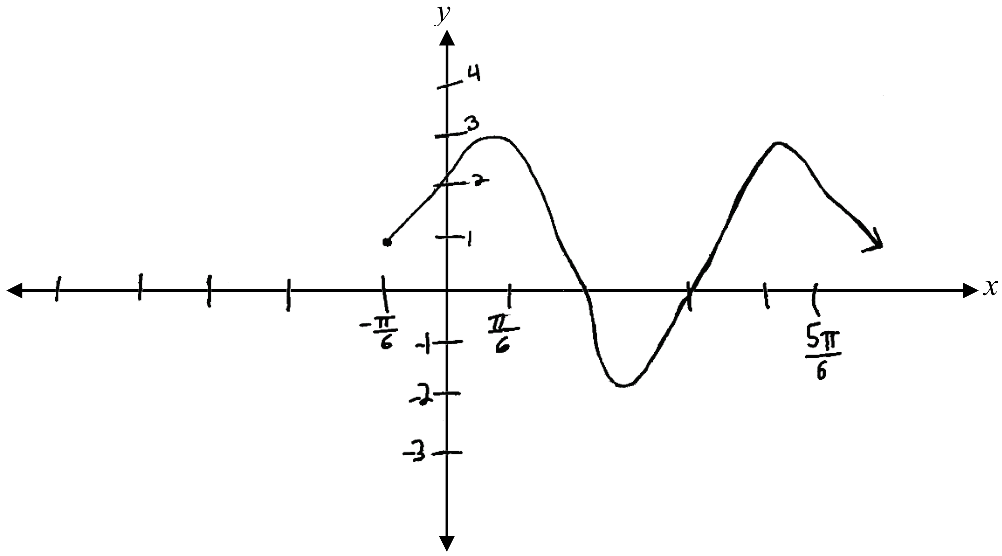
Solution



1 mark for shape of a sinusoidal function with correct amplitude
1 mark for period
1 mark for horizontal translation
1 mark for vertical translation

4 marks

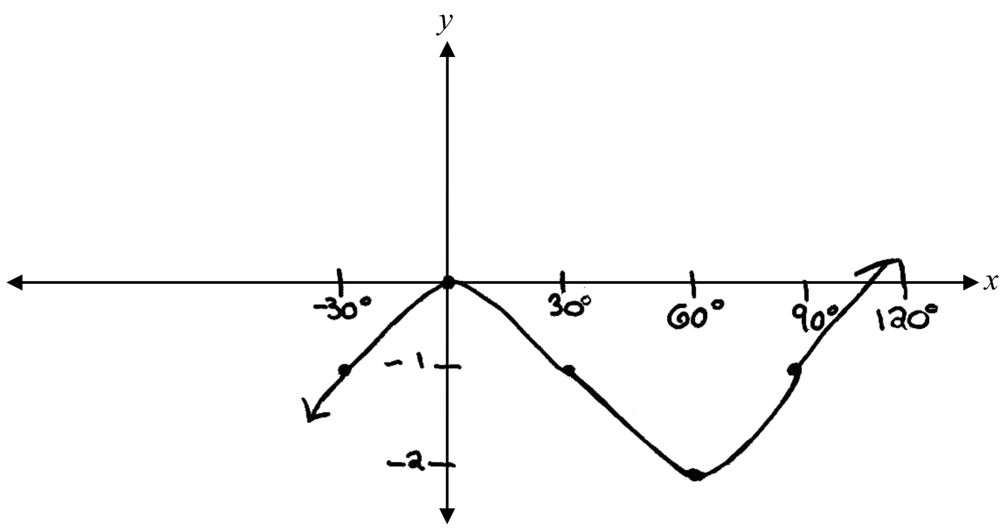
Exemplar 1



1 out of 4

+ 1 mark for horizontal translation
E5 (answer stated in radians instead of degrees)

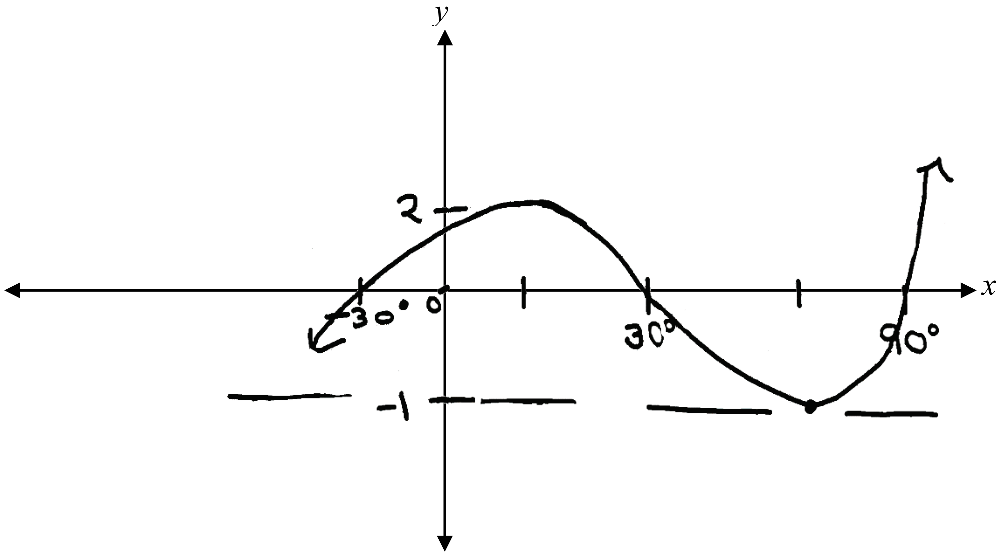
Exemplar 2



3 out of 4

+ 1 mark for period
+ 1 mark for horizontal translation
+ 1 mark for vertical translation

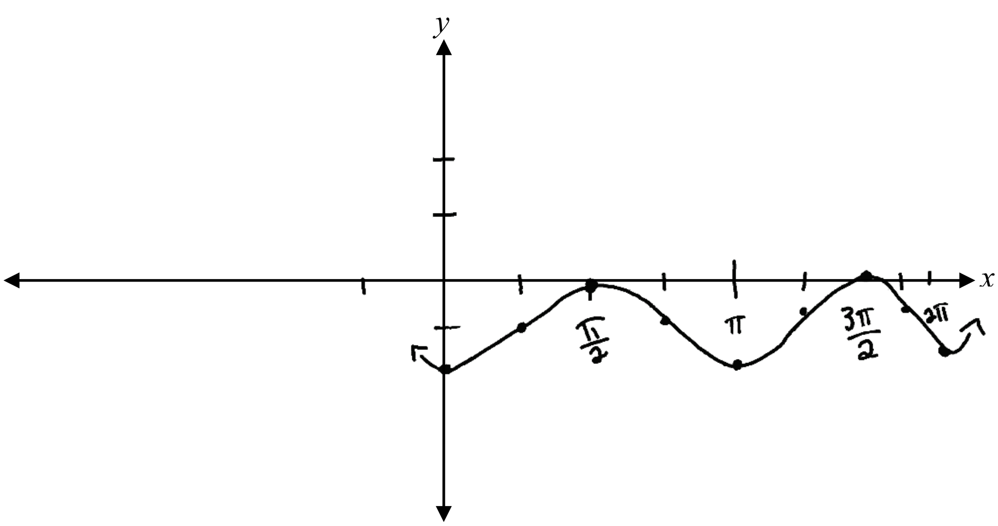
Exemplar 3



1½ out of 4

+ 1 mark for period
+ 1 mark for horizontal translation
– ½ mark for procedural error (inconsistent scale)

Exemplar 4



2 out of 4

+ 1 mark for shape of a sinusoidal function with correct amplitude
+ 1 mark for vertical translation
E9 (scale values on *y*-axis not indicated)

This page was intentionally left blank.

