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General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the Answer/Scoring Sheet are identical
- students and markers use only a pencil to complete the Answer/Scoring Sheets
  - the totals of each of the four parts are written at the bottom
  - each student’s final result is recorded, by booklet number, on the corresponding Answer/Scoring Sheet
  - the Answer/Scoring Sheet is complete
  - a photocopy has been made for school records

Once marking is completed, please forward the Answer/Scoring Sheets to Manitoba Education and Advanced Learning in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of constructed response questions and multiple-choice questions. Constructed response questions are worth 1 to 5 marks each, and multiple-choice questions are worth 1 mark each. An answer key for the multiple-choice questions can be found at the beginning of the section “Booklet 2 Questions.”

To receive full marks, a student’s response must be complete and correct. Where alternative answering methods are possible, the Marking Guide attempts to address the most common solutions. For general guidelines regarding the scoring of students’ responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an Answer/Scoring Sheet is marked with “0” and/or “NR” only (e.g., student was present but did not attempt any questions), please document this on the Irregular Test Booklet Report.
Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Advanced Learning at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

Youyi Sun
Assessment Consultant
Grade 12 Pre-Calculus Mathematics
Telephone: 204-945-7590
Toll-Free: 1-800-282-8069, extension 7590
Email: youyi.sun@gov.mb.ca
Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the Answer/Scoring Sheet that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called “Communication Errors” (see Appendix A) and will be tracked on the Answer/Scoring Sheet in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student’s mark), with a maximum deduction of 5 marks from the total test mark.

The total mark deduction for communication errors for any student response is not to exceed the marks given for that response. When multiple communication errors are made in a given response, any deductions are to be indicated in the order in which the errors occur in the response, without exceeding the given marks.

The student’s final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

<table>
<thead>
<tr>
<th>COMMUNICATION ERRORS / ERREURS DE COMMUNICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shade in the circles below for a maximum total deduction of 5 marks (0.5 mark deduction per error).</td>
</tr>
<tr>
<td>Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).</td>
</tr>
<tr>
<td>E1 ●</td>
</tr>
<tr>
<td>E6 ○</td>
</tr>
</tbody>
</table>

Example: Marks assigned to the student.

<table>
<thead>
<tr>
<th>Marks Awarded</th>
<th>Booklet 1</th>
<th>Multiple Choice</th>
<th>Booklet 2</th>
<th>Communication Errors (Deduct)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
<td>7</td>
<td>40</td>
<td>1½</td>
<td>70½</td>
</tr>
<tr>
<td>Total Marks</td>
<td>36</td>
<td>9</td>
<td>45</td>
<td>maximum deduction of 5 marks</td>
<td>90</td>
</tr>
</tbody>
</table>
Scoring Guidelines
Booklet 1 Questions
Question 1

Convert $\frac{13\pi}{5}$ to degrees.

**Solution**

$$\left(- \frac{13\pi}{5}\right) \times \frac{180^\circ}{\pi} = -468^\circ$$

1 mark
Exemplar 1

\[-\frac{13\pi}{5} \times \frac{180}{\pi}\]

\[-460^\circ\]

\[\frac{1}{2}\] out of 1
award full marks
- \(\frac{1}{2}\) mark for arithmetic error

Exemplar 2

\[-\frac{13\pi}{5} \cdot 180\]

\[= \frac{-2340\pi}{5}\]

\[= -468\pi\]

\[= -468 \cdot \pi\]

\[= -1470.27^\circ\]

0 out of 1
Question 2

a) From a group of 9 people, in how many ways can you select a committee of 4 members?

b) From a group of 9 people, in how many ways can you select a president, a vice president, a secretary, and a treasurer?

c) Explain why the answers in a) and b) are different.

Solution

a) \( \binom{9}{4} = 126 \) ways

1 mark for \( \binom{9}{4} \)

1 mark

b) \( 9P_4 = 3024 \) ways

1 mark for \( 9P_4 \)

1 mark

c) Part a) is a combination because the order does not matter; part b) is a permutation because committee members have specific roles.

1 mark
Exemplar 1

a) 
\[
\frac{n!}{(n-r)!} = \frac{9!}{(9-4)!} = 3024
\]

You can select 4 committee members in 3024 ways.

**0 out of 1**

Concept error (using permutations instead of combinations)

b) 
\[
\frac{n!}{r!(n-r)!} = \frac{9!}{4!(9-4)!} = 126
\]

There are 126 ways to select these people.

**1 out of 1**

Consistent with concept error in a)

c) 
They are different because the 4 committee members can be in any order, until you assign them roles like in question b.

**1 out of 1**
Exemplar 2

a) \[ 9 \times 4! \]
\[ 9 \times 3 \times 4 \times 2 \times 1 = 216 \]

0 out of 1

b) \[ 9 \times 8 \times 7 \times 6 = 3024 \]

1 out of 1

c) They are different because there are less possibilities for choosing 4 committee members. When choosing for a certain role, more people can go for it.

\( \frac{1}{2} \) out of 1

award full marks
- \( \frac{1}{2} \) mark for lack of clarity in explanation
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Question 3

A population of 500 bacteria will triple in 20 hours.

Using the formula given below,

\[ A = Pe^{rt} \]

\( A \) = population after \( t \) hours
\( P \) = initial population
\( r \) = rate of growth
\( t \) = time in hours

a) Determine the rate of growth, \( r \).

b) Determine how many hours it will take for the initial population to double with the same rate of growth.

Solution

a) \[
1500 = 500e^{r(20)}
\]
\[
3 = e^{20r}
\]
\[
\ln(3) = \ln(e^{20r})
\]
\[
\ln(3) = 20r \cdot \ln(e)
\]
\[
r = \frac{\ln(3)}{20}
\]
\[
r = 0.054930614
\]

b) \[
1000 = 500e^{0.054930614t}
\]
\[
2 = e^{0.054930614t}
\]
\[
\ln(2) = \ln(e^{0.054930614t})
\]
\[
\ln(2) = 0.054930614t \cdot \ln(e)
\]
\[
t = \frac{\ln(2)}{0.054930614}
\]
\[
t = 12.619 \text{ hours}
\]
Exemplar 1

a)

\[
1500 = 500 e^{r \cdot 20} \\
\ln(1500) = \ln(500 e^{r \cdot 20}) \\
\ln 1500 = \ln 500 + \ln e^{r \cdot 20} \\
\frac{\ln 1500}{\ln 500} = r \cdot 20 \\
\frac{1.1767...}{20} = \frac{r}{20} \\
r = 0.0588389...
\]

1 out of 2

award full marks
− 1 mark for concept error in line 3

b)

\[
1000 = 500 e^{0.0588389\cdot t} \\
2 = e^{0.0588389\cdot t} \\
\ln 2 = \ln e^{0.0588389\cdot t} \\
\ln 2 = \frac{0.0588389\cdot t}{0.0588389...} \\
11.780 \text{ hours} = t
\]

2 out of 2

award full marks
(work consistent with answer in a)]
Exemplar 2

a)

\[
1500 = 500e^{20r} \\
3 = e^{20r} \\
\ln 3 = 20r \cdot \ln e \\
\frac{\ln 3}{20} = r \\
= 0.055
\]

2 out of 2
award full marks

b)

\[
1000 = 500e^{0.05t} \\
2 = e^{0.05t} \\
\ln 2 = 0.05t \cdot \ln e \\
\frac{\ln 2}{0.05} = t \\
ts = 12.603 \text{ hours}
\]

2 out of 2
award full marks
[work consistent with answer in a)]
E7 (transcription error in line 1)
Question 4

Talla incorrectly solved the following trigonometric equation:

Solve: \( 2 \sec x - 5 = 0; \ 0^\circ \leq x \leq 360^\circ \).