Question 32 R9

Explain why the domain of the function, $f(x) = \log(x - 3)$, is $x > 3$.

Solution

The argument of a logarithm cannot be negative or zero. **1 mark**

Exemplar 1

It is restricted because we cannot take the log of a negative.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 2

because it cannot be negative or zero.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 3

$x - 3 > 0$

$x > 3$

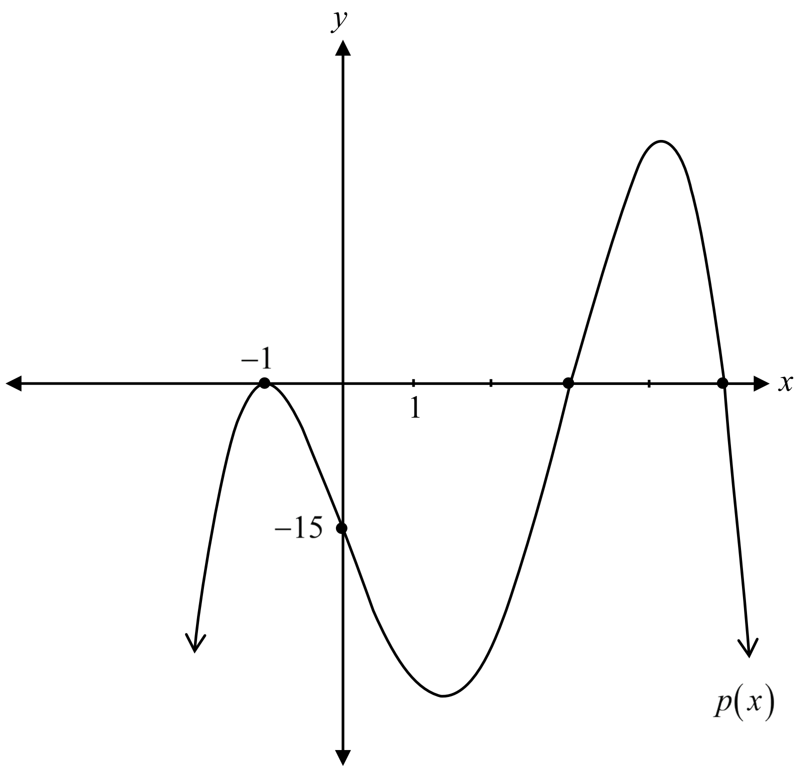
The domain of f(x) is x > 3 because there is a vertical asymptote at x = 3.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Sketch the graph of $p(x) = -(x-3)(x+1)^2(x-5)$.

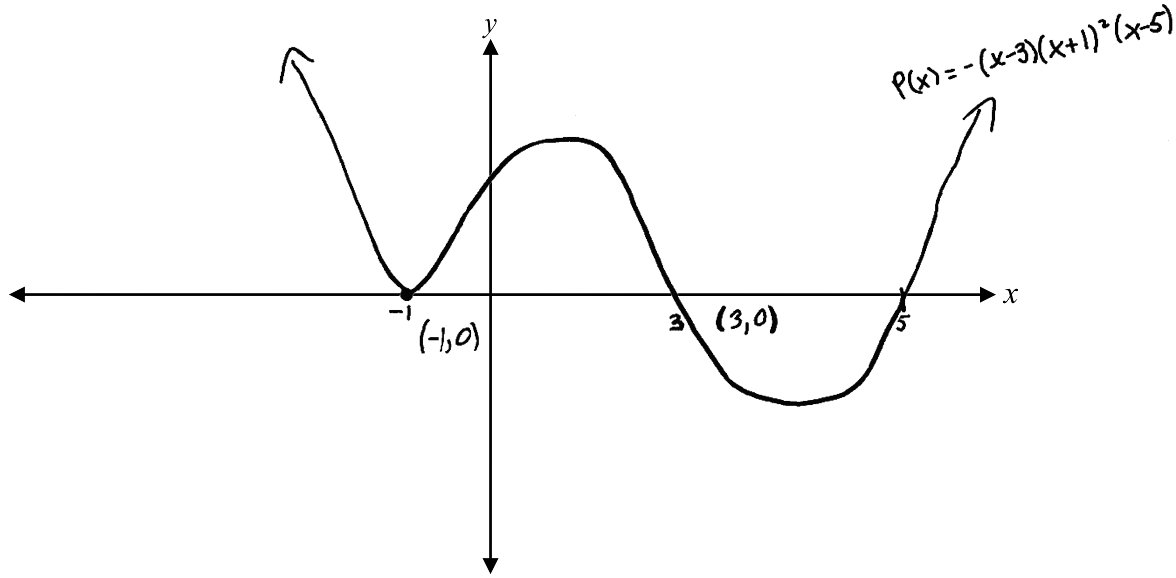
Solution



1 mark for x-intercepts
1 mark for multiplicity of 2 at $x = -1$
½ mark for end behaviour
½ mark for y-intercept

3 marks

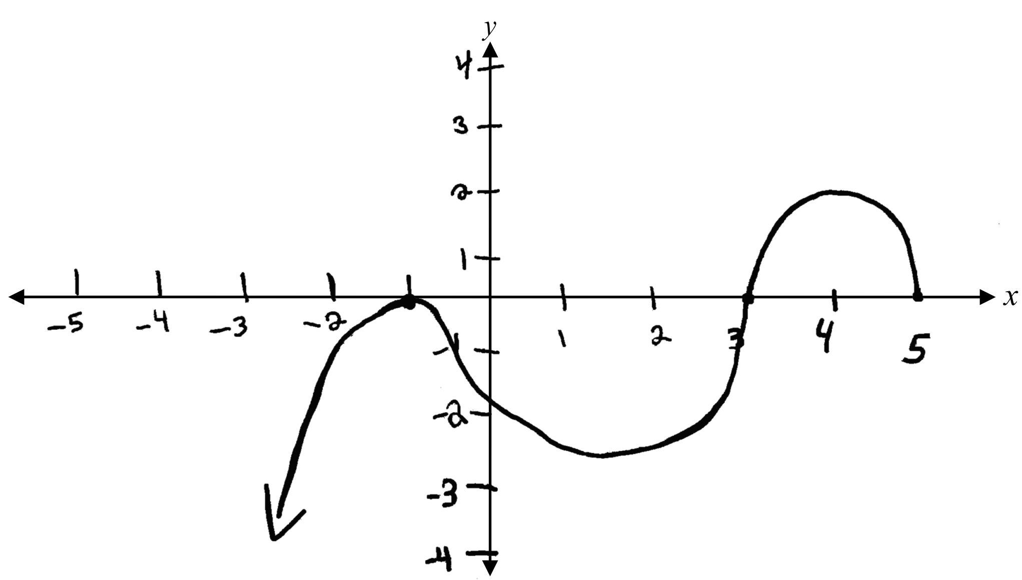
Exemplar 1



2 out of 3

+ 1 mark for x-intercepts
+ 1 mark for multiplicity of 2 at $x = -1$

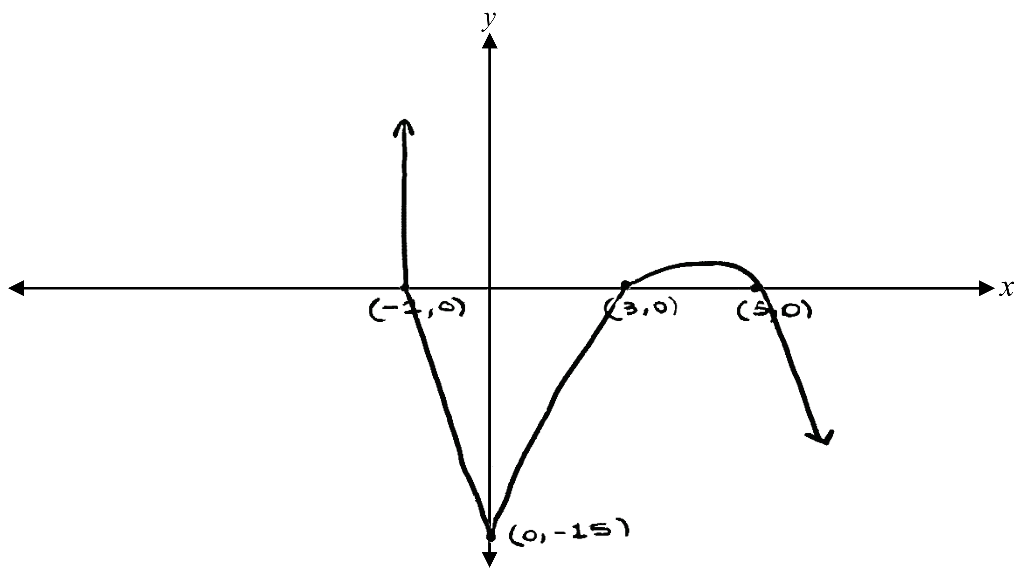
Exemplar 2



2½ out of 3

+ 1 mark for x-intercepts
+ 1 mark for multiplicity of 2 at $x = -1$
+ ½ mark for end behaviour
E9 (arrowhead omitted)

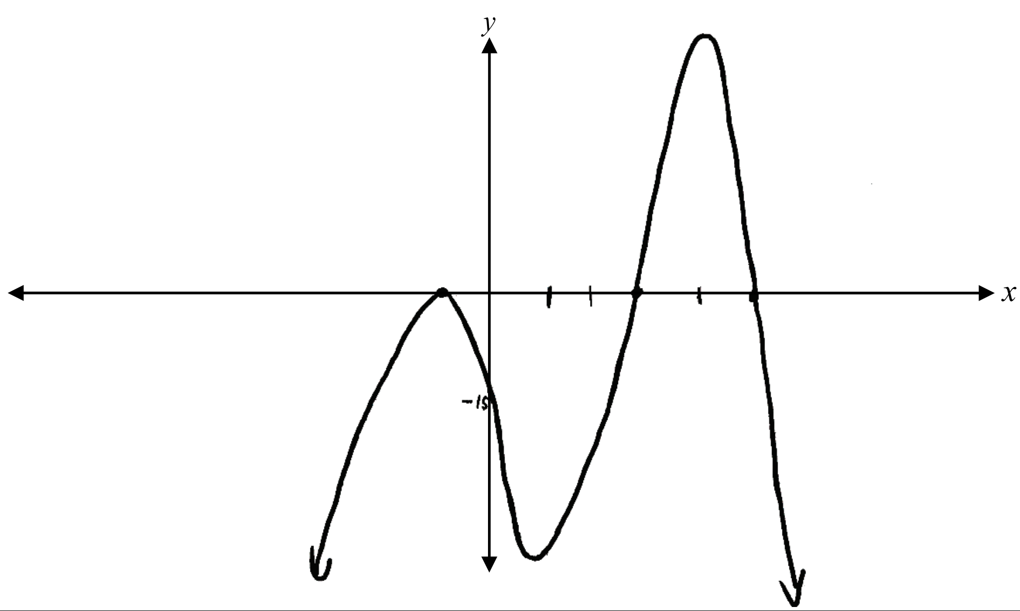
Exemplar 3



1 out of 3

+ 1 mark for x-intercepts
+ ½ mark for y-intercept
– ½ mark for incorrect shape of graph

Exemplar 4



3 out of 3

award full marks
E9 (scale values on x-axis not indicated)

This page was intentionally left blank.

Question 34 T6

Given that $\sin\theta = -\frac{2}{3}$ and $\tan\theta > 0$, determine the exact value of $\sin 2\theta$.

Solution

$x^2 = r^2 - y^2$

$x^2 = 9 - 4$

$x^2 = 5$

$x = \pm\sqrt{5}$ ½ mark for value of x

$x = -\sqrt{5}$

$\cos\theta = -\frac{\sqrt{5}}{3}$ ½ mark for value of $\cos\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(-\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right)$ 1 mark for substitution into correct identity

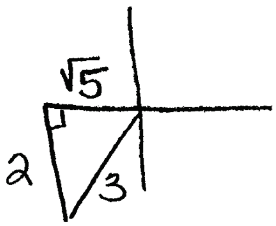
$= \frac{4\sqrt{5}}{9}$

2 marks

Note:

- Accept any of the following values for $x: x = \pm\sqrt{5}$, $x = -\sqrt{5}$, or $x = \sqrt{5}$.