Talla’s work:

\[
\begin{align*}
2 \sec x - 5 &= 0 \\
\sec x &= \frac{5}{2} \\
\text{No solution, } \sec x \text{ cannot be greater than 1.}
\end{align*}
\]

a) Explain her error.

b) Determine the correct solution.

**Solution**

a) Talla incorrectly stated that \( \sec x \) cannot be greater than 1.
   The value of \( \cos x \) cannot be greater than 1.

   \[1 \text{ mark}\]

b) \( \sec x = \frac{5}{2} \)
   \( \cos x = \frac{2}{5} \)
   \( x_r = 66.421 \ 821 \)
   \( x = 66.422^\circ \)
   \( x = 293.578^\circ \)

   1 mark for reciprocal
   1 mark for solving for \( x \) (½ mark for each value of \( x \))

   \[2 \text{ marks}\]
Exemplar 1

a)

her error was she didn’t flip \( \frac{\pi}{2} \) to make it smaller than one.

½ out of 1

– ½ mark for lack of clarity in explanation

b)

\[
\sec x = \frac{8}{2} \\
\cos x = \frac{2}{3}
\]

1 out of 2

+ 1 mark for reciprocal
Exemplar 2

a) She did not change sec to cos in the end.

b) 

\[ 2 \sec x - 5 = 0 \]
\[ 2 \sec x = 5 \]
\[ \frac{2}{2} \]
\[ \sec x = \frac{5}{2} \]
\[ \cos x = \frac{2}{5} \]
\[ \cos = 66.42 \]

\[ \frac{1}{2} \text{ out of 2} \]

+ 1 mark for reciprocal
+ \( \frac{1}{2} \) mark for value of \( x \)
- 1 mark for concept error in line 5
E6 (rounding error in line 5)
This page was intentionally left blank.
Simplify the 6th term in the expansion of:

\[
\left(2x - \frac{3}{x^2}\right)^{10}
\]

**Solution**

\[
t_6 = \binom{10}{5} (2x)^5 \left(-\frac{3}{x^2}\right)^5
\]

\[
= 252 \left(32x^5\right) \left(-\frac{243}{x^{10}}\right)
\]

\[
= -1959552 x^{-5}
\]

2 marks (1 mark for \(\binom{10}{5}\), \(\frac{1}{2}\) mark for each consistent factor)

1 mark for simplification (\(\frac{1}{2}\) mark for coefficient, \(\frac{1}{2}\) mark for exponent)

**3 marks**
Exemplar 1

\[
\begin{align*}
10 \binom{5}{2} &\ (2x)^5 \left(\frac{-3}{x^2}\right)^5 \\
&= 252 \ (2x^5) \left(\frac{-243}{x^{12}}\right) \\
&= 252 \left(\frac{486x^5}{x^{12}}\right) \\
&= 1 \times 2472 \ x^{-5}
\end{align*}
\]

**2½ out of 3**
+ 1 mark for \(10 \binom{5}{2}\)
+ 1 mark for consistent factors
+ \(\frac{1}{2}\) mark for simplification of exponent

Exemplar 2

\[
\begin{align*}
10 \binom{6}{b} &\ (2x)^6 \left(\frac{x^2}{2}\right)^6 \\
&= 210 \ (2^4) (x)^4 (-3)^6 (\frac{1}{4})^6 \\
&= 2494140 \ x^{-8}
\end{align*}
\]

**2 out of 3**
+ 1 mark for consistent factors
+ 1 mark for simplification
**Question 6**

Determine the arc length subtended by a central angle if the diameter is 19 cm and the central angle is 1.6 radians.

**Solution**

\[ s = \theta r \]

\[ s = (1.6)(9.5) \]

\[ s = 15.2 \text{ cm} \]

1 mark
Exemplar 1

\[ S = \theta r \]
\[ S = (1.6)(19) \]
\[ S = 30.4 \text{ cm} \]

½ out of 1

- ½ mark for procedural error

Exemplar 2

\[ S = \theta r \]
\[ S = (1.6)(9.5) \]
\[ S = 15.2 \]

1 out of 1

award full marks
E5 (missing units of measure in line 3)
Question 7

Solve the following equation algebraically for $x$, where $0 \leq x \leq 2\pi$.

$$2\cos^2 x = -3\sin x$$

**Solution**

$$2\left(1 - \sin^2 x\right) = -3\sin x$$

$$2 - 2\sin^2 x = -3\sin x$$

$$0 = 2\sin^2 x - 3\sin x - 2$$

$$0 = (2\sin x + 1)(\sin x - 2)$$

$$\sin x = -\frac{1}{2}$$

No Solution

$$x = \frac{7\pi}{6}$$

$$x = \frac{11\pi}{6}$$

1 mark for identity

1 mark for solving for $\sin x$

1 mark for indicating no solution

1 mark for solving for $x$ (½ mark for each value)

4 marks
Exemplar 1

\[
\begin{align*}
\frac{2\cos^2 x}{2} &= -3\sin x \\
1 - \sin^2 x &= -\frac{3\sin x}{2} \\
2(1 - \sin^2 x) + 3\sin x &= 0 \\
2 - 2\sin^2 x + 3\sin x &= 0 \\
2\sin^2 x - 3\sin x + \frac{3}{2} &= 0 \\
2\sin x - 4\sin x + 1\sin x - 2 &= 0 \\
2\sin x(\sin x - 2) + 1(\sin x - 2) &= 0 \\
(2\sin x + 1)(\sin x - 2) &= 0 \\
2\sin x + 1 &= 0 \quad \text{or} \quad \sin x - 2 = 0 \\
\sin x &= -\frac{1}{2} \quad \text{or} \quad \sin x = 2 \\
\text{Reject} \quad \text{or} \\
\boxed{x = -0.5235} &
\end{align*}
\]

3 out of 4

+ 1 mark for identity
+ 1 mark for solving for \( \sin x \)
+ 1 mark for indicating no solution
Exemplar 2

\[ 2 \cos^2 x = -3 \sin x \]
\[ 2(\cos^2 x) = -3(\sin x) \]
\[ 2(1 - \sin^2 x) = -3(\sin x) \]
\[ \frac{2(1 - \sin x)}{2} = \frac{-3}{2} \]
\[ 1 - \sin x = \frac{-3}{2} \]
\[ \cos x = \frac{-\sqrt{3}}{2} \]

\[ x = \frac{\pi}{6} \]

1 out of 4
+ 1 mark for identity

Exemplar 3

\[ 2 \left( 1 - \sin^2 x \right) = -3 \sin x \]
\[ 2 - 2 \sin^2 x = -3 \sin x \]
\[ 2 \sin^2 x - 3 \sin x - 2 = 0 \]
\[ (2 \sin x + 1)(\sin x - 2) = 0 \]
\[ \sin x = -\frac{1}{2}, \frac{\pi}{2} \]
\[ x = \frac{5\pi}{6} \]
\[ x = \frac{\pi}{2} \]

4 out of 4
award full marks
E5 (answer stated in degrees instead of radians)
E7 (notation error in line 3)
This page was intentionally left blank.
In how many different ways can you arrange the letters in the word VOLLEYBALL?
State your answer as a factorial.