Exemplar 1



$\cos\theta = \frac{\sqrt{5}}{3}$

$\sin 2\theta = 2\sin\theta\cos\theta$

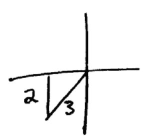
$= 2\sin\left(\frac{-2}{3}\right)\cos\left(\frac{\sqrt{5}}{3}\right)$

½ out of 2

+ ½ mark for value of *x*
+ 1 mark for substitution into correct identity
– 1 mark for concept error

Exemplar 2

$x^2 + 2^2 = 3^2$



$x = \pm\sqrt{9-4}$

$x = \pm\sqrt{5}$

$x = \sqrt{5}$

$\sin 2a = 2\sin\left(\frac{-2}{3}\right)\cos\left(\frac{\sqrt{5}}{3}\right)$

$2\left(\frac{-2}{3}\right)\left(\frac{\sqrt{5}}{3}\right)$

$\sin 2a = \frac{-4\sqrt{5}}{9}$

1 out of 2

+ ½ mark for value of *x*
+ 1 mark for substitution into correct identity
– ½ mark for procedural error in line 5
E3 (variable introduced without being defined)

Question 35 T1

Justify whether $\frac{5\pi}{8}$ and $-\frac{11\pi}{4}$ are coterminal angles.

Solution

Method 1

$$\frac{5\pi}{8}-\left(-\frac{11\pi}{4}\right)$$

$$\frac{5\pi}{8}+\frac{11\pi}{4}$$

$$\frac{5\pi}{8}+\frac{22\pi}{8}$$

$$\frac{27\pi}{8}$$

Since $\frac{27\pi}{8}$ is not a multiple of 2π they are not coterminal angles.

1 mark

Method 2

Coterminal angles of $\frac{5\pi}{8}$:

$$\frac{5\pi}{8}-\frac{16\pi}{8}=-\frac{11\pi}{8}$$

$$-\frac{11\pi}{8}-\frac{16\pi}{8}=-\frac{27\pi}{8}$$

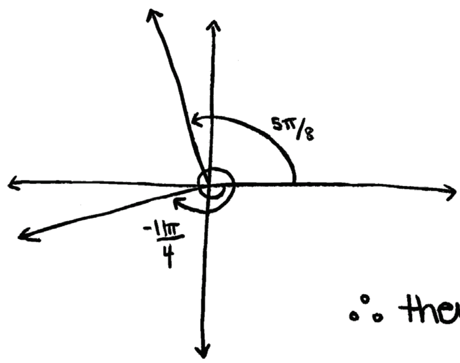
Coterminal angles of $-\frac{11\pi}{4}$:

$$-\frac{11\pi}{4}\left(\frac{2}{2}\right)=-\frac{22\pi}{8}$$

$\therefore$ Angles are not coterminal since $-\frac{27\pi}{8}\neq-\frac{22\pi}{8}$.

1 mark

Exemplar 1



∴ they are not coterminal angles.

1 out of 1

Exemplar 2

$\frac{5\pi}{8} - 2\pi$

$\frac{5\pi}{8} - \frac{16\pi}{8}$

$-\frac{11\pi}{8}$

½ out of 1

award full marks
– ½ mark for lack of clarity in justification

Exemplar 3

$\frac{-11\pi \cdot 2}{4 \cdot 2} = \frac{-22\pi}{8}$

$-\frac{22\pi}{8} + \frac{16\pi}{8}$

$= -\frac{5\pi}{8} + \frac{16\pi}{8}$

$= \frac{11\pi}{8}$

By adding 2 rotations $-\frac{11\pi}{4}$ doesn't equal $\frac{5\pi}{8}$ ∴ They are not coterminal angles.

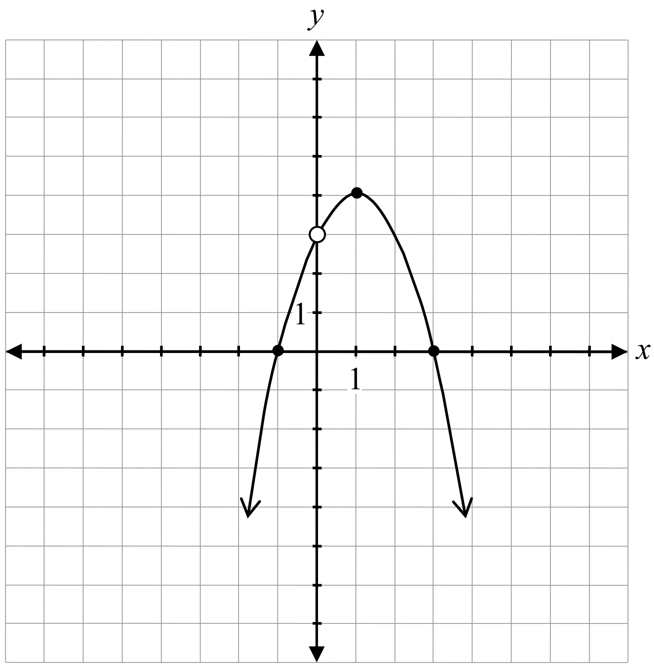
½ out of 1

award full marks
– ½ mark for arithmetic error in line 2

Question 36 R14

Sketch the graph of $f(x) = \dfrac{-2x(x+1)(x-3)}{2x}$.

Solution



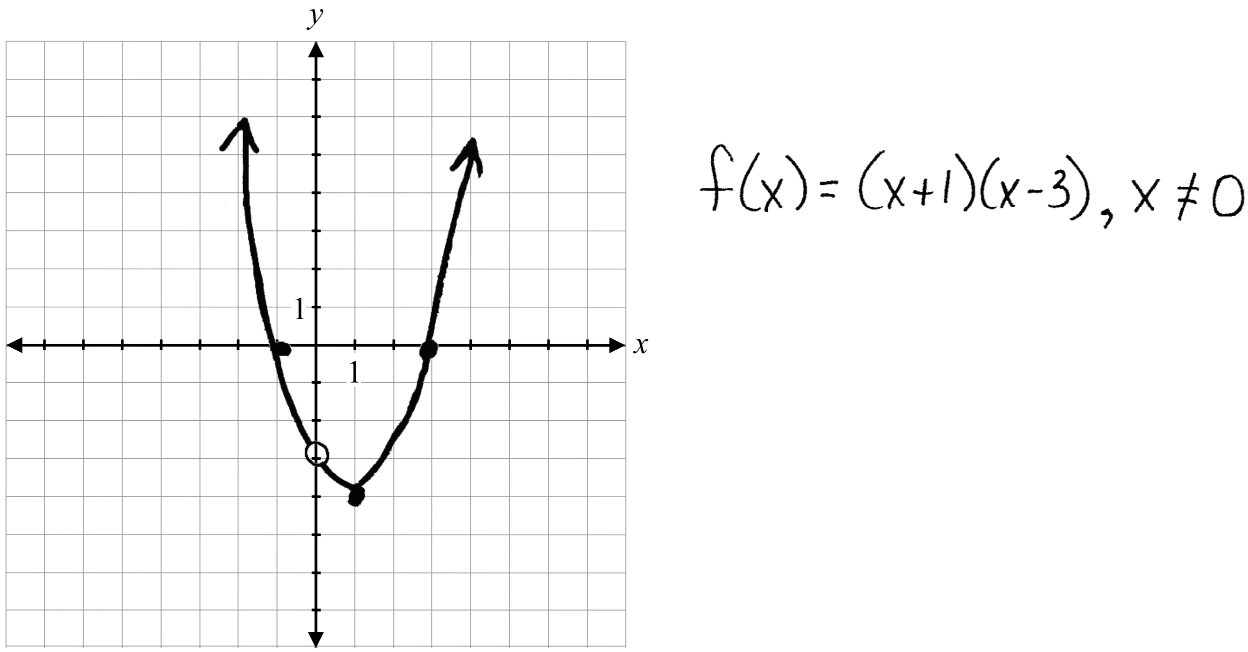
1 mark for point of discontinuity (hole) at $x = 0$
½ mark for shape of a parabola with correct x-intercepts
½ mark for end behaviour

2 marks

Note:

- Deduct ½ mark for procedural error (incorrect y-value for point of discontinuity (hole)).

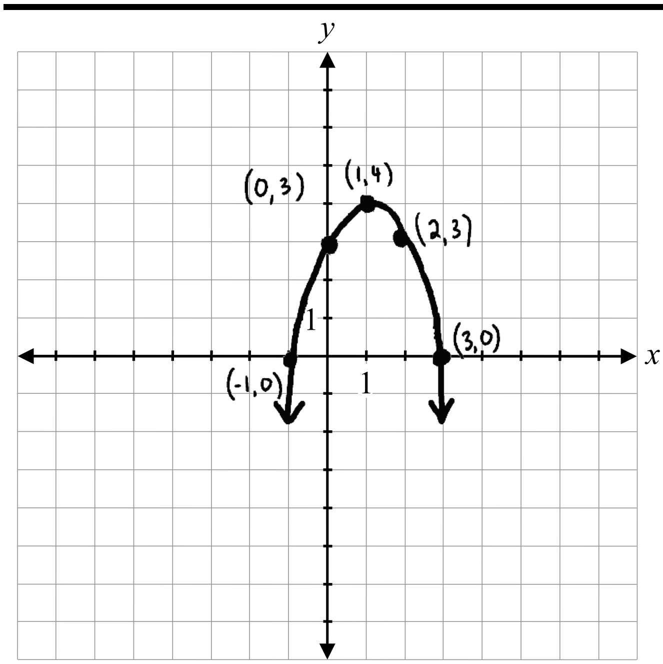
Exemplar 1



1½ out of 2

Award full marks
– ½ mark for procedural error (omitted negative leading coefficient)

Exemplar 2



1 out of 2

+ ½ mark for shape of a parabola with correct x-intercepts
+ ½ mark for end behaviour

Question 37 T5, T6

Given $\frac{\sin\theta + \cos\theta \csc\theta}{\sin\theta}$, determine the non-permissible values of θ, where $\theta \in \mathbb{R}$.

Solution

Method 1

$\sin\theta = 0$	½ mark for $\sin\theta = 0$
$\theta = 0, \pi, 2\pi$	½ mark for any non-permissible value of θ
$\theta = \pi k,\ k \in \mathbb{Z}$	1 mark for all non-permissible values of θ

2 marks

Method 2

$\sin\theta = 0$	½ mark for $\sin\theta = 0$
$\theta = 0, \pi, 2\pi$	½ mark for any non-permissible value of θ
$\left.\begin{array}{l}\theta = 2\pi k,\ k \in \mathbb{Z} \\ \theta = \pi + 2\pi k,\ k \in \mathbb{Z}\end{array}\right\}$	1 mark for all non-permissible values of θ

2 marks

Exemplar 1

$\sin\theta = 0$

$\theta = 0°, 180°, 360°$

$\therefore$ NPV: $\left.\begin{array}{l}\theta \neq 360°k \\ \theta \neq 180° \pm 360°k\end{array}\right\} k \in I$

2 out of 2

Exemplar 2

$\pi + 2\pi n$

$2\pi + 2\pi n$ — non permissible values

1 out of 2

+ ½ mark for $\sin\theta = 0$
+ ½ mark for any non-permissible value of θ

Exemplar 3

$\theta = \pi + \pi k, \; k \in \mathbb{R}$

1 out of 2

+ ½ mark for $\sin\theta = 0$
+ ½ mark for any non-permissible value of θ

Exemplar 4

$\sin\theta = 0$

$\theta = \frac{\pi}{2}$ $\qquad \theta = \frac{\pi}{2} + 2\pi k, \; k \in Z$

1½ out of 2

+ ½ mark for $\sin\theta = 0$
+ 1 mark for all non-permissible value of θ

Question 38 R14

Write an equation of a rational function that has a horizontal asymptote at $y = 0$ and a vertical asymptote at $x = 6$.

Solution

$f(x) = \dfrac{1}{x-6}$

1 mark for horizontal asymptote
1 mark for vertical asymptote

2 marks

Note:

- Other answers are possible.

Exemplar 1

$$\frac{x^2}{(x-6)}$$

½ out of 2

+ 1 mark for vertical asymptote
– ½ mark for procedural error (not written as an equation)

Exemplar 2

$$f(x) = \frac{2x}{x^2(x-6)}$$

2 out of 2

Question 39 R1

Given the functions $f(x)=\sqrt{x-1}$ and $g(x)=x^2$,

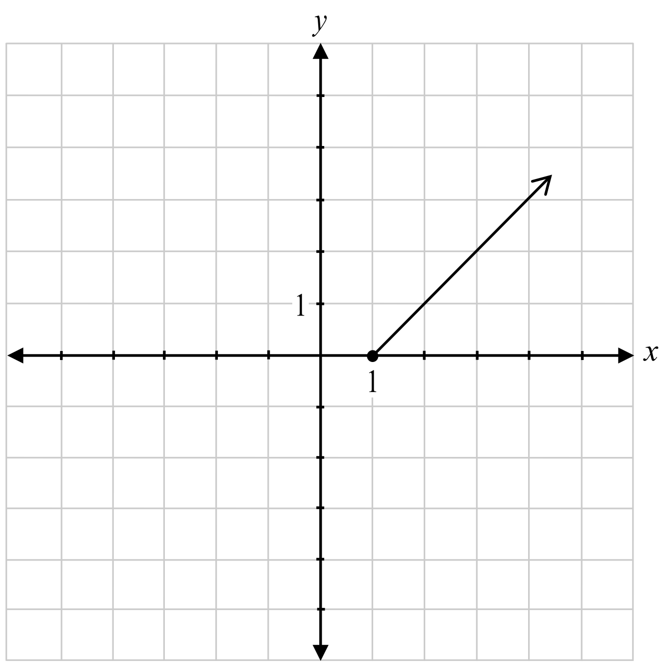
a) state the equation of $g(f(x))$.

b) sketch the graph of $g(f(x))$.

Solution

a) $g(f(x))=$ $\underline{x-1,\ x\geq 1}$ **1 mark**

b)



1 mark for shape of graph consistent with a)
1 mark for restricted domain

2 marks

Notes:

- Deduct a maximum of 1 mark for the concept error of not restricting the domain.
- Deduct ½ mark for procedural error (not stating domain) if graph shows correct domain.