**Solution**

\[
\frac{10!}{4!}
\]

1 mark

---

**Note(s):**

- award full marks for 151 200
Exemplar 1

\[ \frac{10!}{4!} = 151200 \]

0 out of 1

Exemplar 2

\[ \frac{10!}{4!} = 151200 \]

1 out of 1

Exemplar 3

\[ \frac{10!}{2!2!} \]

0 out of 1

Exemplar 4

1 out of 1
Question 9

Is \((x - 2)\) a factor of the polynomial \(p(x) = -x^4 - 3x^3 + 11x^2 + 3x - 10\)?

Justify your response.

**Solution**

**Method 1**

\[
p(2) = -(2)^4 - 3(2)^3 + 11(2)^2 + 3(2) - 10 = -16 - 24 + 44 + 6 - 10 = 0
\]

The remainder is zero, so \((x - 2)\) is a factor.

½ mark for \(p(2)\)
1 mark for the remainder theorem
½ mark for justification

2 marks

**Method 2**

\[
\begin{array}{c|cccc}
  & -1 & -3 & 11 & 3 \\
\hline
2 & -2 & -10 & 2 & 10 \\
\end{array}
\]

\[
\begin{array}{c|cccc}
  & -1 & -5 & 1 & 5 \\
\hline
  & 0 & 0 & 0 & 0 \\
\end{array}
\]

The remainder is zero, so \((x - 2)\) is a factor.

½ mark for \(x = 2\)
1 mark for synthetic division
(or for any equivalent strategy)
½ mark for justification

2 marks

**Method 3**

I entered \(y = -x^4 - 3x^3 + 11x^2 + 3x - 10\) into my calculator and located the zeroes.

\(x = 2\) was a zero, which means \((x - 2)\) is a factor.

1 mark for graphing calculator method
1 mark for relating the zeroes to the factors

2 marks
Exemplar 1

\[ p(2) = (-2)^2 - 3(2)^3 + 11(2)^2 + 3(2) - 10 \]
\[ = 4 - 3(8) + 11(4) + 6 - 10 \]
\[ = 16 - 24 + 44 + 6 - 10 \]
\[ = 66 - 34 \]
\[ = 32 \]

No, it’s not a factor because it doesn’t equal zero.

1½ out of 2
award full marks
- ½ mark for procedural error in line 1

Exemplar 2

\[ 2 \left\{ \begin{array}{c}
-1 & -3 & 11 & 3 & -10 \\
\cline{2-5}
\hline
-1 & -4 & 7 & 10 \\
\cline{2-5}
-1 & -4 & 7 & 10 & 0 \\
\end{array} \right. \]

\((x-2)\) is not a factor.

½ out of 2
+ ½ mark for \(p(2)\)
Question 10

Determine the period of the sinusoidal function \( y = \frac{1}{2} \sin \left( \frac{1}{3} x \right) \).

State your answer in radians.

**Solution**

\[
p = \frac{2\pi}{|b|}
\]

\[
p = \frac{2\pi}{\frac{1}{3}}
\]

\[
p = 6\pi \quad \text{or} \quad p = 18.850
\]

\( \frac{1}{2} \) mark for correct value of \( b \)

\( \frac{1}{2} \) mark for period consistent with \( b \)

1 mark
Exemplar 1

\[ \text{period} = \frac{2\pi}{b} = \frac{2\pi}{1/2} = 2\pi \cdot 2 = 4\pi \]

\[ \text{period} = 4\pi \]

\( \frac{1}{2} \text{ out of } 1 \)
The domain of $f(x)$ is $x \leq 2$. The domain of $g(x)$ is $x \geq -7$.

State the domain of $f(x) + g(x)$.

Justify your answer.

**Solution**

Both $f(x)$ and $g(x)$ have restricted domains, so both domains need to be considered.

The solution is the intersection of the two domains.

$$\{x | x \in \mathbb{R}, -7 \leq x \leq 2\} \text{ or } [-7, 2]$$

1 mark for justification

1 mark for domain

2 marks
Exemplar 1

\[ D = -7 \leq x \leq 2 \]

The domain of \( f(x) + g(x) \) is only even through the domain \(-7 \leq x \leq 2\).

1 out of 2
+ 1 mark for domain

Exemplar 2

The domain of \( f(x) + g(x) \) is \([-7, 2]\).

Because \( x \) cannot be greater than 2 in the \( f(x) \) function, and \( x \) cannot be less than \(-7\) in the \( g(x) \) function. So when you put the 2 together those will be your restrictions.

2 out of 2
award full marks
E8 (domain written in incorrect order)

Exemplar 3

\[ x \leq 2, \quad x \geq -7, \]

You have to combine the two domains when adding functions.

½ out of 2
+ 1 mark for justification
− ½ mark for lack of clarity
Question 12

Prove the identity below for all permissible values of $\theta$.

$$\frac{1}{1 + \cos \theta} = \csc^2 \theta - \frac{\cot \theta}{\sin \theta}$$

**Solution**

**Method 1**

<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{1 + \cos \theta}$</td>
<td>$\csc^2 \theta - \frac{\cot \theta}{\sin \theta}$</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1}{\sin^2 \theta} - \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\sin \theta}$</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1}{\sin^2 \theta} - \frac{\cos \theta}{\sin^2 \theta} \cdot \frac{1}{\sin \theta}$</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1 - \cos \theta}{\sin^2 \theta}$</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1 - \cos \theta}{1 - \cos^2 \theta}$</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1 - \cos \theta}{(1 - \cos \theta)(1 + \cos \theta)}$</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1}{1 + \cos \theta}$</td>
</tr>
</tbody>
</table>

1 mark for correct substitution of identities

1 mark for algebraic strategies

1 mark for logical process to prove an identity

**3 marks**
<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{1 + \cos \theta} \cdot (1 - \cos \theta)$</td>
<td>$\csc^2 \theta - \frac{\cot \theta}{\sin \theta}$</td>
</tr>
<tr>
<td>$\frac{1 - \cos \theta}{1 - \cos^2 \theta}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1 - \cos \theta}{\sin^2 \theta}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{\sin^2 \theta} - \frac{\cos \theta}{\sin^2 \theta} - \frac{1}{\sin \theta}$</td>
<td>$\csc^2 \theta - \frac{\cot \theta}{\sin \theta}$</td>
</tr>
</tbody>
</table>

1 mark for algebraic strategies

1 mark for correct substitution of identities

1 mark for logical process to prove an identity

3 marks
# Exemplar 1

<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L.S. = \frac{1}{1 + \cos \theta} )</td>
<td>( R.S. = \csc^2 \theta - \frac{\cot \theta}{\sin \theta} )</td>
</tr>
</tbody>
</table>

\[
= \frac{1}{\sin^2 \theta} - \frac{\cos \theta}{\sin \theta} \\
= \frac{1}{\sin^2 \theta} - \frac{\cos \theta}{\sin \theta} \left( \frac{\sin \theta}{\sin \theta} \right) \\
= \frac{1 - \cos \theta \sin \theta}{\sin^2 \theta} \frac{\sin \theta}{\sin \theta} \left( \frac{\sin \theta}{\sin \theta} \right) \\
= \frac{1 - \cos \theta \sin^2 \theta}{\sin^2 \theta} \\
= \frac{1 - \cos \theta \sin^2 \theta}{\sin^2 \theta}
\]