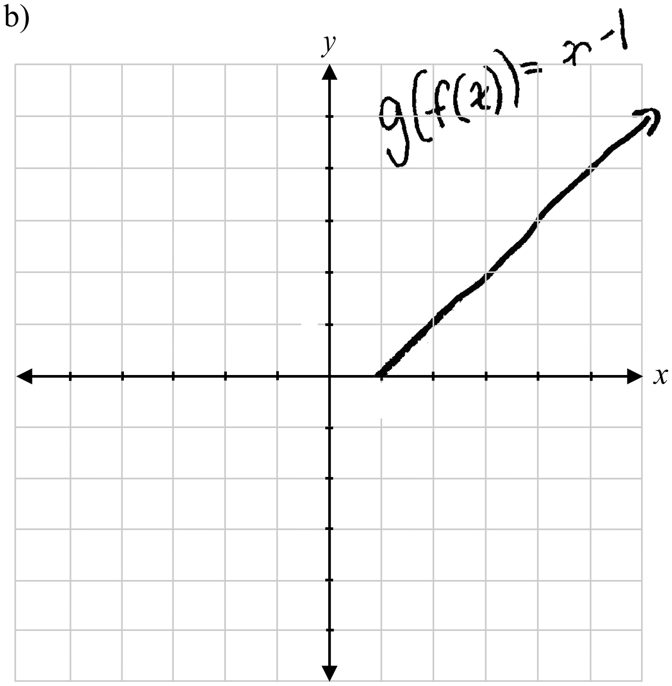
Exemplar 1

a) $g(f(x)) =$ $x - 1$

½ out of 1

award full marks
– ½ mark for procedural error (not stating domain)

b)



2 out of 2

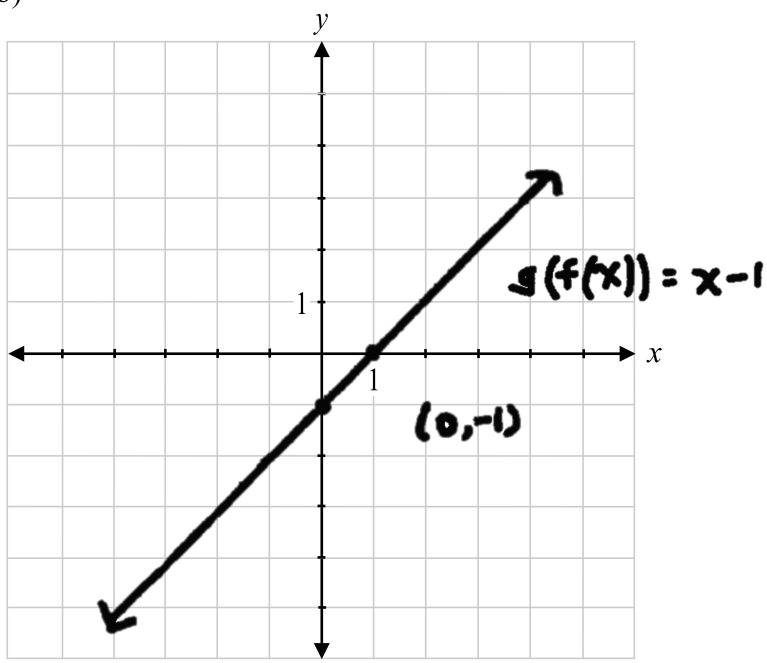
Exemplar 2

$= (\sqrt{x-1})^2$

a) $g(f(x)) =$ $x - 1$

1 out of 1

b)



1 out of 2

+ 1 mark for shape of graph consistent with a)

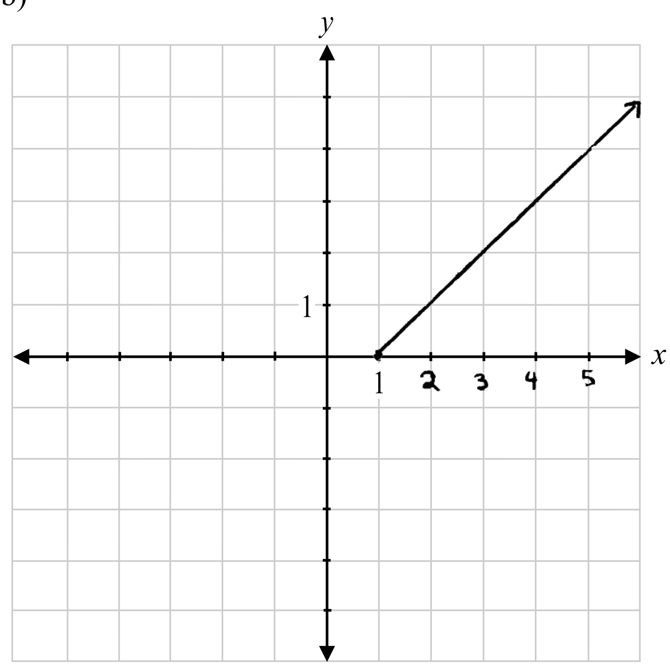
Exemplar 3

$g(f(x)) = (\sqrt{x-1})^2$

a) $g(f(x)) = $ $(\sqrt{x-1})^2$

1 out of 1

b)



2 out of 2

Suzanne was asked to determine the value of $\tan\theta$, given that $\sec\theta = -\frac{8}{3}$ and θ terminates in quadrant II.

Her solution:

$$(-3)^2 + y^2 = (8)^2$$
$$y^2 = 55$$
$$y = \sqrt{55}$$
$$\tan\theta = \frac{\sqrt{55}}{3}$$

Describe her error.

Solution

Suzanne did not consider that the value of $\tan\theta$ is negative in quadrant II.

1 mark

Exemplar 1

Her error is that tanθ should be $-\frac{\sqrt{55}}{8}$ she messed up writing what her correct hypotenuse was in the answer as well as she forgot to write the negative sign.

0 out of 1

Exemplar 2

Her answer is not found in quad 2.

½ out of 1

award full marks
– ½ mark for lack of clarity in description

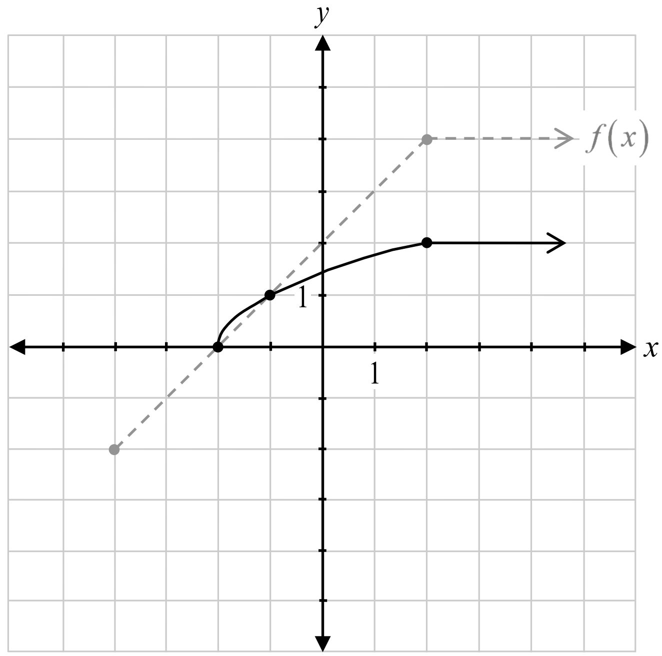
Exemplar 3

Her answer is tanθ positive but the θ needed to be in quadrant II.