**1 out of 3**

+ 1 mark for correct substitution of identities
Exemplar 2

<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
</table>
|                | \[
1 - \frac{\cos \Theta}{\sin^2 \Theta} - \frac{\cos \Theta}{\sin \Theta} = \frac{1 - \cos \Theta}{\sin \Theta}
\]|

2½ out of 3

award full marks
- ½ mark for procedural error in line 4
Exemplar 3

<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{1 + \cos \theta} )</td>
<td>( \csc^2 \theta - \frac{\cot \theta}{\sin \theta} )</td>
</tr>
<tr>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>= ( \frac{\sin^2 \theta}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \left( \frac{1}{\sin \theta} \right) )</td>
<td>= ( 1 + \frac{\sin^2 \theta}{\cos^2 \theta} - \frac{1}{\cos \theta} )</td>
</tr>
</tbody>
</table>

0 out of 3
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Explain how the end behaviours of the graphs of polynomial functions with an even degree and with an odd degree are different.

**Solution**

When the degree is odd, the end behaviour is in opposite directions. When the degree is even, the end behaviour is in the same direction.
Exemplar 1

If it is an odd degree one end will point up and one will point down

½ out of 1

Award full marks
– ½ mark for lack of clarity in explanation

Exemplar 2

The graph can be used to determine it because depending which way the graph starts (goes up or goes down) determine if it's even or odd. If it's odd it will go down, even will go up. The number of bumps on it will also help you read a graph.

0 out of 1
Given the graphs of $f(x)$ and $g(x)$, sketch the graph of $g(x) - f(x)$.

**Solution**

<table>
<thead>
<tr>
<th>$x$</th>
<th>$g(x)$</th>
<th>$f(x)$</th>
<th>$(g - f)(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>−4</td>
<td>4</td>
<td>−1</td>
<td>5</td>
</tr>
<tr>
<td>−2</td>
<td>6</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>5</td>
<td>−1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>8</td>
<td>−4</td>
</tr>
</tbody>
</table>

1 mark for subtraction of $g(x) - f(x)$

1 mark for restricting domain on graph

2 marks
Exemplar 1

1 out of 2
+ 1 mark for subtraction of $g(x) - f(x)$

Exemplar 2

1 out of 2
+ 1 mark for restricting domain
Exemplar 3

1½ out of 2

award full marks
– ½ mark for arithmetic error (1 incorrect point)
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Given $f(x) = -3x + 7$, evaluate $f^{-1}(-2)$.

**Solution**

Let $y = f(x)$

$f(x) = -3x + 7$

$y = -3x + 7$

$x = -3y + 7$

$x - 7 = -3y$

$y = \frac{x - 7}{-3}$

$f^{-1}(x) = \frac{x - 7}{-3}$

$\frac{1}{2}$ mark for $f^{-1}(x)$

$f^{-1}(-2) = \frac{-2 - 7}{-3}$

$f^{-1}(-2) = 3$

$\frac{1}{2}$ mark for $f^{-1}(-2)$

2 marks
Exemplar 1

\[ f^{-1}(-2) = \]
\[ f^{-1}(x) = \]
\[ y = -3x + 7 \]
\[ x = -3y + 7 \]
\[ x - 7 = -3y \]
\[ \frac{x - 7}{3} = y \]
\[ \frac{x - 7}{3} = f^{-1}(x) \]
\[ \frac{-2}{3} = f^{-1}(-2) \]
\[ \frac{-9}{3} = f^{-1}(-2) \]
\[ -3 = f^{-1}(-2) \]

2 out of 2
E7 (transcription error in line 6)

Exemplar 2

\[ f^{-1}(-2) = -3(2) + 7 \]
\[ f(-2) = 6 + 7 \]
\[ f^{-1}(\text{x}) = \boxed{13} \]

0 out of 2
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## Answer Key for Multiple-Choice Questions

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Learning Outcome</th>
</tr>
</thead>
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<td>16</td>
<td>C</td>
<td>P4</td>
</tr>
<tr>
<td>17</td>
<td>B</td>
<td>T1</td>
</tr>
<tr>
<td>18</td>
<td>B</td>
<td>R9</td>
</tr>
<tr>
<td>19</td>
<td>C</td>
<td>P3</td>
</tr>
<tr>
<td>20</td>
<td>C</td>
<td>R13</td>
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<tr>
<td>21</td>
<td>B</td>
<td>T6</td>
</tr>
<tr>
<td>22</td>
<td>A</td>
<td>R8</td>
</tr>
<tr>
<td>23</td>
<td>D</td>
<td>T2</td>
</tr>
<tr>
<td>24</td>
<td>C</td>
<td>R12</td>
</tr>
<tr>
<td>25</td>
<td>A</td>
<td>R5</td>
</tr>
</tbody>
</table>
Question 16

How many terms are there in the expansion of \((x^{12} + 3)^{10}\)?

a) 9  

b) 10  

c) 11  

d) 12 

Question 17

A co-terminal angle for \(\theta = \frac{11\pi}{3}\) in the domain \(-2\pi \leq \theta \leq 0\) would be:

a) \(-\frac{5\pi}{3}\)  

b) \(-\frac{\pi}{3}\)  

c) \(\frac{\pi}{3}\)  

d) \(\frac{5\pi}{3}\) 

Question 18

The x-intercept of the graph of \(y = 3^x - 1\) is:

a) -1  

b) 0  

c) 1  

d) 2
Question 19

If \( \binom{n}{5} = \binom{n}{3} \), the value of \( n \) must be:

a) 3  
b) 5  
c) 8  
d) 15

Question 20

What is the domain of the function \( f(x) = \sqrt{-(x+1)} \)?

a) \( \{x \mid x \in \mathbb{R}, x \neq -1\} \)

b) \( \{x \mid x \in \mathbb{R}, x \geq -1\} \)

c) \( \{x \mid x \in \mathbb{R}, x \leq -1\} \)

d) \( \{x \mid x \in \mathbb{R}\} \)

Question 21

Identify a non-permissible value of \( x \) for the expression \( \frac{1}{\cos 2x} \).

a) 0  
b) \( \frac{\pi}{4} \)  
c) \( \frac{\pi}{2} \)  
d) \( \pi \)
Question 22

The expression $2 \log x - \frac{1}{3} \log y$ as a single logarithm is:

a) $\log \frac{x^2}{3\sqrt{y}}$

b) $\log \frac{2x}{3y}$

c) $-\log x^2 3\sqrt{y}$

d) $\log (x^2 - 3\sqrt{y})$

Question 23

The point $P(\theta)$ lies on the unit circle. What are the coordinates of the point if $\theta = 300^\circ$?

a) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

b) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

c) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

d) $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
Question 24
What is the degree of the polynomial function represented by the graph below?

a) 2
b) 3
c) 4
D) 5

Question 25
When the point \((-4, -3)\) is reflected over the line \(y = x\), the coordinates of the new point are:

a) \((-3, -4)\)
b) \((3, 4)\)
c) \((4, -3)\)
d) \((-4, 3)\)
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Question 26

Sketch the graphs of:

a) \( y = \left(\frac{1}{4}\right)^x \)

b) \( y = 2\left(\frac{1}{4}\right)^x \)

Solution

a) ½ mark for decreasing exponential function
½ mark for \( y \)-intercept \((0, 1)\)
½ mark for consistent point of an exponential function
½ mark for horizontal asymptote at \( y = 0 \)

1 mark for a vertical stretch by a factor of 2 of the graph consistent with a)

b) 2 marks
Exemplar 1

a) 

1½ out of 2
+ ½ mark for decreasing exponential function
+ ½ mark for y-intercept (0, 1)
+ ½ mark for consistent point of an exponential function
E9 (missing arrowhead)

b) 

1 out of 1
consistent with a)
Exemplar 2

a) 

1½ out of 2

+ ½ mark for decreasing exponential function
+ ½ mark for \( y \)-intercept \((0, 1)\)
+ ½ mark for consistent point of an exponential function

b) 

1 out of 1
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Question 27

Determine all of the zeroes of the function \( p(x) = x^3 - 5x^2 - 2x + 24 \), given one of the factors of \( p(x) \) is \( x - 3 \).

**Solution**

\[
0 = x^3 - 5x^2 - 2x + 24
\]

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>-5</th>
<th>-2</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
<td>-6</td>
<td>-24</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
<td>-8</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

\[
x^2 - 2x - 8 = 0
\]

\[
(x - 4)(x + 2) = 0
\]

zeroes: 3, 4, -2

½ mark for \( x = 3 \)

1 mark for synthetic division (or for any equivalent strategy)

½ mark for consistent zeroes

2 marks
Exemplar 1

\[ \begin{array}{c|cccc}
 & 1 & -5 & -2 & 24 \\
3 & \hline & 3 & -6 & 24 \\
 & 1 & -2 & -8 & \text{(OR)} \\
\end{array} \]

\[ (x-3)(x^2-2x-8) \]

\[ (x-3)x-4x+2) \]

\textbf{1½ out of 2}

+ ½ mark for \( x = 3 \)
+ 1 mark for synthetic division
E2 (changing an equation to an expression)
E7 (transcription error in line 2)
Given the graph of \( f(x) \), sketch the graph of \( y = \sqrt{f(x)} \).

**Solution**

1 mark for restricting domain
½ mark for shape between invariant points
½ mark for shape to the right of the invariant points

2 marks
Exemplar 1

0 out of 2

Exemplar 2

1 out of 2

+ 1 mark for restricting domain
Question 29

Sketch the graph of at least one period of the function $y = -2\sin(4x)$.

**Solution**

![Graph of $y = -2\sin(4x)$]

- 1 mark for amplitude
- 1 mark for period
- 1 mark for reflection in the $x$-axis

3 marks
Exemplar 1

3 out of 3
+ 1 mark for amplitude
+ 1 mark for period
+ 1 mark for reflection in the $x$-axis
E9 (scale values on $y$-axis not indicated)

Exemplar 2

2 out of 3
+ 1 mark for amplitude
+ 1 mark for reflection in the $x$-axis
Exemplar 3

2 out of 3

award full marks
– 1 mark for concept error, sketched $y = -2 \cos(4x)$
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Question 30

Evaluate:

\[ \frac{1}{2} \log_3 144 - \log_3 4 + 2 \log_3 3 \]

Solution

\[ \log_3 \left( \frac{144}{2} \right) - \log_3 4 + \log_3 9 \]

\[ \log_3 \left( \frac{12 \cdot 9}{4} \right) \]

\[ \log_3 27 \]

1 mark for power rule

\( \frac{1}{2} \) mark for product rule

\( \frac{1}{2} \) mark for quotient rule

1 mark for evaluating a logarithm

3 marks
Exemplar 1

\[
\log_3 \sqrt[4]{4} - \log_3 4 + \log_3 3^2 \\
\log_3 12 - \log_3 4 + \log_3 9 \\
\log_3 \left( \frac{12}{4 \cdot 9} \right) \\
\log_3 \frac{1}{3} \\
= -1
\]

2½ out of 3
+ 1 mark for power rule
+ ½ mark for quotient rule
+ 1 mark for evaluating a logarithm
Exemplar 2

\[
\log_3 144^{\frac{1}{2}} - \log_3 4 + \log_3 3^2
\]

\[
144^{\frac{1}{2}} - 4 + 3^2
\]

\[
144^{\frac{1}{2}} - 4 + 9
\]

\[
144^{\frac{1}{2}} + 5
\]

0 out of 3
+ 1 mark for power rule
− 1 mark for concept error in line 1

Exemplar 3

\[
\log_3 144^{\frac{1}{2}} - \log_3 4 + \log_3 3^2
\]

\[
\log_3 12 - \log_3 4 + \log_3 9
\]

\[
\log_3 \left( \frac{12}{4} \right) + 2
\]

\[
\log_3 3 + 1
\]

\[
1 + 1
\]

\[
= 2
\]

3 out of 3
+ 1 mark for power rule
+ \( \frac{1}{2} \) mark for product rule
+ \( \frac{1}{2} \) mark for quotient rule
+ 1 mark for evaluating a logarithm
E7 (transcription error in line 4)
This page was intentionally left blank.
Match each function with its correct description.

a) The graph of this function has a vertical asymptote at $x = -1$.

b) The graph of this function has a point of discontinuity (hole) at $x = 3$.

c) The graph of this function has a horizontal asymptote at $y = 4$.

d) The domain of this function is $x \in \mathbb{R}$.

Place the appropriate letter in this column.

**Solution**

\[
\begin{align*}
  f(x) &= \frac{4}{x^2 + 1} & \text{d)} \\
  g(x) &= \frac{4x}{x + 3} & \text{c)} \\
  h(x) &= \frac{4(x - 3)(x + 2)}{(x - 3)} & \text{b)} \\
  k(x) &= \frac{4(x - 3)}{(x + 3)(x + 1)} & \text{a)}
\end{align*}
\]
The point $(-3, 4)$ is on the graph of $y = \frac{1}{2} f(3x)$.

State the coordinates of the corresponding point on the graph of $y = f(x)$.

**Solution**

$(-9, 8)$  
$\frac{1}{2}$ mark for each coordinate

1 mark
Exemplar 1

\[ y_{\text{coord}} : \quad \frac{1}{2} (4) = 2 \]

\[ x_{\text{coord}} : \quad \frac{1}{3} (-3) = -1 \]

\[ \text{Soln} : \quad (-1, 2) \]

0 out of 1

Exemplar 2

\[ y = \frac{1}{2} f(3x) \quad (-3, 4) \]

\[ y = f(x) \]

\( (x, y) \rightarrow (3x, \frac{1}{2}y) \)

The coordinates of \( y = f(x) \)

would be \((-1, 8)\)

\( \frac{1}{2} \) out of 1

+ \( \frac{1}{2} \) mark for the \( y \)-coordinate
Sketch the graph of \( y = -2(x - 1)(x - 3)(x + 1) \).

**Solution**

![Graph of the function](image)

1 mark for \( x \)-intercepts
1 mark for \( y \)-intercept
1 mark for end behaviour

3 marks
Exemplar 1

\[ y = (0-1)(0-3)(0+1) \]
\[ = (-1)(-3)(1) \]
\[ = 3 \]

1 out of 3
+ 1 mark for \(x\)-intercepts
Exemplar 2

2 out of 3
+ 1 mark for $x$-intercepts
+ 1 mark for $y$-intercept

Exemplar 3

2 out of 3
+ 1 mark for $x$-intercepts
+ 1 mark for end behaviour
Question 34

a) Verify that the equation \( \frac{1 - \sin^2 x}{\cos x} = \frac{\sin 2x}{2 \sin x} \) is true for \( x = \frac{\pi}{3} \).

b) Explain why verifying the equation for \( x = \frac{\pi}{3} \) is insufficient to conclude that the equation is an identity.