1 out of 1

Given the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$.

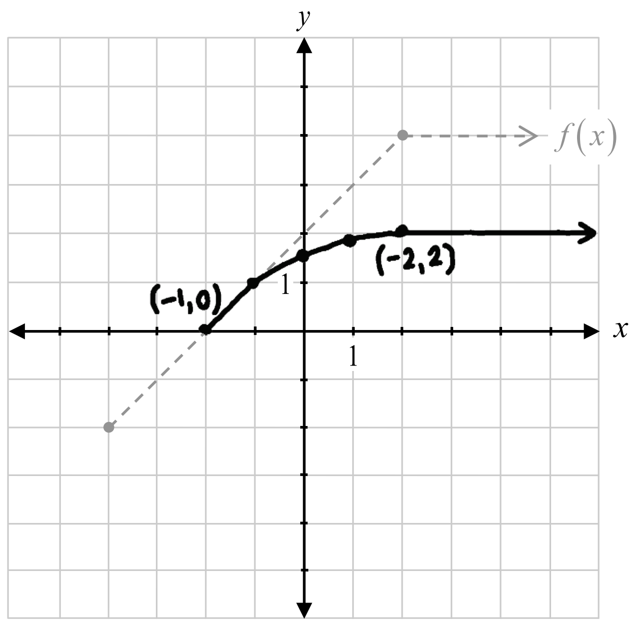
Solution



1 mark for restricting domain
½ mark for shape between invariant points
½ mark for shape to the right of invariant points

2 marks

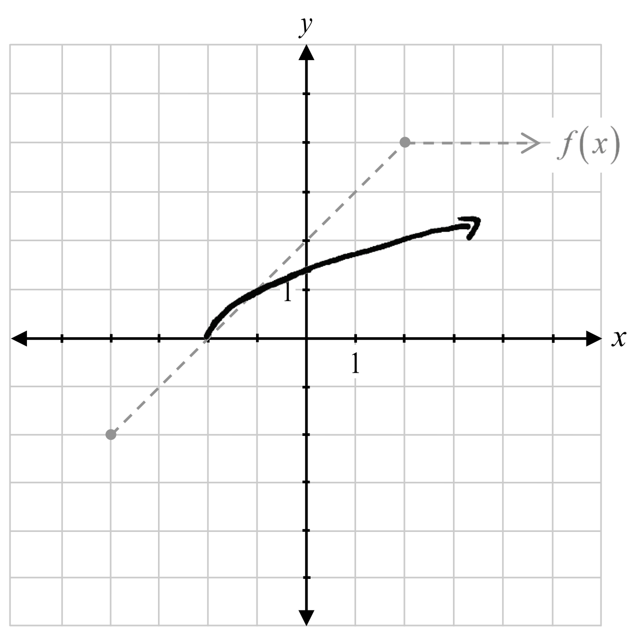
Exemplar 1



1½ out of 2

+ 1 mark for restricting domain
+ ½ mark for shape to the right of invariant points
E9 (coordinate points labelled incorrectly)

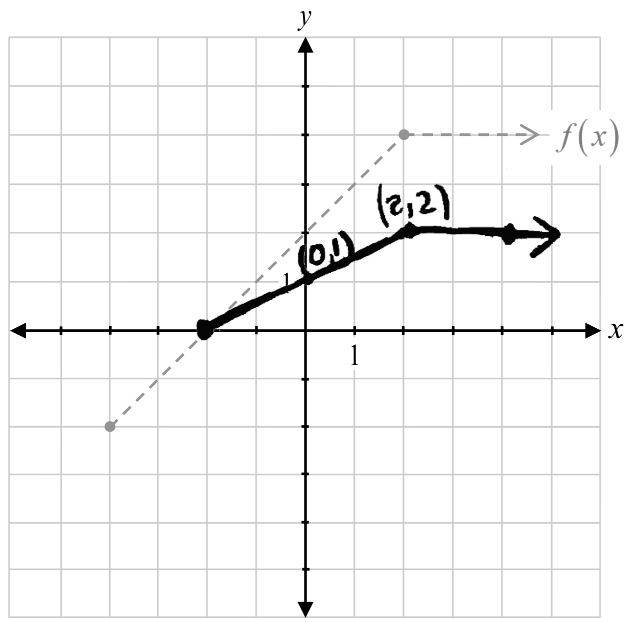
Exemplar 2



1½ out of 2

+ 1 mark for restricting domain
+ ½ mark for shape between invariant points

Exemplar 3



1 out of 2

+ 1 mark for restricting domain

This page was intentionally left blank.

Question 42

T1

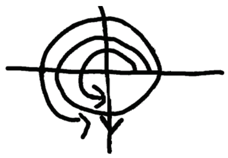
The point $P(\theta) = (0, -1)$ lies on the unit circle. State the angle θ, over the interval $[2\pi, 4\pi]$.

Solution

$$\theta = \frac{7\pi}{2}$$

1 mark

Exemplar 1



$\frac{3\pi}{2} + \frac{4\pi}{2} = \frac{7\pi}{2}$

$\frac{3\pi}{2} + \frac{2(4\pi)}{2} = \frac{11\pi}{2}$

$\theta = \frac{7\pi}{2}, \frac{11\pi}{2}$

1 out of 1

award full marks
E8 (answer outside the given domain)

Exemplar 2

270°
$+\,360^\circ$
630°

1 out of 1

award full marks
E5 (answer stated in degrees instead of radians)

Exemplar 3

$(0,-1) = \frac{3\pi}{2}$

2π —— 4π

$\theta = \frac{5\pi}{2} \quad \frac{7\pi}{2}$

0 out of 1

Question 43 R2

Describe how the transformations of $f(x)$ on the graphs of $g(x) = f(3x-6)$ and $h(x) = f(3(x-6))$ are different.

Solution

The graph of $g(x)$ is a horizontal translation of $f(x)$ two units to the right and the graph of $h(x)$ is a horizontal translation of $f(x)$ six units to the right.

1 mark

Exemplar 1

since the 3 is outside of the bracket on h(x), the other bracket is multiplied by 3 So h(x) has a translation of 18 units right whereas g(x) has a translation of 6 units right.

0 out of 1

Exemplar 2

$g(x) = f(3x-6)$

factor out the 3

$g(x) = f(3(x-2))$

$H(x) = f(3(x-6))$

When you factor out the three they become different because of the shifts.