**Solution**

\[
\begin{align*}
\text{a) } & \quad \frac{1 - \sin^2 \left( \frac{\pi}{3} \right)}{\cos \left( \frac{\pi}{3} \right)} = \frac{\sin 2 \left( \frac{\pi}{3} \right)}{2 \sin \left( \frac{\pi}{3} \right)} \\
& \quad \frac{1 - \left( \frac{\sqrt{3}}{2} \right)^2}{\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} \\
& \quad \frac{1 - \frac{3}{4}}{\frac{1}{2}} = 1 \\
& \quad \frac{1}{2} = \frac{1}{2}
\end{align*}
\]

LHS = RHS

b) Proving that it is true for one value does not mean that it is true for all values.
Exemplar 1

a)

\[
\begin{align*}
1 - \sin^2 \left( \frac{\pi}{3} \right) &= \frac{1 - (\sqrt{3}/3)^2}{1/2} \\
&= \frac{1 - \frac{3}{9}}{1/2} \\
&= \frac{1 - \frac{1}{3}}{1/2} \\
&= \frac{2}{2} \\
&= 1
\end{align*}
\]

\[
\sin 2 \left( \frac{\pi}{3} \right) = \frac{\sqrt{3}}{2}
\]

\[
\frac{\sqrt{3}}{2} \neq 1
\]

\[x = \frac{\pi}{3} \text{ is not a solution}\]

1½ out of 2

award full marks

− ½ mark for arithmetic error in line 4

b)

In an identity you only solve on one side.

0 out of 1
Exemplar 2

a)

\[
\begin{align*}
1 - \sin^2 x &= \frac{\cos^2 x}{\cos x} \\
&= \frac{2 \sin x \cos x}{2 \sin x} \\
&= \cos x \\
&= \cos \left( \frac{\pi}{3} \right) \\
&= \frac{1}{2}
\end{align*}
\]

\[
\begin{align*}
2 \sin x &= \frac{\sin 2x}{2 \sin x} \\
&= \frac{2 \sin x \cos x}{2 \sin x} \\
&= \cos x \\
&= \cos \left( \frac{\pi}{3} \right) \\
&= \frac{1}{2}
\end{align*}
\]

2 out of 2

b)

Because you'd have to check all the values

½ out of 1

award full marks
- ½ mark for lack of clarity
This page was intentionally left blank.
Evaluate:

$$\frac{7 \cdot P_2}{7 \cdot P_5}$$

**Solution**

$$\frac{7!}{(7 - 2)!}$$

$$\frac{7!}{7!}$$

$$\frac{(7 - 5)!}{(7 - 2)!}$$

$$\frac{7!}{5!}$$

$$\frac{7!}{7!}$$

$$\frac{2!}{2!}$$

$$\frac{5!}{5!}$$

$$\frac{2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$$

$$\frac{1}{60}$$

½ mark for substitution

½ mark for simplification

1 mark for expanding factorials

2 marks
Exemplar 1

\[
\frac{\binom{7}{2}}{\binom{7}{5}} = \frac{\frac{7!}{(7-2)!}}{\frac{7!}{(7-5)!}} = 1
\]

1 out of 2
award full marks
– 1 mark for concept error (used combinations)

Exemplar 2

\[
\begin{align*}
\binom{7}{2} &= \frac{7!}{(7-2)!} \\
&= \frac{7!}{5!} \\
&= \frac{7 \cdot 6 \cdot 5!}{5!} \\
&= 42
\end{align*}
\]

\[
\begin{align*}
\binom{7}{5} &= \frac{7!}{(7-5)!} \\
&= \frac{7!}{2!} \\
&= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!} \\
&= 2 \cdot 5 \cdot 20 \\
&= 2520
\end{align*}
\]

2 out of 2
award full marks
E1 (final answer not stated)
Question 36

Use the graph of \( y = f(x) \) to sketch the graph of \( y = f(3x) + 1 \).

Solution

1 mark for horizontal compression
1 mark for vertical translation

2 marks
Exemplar 1

1 out of 2

+ 1 mark for vertical translation
E9 (coordinate point labelled incorrectly)
1 out of 2

+ 1 mark for horizontal compression
E9 (incorrect endpoint)
This page was intentionally left blank.
Solve the following equation:

\[ \log_4 (x + 2) + \log_4 3 = \log_4 x \]

**Solution**

**Method 1**

\[ \log_4 (x + 2) + \log_4 3 = \log_4 x \]
\[ \log_4 [(x + 2) 3] = \log_4 x \]
\[ 3(x + 2) = x \]
\[ 3x + 6 = x \]
\[ x = -3 \]

No solution

1 mark for product rule  
1 mark for equating arguments  
½ mark for solving for \( x \)  
½ mark for rejecting extraneous root

3 marks

**Method 2**

\[ \log_4 (x + 2) + \log_4 3 = \log_4 x \]
\[ \log_4 (x + 2) + \log_4 3 - \log_4 x = 0 \]
\[ \log_4 \left[ \frac{3(x + 2)}{x} \right] = 0 \]
\[ 4^0 = \frac{3x + 6}{x} \]
\[ x = -3 \]

½ mark for logarithmic rules (½ mark for product rule; ½ mark for quotient rule)  
1 mark for exponential form  
½ mark for solving for \( x \)  
½ mark for rejecting extraneous root

3 marks
Exemplar 1

\[
\log_y (x+2)(3x) = \log_y x \\
\log_y (3x+6) = \log_y x \\
3x+6 = x \\
\frac{3x}{2} = -\frac{6}{2} \\
\underline{x = -3}
\]

2½ out of 3
+ 1 mark for product rule
+ 1 mark for equating arguments
+ ½ mark for solving for \(x\)

Exemplar 2

\[
\log_y (x+2)(3x) = \log_y x \\
\log_y (3x+6) = \log_y x \\
3x+6 = x \\
6 = -2x \\
x = -3
\]

no solution
because the argument of one log.
you can't be negative.

2½ out of 3
award full marks
- ½ mark for procedural error in line 2
Exemplar 3

\[ \log_4 (x+2)(3) \]

1 out of 3
+ 1 mark for product rule
E2 (changing an equation to an expression)

Exemplar 4

\[ \log_4 ((x+2)(3)) = \log_4 x \]
\[ \log_4 (3x + 6) - \log_4 x = 0 \]
\[ \frac{\log_4 (3x + 6)}{x} = 0 \]
\[ 4^0 = \frac{3x + 6}{x} \]
\[ x = \frac{3x + 6}{3x} \]
\[ \frac{x}{3x} = \frac{10}{3} \]
\[ \frac{x}{x} = 18 \]

2 out of 3
+ 1 mark for logarithmic rules
+ 1 mark for exponential form
This page was intentionally left blank.
Question 38

Determine the coordinates of the point of discontinuity (hole) for the graph of the function
\[ y = \frac{(2 - x)(x - 3)}{(x - 2)}. \]

**Solution**

\[ x \neq 2 \]
\[ y = -(x - 3) \]
\[ y = -(2 - 3) \]
\[ y = 1 \]

(2, 1)  
1 mark for point of discontinuity (hole) at (2, 1)  
(½ mark for \( x = 2 \), ½ mark for consistent \( y \)-coordinate)
Exemplar 1

\[ y = 2 \]

\( \frac{1}{2} \) out of 1
+ \( \frac{1}{2} \) mark for \( x = 2 \)

Exemplar 2

\[ y = \frac{(x-2)(x-3)}{(x-2)} \]

\[ y = x - 3 \]
\[ y = 2 - 3 \]
\[ y = -1 \]
\[ (2, -1) \]

\( \frac{1}{2} \) out of 1
+ \( \frac{1}{2} \) mark for \( x = 2 \)
Question 39

Evaluate and simplify \( \sec\left(\frac{5\pi}{6}\right) \cdot \tan\left(-\frac{\pi}{6}\right) \).

**Solution**

\[
\left(-\frac{2}{\sqrt{3}}\right) \cdot \left(-\frac{1}{\sqrt{3}}\right)
\]

1 mark for \( \sec\left(\frac{5\pi}{6}\right) \) (½ mark for value, ½ mark for quadrant)

\[
\frac{2}{3}
\]

1 mark for \( \tan\left(-\frac{\pi}{6}\right) \) (½ mark for value, ½ mark for quadrant)

2 marks
Exemplar 1

\[
\begin{align*}
\cos \frac{5\pi}{6} &= -\frac{\sqrt{3}}{2} \\
\sec \frac{5\pi}{6} &= -\frac{2}{\sqrt{3}}
\end{align*}
\]

\[
\frac{\sqrt{3}}{2} \div \frac{1}{2} = \frac{\sqrt{3}}{2} \times 2 = \frac{\sqrt{3}}{1}
\]

\[
\left( \frac{2}{3} \right) \left( -\frac{5}{3} \right)
\]

\[
= 2
\]

1½ out of 2

+ 1 mark for \( \sec \left( \frac{5\pi}{6} \right) \)

+ ½ mark for quadrant of \( \tan \left( -\frac{\pi}{6} \right) \)
Exemplar 2

\[ \frac{1}{\cos\left(\frac{\pi}{6}\right)} \cdot \frac{1}{\frac{1}{2}} \cdot \frac{\sin\left(\frac{\pi}{6}\right)}{\cos\left(-\frac{\pi}{6}\right)} = -\frac{1}{2} \cdot \frac{-\frac{\sqrt{3}}{2}}{-\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3} \]

\[ 2 \cdot \left( \frac{-\frac{1}{2}}{2} \times \frac{\sqrt{3}}{3} \right) = 2 \cdot \frac{+1}{+\sqrt{3}} = \frac{2}{\sqrt{3}} \]

½ out of 2

+ ½ mark for value of \( \tan\left( -\frac{\pi}{6} \right) \)
Exemplar 3

\[ \left( -\frac{2}{\sqrt{2}} \right) \left( -\sqrt{3} \right) = \frac{2\sqrt{3}}{\sqrt{2}} \]

1 out of 2

+ \frac{1}{2} \text{ mark for quadrant of } \sec \left( \frac{5\pi}{6} \right)

+ \frac{1}{2} \text{ mark for quadrant of } \tan \left( -\frac{\pi}{6} \right)
Question 40

Sketch the graph of the following function:

\[ y = -2\sqrt{x - 3} \]

**Solution**

**Method 1**

1 mark for shape (graph of a radical function)
1 mark for vertical reflection
1 mark for horizontal shift
1 mark for vertical stretch

1 mark for invariant points where \( y = 0 \) and \( y = 1 \) (½ mark for each point)
1 mark for domain of \([3, \infty)\)
½ mark for shape between invariant points
½ mark for shape to the right of the invariant points
1 mark for applying transformations (vertical stretch, vertical reflection)

4 marks

**Method 2**

1 mark for invariant points

1 mark for shape between invariant points
1 mark for shape to the right of the invariant points
1 mark for applying transformations (vertical stretch, vertical reflection)

4 marks
Exemplar 1

3 out of 4
+ 1 mark for shape
+ 1 mark for vertical reflection
+ 1 mark for vertical stretch

Exemplar 2

3 out of 4
+ 1 mark for shape
+ 1 mark for horizontal shift
+ 1 mark for vertical stretch
Exemplar 3

2 out of 4

+ 1 mark for shape
+ 1 mark for horizontal shift
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Question 41

Sketch the graph of $f(x) = \frac{2x + 3}{x + 2}$.

Solution

1 mark for vertical asymptote
1 mark for horizontal asymptote
½ mark for graph left of the vertical asymptote
½ mark for graph right of the vertical asymptote

3 marks
Exemplar 1

1 out of 3
+ 1 mark for vertical asymptote
Exemplar 2

1½ out of 3
+ 1 mark for vertical asymptote
+ ½ mark for graph right of vertical asymptote
Exemplar 3

2½ out of 3
+ 1 mark for vertical asymptote
+ 1 mark for horizontal asymptote
+ ½ mark for graph right of vertical asymptote
E10 (graph crosses asymptote)
Question 42

a) Given the functions \( f(x) = \sqrt{4 + x} \) and \( g(x) = |3x - 6| \), evaluate \( f(g(-5)) \).

b) Is it possible to evaluate \( g(f(-5)) \)?

Justify your answer.

**Solution**

a) \( g(-5) = |3(-5) - 6| \)

\[ g(-5) = 21 \]

\[ f(21) = \sqrt{4 + 21} \]

\[ = \sqrt{25} \]

\[ = 5 \]

1 mark for consistent value of \( f(g(-5)) \)

2 marks

b) No, because \( f(x) \) is undefined when \( x = -5 \)

\[ f(-5) = \sqrt{4 + (-5)} \]

\[ f(-5) = \sqrt{-1} \]

\( f(-5) \) is undefined because you cannot evaluate the square root of a negative number.

1 mark for justification
Exemplar 1

a) 

\[ g(x) = \left| 3(-5) - 6 \right| \]
\[ g(x) = \left| -15 - 6 \right| \]
\[ g(x) = \left| -21 \right| \]
\[ g(x) = 21 \]

1 out of 2

+ 1 mark for \( g(-5) \)

E7 (notation error in line 1)

b) 

\[ f(-5) = \sqrt{4 + (-5)} \]
\[ f(-5) = \sqrt{1} \]
\[ f(-5) = 1 \]

\[ g(f(-5)) = \left| 3 \times -6 \right| \]
\[ g(f(-5)) = |3(1) - 6| \]
\[ g(f(-5)) = 3 - 6 \]
\[ g(f(-5)) = 3 \]

½ out of 1

award full marks

− ½ mark for arithmetic error
Exemplar 2

a)
\[ g(x) = |3(-5) - 6| = |-3| = 3 \]
\[ f(x) = \sqrt{4 + 9} \]
\[ f(x) = \sqrt{13} \]

1½ out of 2

- award full marks
- ½ mark for arithmetic error in line 2
- E7 (notation error in line 1)

b)

Yes it is possible:
\[ |3(\sqrt{4 + 5}) - 6| = |3(\sqrt{9}) - 6| = |3(3) - 6| = |9 - 6| = 3 \]