½ out of 1

award full marks
– ½ mark for lack of clarity in description

Exemplar 3

In the first, the equation is not in the form $g(x) = af(b(x-h))+k$, so to make the transformations we should put it in this form.
The 2nd equation is already in the form $h(x) = af(b(x-h))+k$.

0 out of 1

a) Solve.

$$\sqrt{2x+5}-3=0$$

b) Describe how the solution in a) relates to the graph of $y=\sqrt{2x+5}-3$.

Solution

a) $\left(\sqrt{2x+5}\right)^2=3^2$

$$2x+5=9$$

$$2x=4$$

$$x=2$$

1 mark

b) The solution is the *x*-intercept of the graph. **1 mark**

Exemplar 1

a)

$$\sqrt{2x+5}^{\,2} = 3^2$$
$$2x+5 = 6$$
$$2x = 1$$
$$\boxed{x = \frac{1}{2}}$$

½ out of 1

award full marks
– ½ mark for arithmetic error in line 2

b)

Its the x intercept

1 out of 1

Exemplar 2

a)

$$x = 2$$

1 out of 1

b) $x = 2$ is the point where $y = 0$ in the graph $\sqrt{2x+5} - 3$

$x = 2$ is the solution to the graph of $y = \sqrt{2x+5} - 3$.

½ out of 1

award full marks
– ½ mark for lack of clarity in description

Question 45 R11

Determine all of the zeros of the function $p(x) = x^3 - 2x^2 - 9x + 18$.

Solution

$p(2) = 2^3 - 2(2)^2 - 9(2) + 18$ — 1 mark for identifying one possible zero of $p(x)$

$= 0$

$\therefore (x-2)$ is a factor of $p(x)$

2	1	–2	–9	18
		2	0	–18
	1	0	–9	0

1 mark for synthetic division (or equivalent strategy)

$(x-2)(x^2-9) = 0$ — ½ mark for consistent factors

$(x-2)(x-3)(x+3) = 0$

$x = 2,\ x = 3,\ x = -3$ — ½ mark for consistent zeros

3 marks

Exemplar 1

$0 = x^3 - 9x - 2x^2 + 18$

$0 = x(x^2 - 9) - 2(x^2 - 9)$

$0 = (x^2 - 9)(x - 2)$

$0 = (x + 3)(x - 3)(x - 2)$

$x = -3$

$x = 3$

$x = 2$

3 out of 3

Exemplar 2

2	1	−2	−9	+18
		2	0	−18
	1	0	−9	0

$(x - 2)(x^2 - 9)$

$(x - 2)(x - 3)(x + 3)$

Zeros are

$2, \pm 3$

2½ out of 3

award full marks
– ½ mark for procedural error (not equating factors to zero)

Question 46 T2

Given that the point $\left(\frac{\sqrt{23}}{6}, y\right)$ is on the unit circle, determine the exact value(s) of y.

Solution

$\left(\frac{\sqrt{23}}{6}\right)^2 + y^2 = 1$ ½ mark for substitution

$\frac{23}{36} + y^2 = 1$

$y^2 = \frac{36}{36} - \frac{23}{36}$

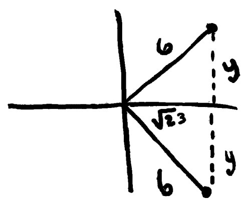
$y = \pm\frac{\sqrt{13}}{6}$ ½ mark for exact values of y

1 mark

Exemplar 1

$\cos\theta = x$

$\cos\theta = \frac{\sqrt{23}}{6}$



$y^2 = 6^2 - \sqrt{23}^2$

$y^2 = 36 - 23$

$y^2 = 13$

$\boxed{y = \pm\sqrt{13}}$

½ out of 1

+ ½ mark for substitution

Exemplar 2

$\sqrt{23} = x$

$6 = r$

$y = ?$

$\sqrt{23}^2 + y^2 = 6^2$

$23 + y^2 = 36$

$y^2 = 13$

$\sqrt{y} = \sqrt{13}$

$y = \sqrt{13}$

$\boxed{y = \frac{\sqrt{13}}{6}}$

½ out of 1

+ ½ mark for substitution

Question 47 T4

State one zero of the function $y = \tan x$.

Solution

Any solution where $x = \pi k,\ k \in \mathbb{Z}$, is acceptable.

1 mark

Appendices

Appendix A: Marking Guidelines

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply:

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation, description, or justification
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet*.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated ▪ impossible solution(s) not rejected in final answer and/or in step leading to final answer
E2 equation/expression	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined
E4 brackets	▪ " $\sin x^2$ " written instead of " $\sin^2 x$ " ▪ missing brackets but still implied
E5 units	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa
E6 rounding	▪ rounding error ▪ rounding too early
E7 notation/transcription	▪ notation error ▪ transcription error
E8 domain/range	▪ answer outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order
E9 graphing	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated or inconsistently spaced ▪ coordinate points labelled incorrectly
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes

Appendix B: Irregularities in Provincial Tests

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.

Irregular Test Booklet Report

Test: ____________________

Date marked: ____________________

Booklet No.: ____________________

Problem(s) noted: ____________________

Question(s) affected: ____________________

Action taken or rationale for assigning marks: ____________________

Follow-up: ____

____

____

____

____

____

Decision: ____

____

____

____

____

____

____

Marker's Signature: ____

Principal's Signature: ____

For Department Use Only—After Marking Complete

Consultant: ____

Date: ____

Appendix C: Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions		
Question	**Learning Outcome**	**Mark**
6	R4, R5	2
9	R6	2
14	R4, R5	3
16	R3	1
23	R1	1
27	R1	3
39a)	R1	1
39b)	R1	2
43	R2	1
Unit B: Trigonometric Functions		
Question	**Learning Outcome**	**Mark**
1	T1	2
18	T1	1
25	T3	2
31	T4	4
35	T1	1
40	T3	1
42	T1	1
46	T2	1
47	T4	1
Unit C: Binomial Theorem		
Question	**Learning Outcome**	**Mark**
3	P2	2
5	P3	2
7	P1	1
8	P4	2
10	P2	1
17	P4	1
22	P3	1

Unit D: Polynomial Functions		
Question	**Learning Outcome**	**Mark**
13	R11	1
19	R12	1
33	R12	3
45	R11	3
Unit E: Trigonometric Equations and Identities		
Question	**Learning Outcome**	**Mark**
2	T5	4
11	T6	3
28	T6	2
34	T6	2
37	T5, T6	2
Unit F: Exponents and Logarithms		
Question	**Learning Outcome**	**Mark**
4	R10	3
12	R8, R10	3
15	R8	2
21	R7	1
26	R9	3
30	R10	3
32	R9	1
Unit G: Radicals and Rationals		
Question	**Learning Outcome**	**Mark**
20	R14	1
24	R13	3
29	R14	1
36	R14	2
38	R14	2
41	R13	2
44a)	R13	1
44b)	R13	1