1 out of 1

- award full marks
- E7 (transcription error in line 1)
Exemplar 3

a) 
\[ f(g(-5)) = 5 \]

2 out of 2
award full marks

b) 

No because \[ f(x) = \sqrt{4 + 9} \Rightarrow f(x) = \sqrt{13} \]
and you can't take the square of -1

½ out of 1
award full marks
- ½ mark for terminology error in line 2
Question 43

Identify which of these values is greater. Justify your answer.

\[ \log_5 80 \quad \text{or} \quad \log_3 30 \]

Solution

\[ 5^2 = 25 \quad \log_5 80 \quad \text{is less than} \quad 3 \]
\[ 5^3 = 125 \]

\[ 3^3 = 27 \quad \log_3 30 \quad \text{is more than} \quad 3 \]
\[ 3^4 = 81 \]

\[ \therefore \quad \log_3 30 \quad \text{is greater} \]

1 mark for justification
Exemplar 1

\[ \log_5 25 = 2 \quad \text{and} \quad \log_3 27 = 3 \quad \log_3 30 \text{ is a little bit greater than } 3 \]

\[ \frac{3}{2} \log_5 125 = 3 \]

So \( \log_5 80 \) is close to 3, but less than 3

\( \Rightarrow \) around 2.7

\( \therefore \) \( \log_3 30 \) is greater.

1 out of 1

Exemplar 2

\[ 5^x = 80 \quad 3^x = 30 \]

\( x \approx 3.3 \quad x \approx 3.5 \)

0 out of 1
Exemplar 3

\[ 5^2 = 25 \quad 3^3 = 27 \]

\[ 5^3 = 125 \quad 3^4 = 81 \]

\[ \log_3 30 \text{ is Greater.} \]

1 out of 1
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Given \( \cos \alpha = \frac{3}{5}, \) where \( \alpha \) is in quadrant IV, and \( \cos \beta = -\frac{2}{3}, \) where \( \beta \) is in quadrant II, determine the exact value of \( \sin(\alpha - \beta). \)

**Solution**

\[
\begin{align*}
\sin \alpha &= -\frac{4}{5} \\
\sin \beta &= \frac{\sqrt{5}}{3} \\
\sin(\alpha - \beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\
&= \left(-\frac{4}{5}\right)\left(-\frac{2}{3}\right) - \left(\frac{3}{5}\right)\left(\frac{\sqrt{5}}{3}\right) \\
&= \frac{8 - 3\sqrt{5}}{15} \\
\end{align*}
\]
Exemplar 1

\[
\sin (\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \\
= \sin \left( -\frac{\pi}{3} \right) \cos \left( -\frac{\pi}{6} \right) - \cos \left( -\frac{\pi}{3} \right) \sin \left( \frac{\sqrt{5}}{3} \right) \\
= \left( -\frac{\sqrt{3}}{2} \right) \left( -\frac{2}{3} \right) - \left( -\frac{\sqrt{3}}{2} \right) \left( \frac{\sqrt{5}}{3} \right) \\
= \frac{8}{15} - \frac{3\sqrt{5}}{15} \\
= \frac{8 - 3\sqrt{5}}{15}
\]

3 out of 3
award full marks
E7 (notation error in line 2)
Exemplar 2

\[
\sin^2 \alpha + \left(\frac{3}{5}\right)^2 = 1 \\
\sin^2 \alpha + \frac{9}{25} = 1 \\
\sin^2 \alpha = \frac{16}{25} \\
\sin \alpha = \frac{4}{5}
\]

\[
\sin^2 \beta + \left(\frac{-2}{3}\right)^2 = 1 \\
\sin^2 \beta + \frac{4}{9} = 1 \\
\sin^2 \beta = \frac{5}{9} \\
\sin \beta = \frac{\sqrt{5}}{3}
\]

\[
\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \\
\left(\frac{4}{5}\right)\left(\frac{-2}{3}\right) - \left(\frac{3}{5}\right)\left(\frac{\sqrt{5}}{3}\right)
\]

\[
-\frac{8}{15} - \frac{3\sqrt{5}}{15}
\]

2½ out of 3

+ ½ mark for \( y = 4 \)
+ ½ mark for \( y = \sqrt{5} \)
+ ½ mark for \( \sin \beta \)
+ 1 mark for substitution into correct identity
E1 (final answer not stated in line 7)
Exemplar 3

\[ \sin d = -\frac{4}{5} \]

\[ 3^2 - 2^2 = y^2 \]
\[ 9 - 4 = y^2 \]
\[ \sqrt{5} = y^2 \]

\[ \sin B = \frac{\sqrt{5}}{3} \]

\[ \sin (a - B) = \left( -\frac{4}{5} \right) \left( -\frac{2}{3} \right) - \left( \frac{3}{5} \right) \left( \frac{\sqrt{5}}{3} \right) \]
\[ = \frac{-8}{15} - \frac{3\sqrt{5}}{15} \]
\[ = \frac{-8 - 3\sqrt{5}}{15} \]

2½ out of 3

Award full marks
– ½ mark for arithmetic error in line 6
E7 (notation error in line 4)
Question 45

Determine the number of possible sandwiches from the following menu.

MENU

Select one item from each column:

<table>
<thead>
<tr>
<th>Bread</th>
<th>Sauce</th>
<th>Meat</th>
<th>Vegetable</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>Mayo</td>
<td>Turkey</td>
<td>Tomato</td>
</tr>
<tr>
<td>Rye</td>
<td>Mustard</td>
<td>Ham</td>
<td>Onion</td>
</tr>
<tr>
<td>Brown</td>
<td></td>
<td>Roast Beef</td>
<td>Lettuce</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Chicken</td>
<td></td>
</tr>
</tbody>
</table>

Solution

$3 \times 2 \times 4 \times 3$

72 sandwiches

1 mark
Exemplar 1

\[ \frac{3!}{2!} - \frac{4!}{3!} \]

1 out of 1
award full marks
E1 (final answer not stated)

Exemplar 2

\[ \frac{3!}{2!} \cdot \frac{4!}{3!} \]

\[ 6 \cdot 2 \cdot 24 + 6 \]

38 sandwich options

0 out of 1
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MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.
- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation
- incorrect shape of graph (only when marks are not allocated for shape)

**Communication Errors**
The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the Answer/Scoring Sheet.

<table>
<thead>
<tr>
<th>Error Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>answer given as a complex fraction</td>
</tr>
<tr>
<td>E2</td>
<td>changing an equation to an expression</td>
</tr>
<tr>
<td>E3</td>
<td>variable omitted in an equation or identity</td>
</tr>
<tr>
<td>E4</td>
<td>“sin x^2” written instead of “sin^2 x”</td>
</tr>
<tr>
<td>E5</td>
<td>missing units of measure</td>
</tr>
<tr>
<td>E6</td>
<td>rounding error</td>
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<tr>
<td>E7</td>
<td>notation error</td>
</tr>
<tr>
<td>E8</td>
<td>answer included outside the given domain</td>
</tr>
<tr>
<td>E9</td>
<td>incorrect or missing endpoints or arrowheads</td>
</tr>
<tr>
<td>E10</td>
<td>asymptotes drawn as solid lines</td>
</tr>
</tbody>
</table>
IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all “NR”) or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.
Irregular Test Booklet Report

Test: ________________________________________________________________

Date marked: _________________________________________________________

Booklet No.: _________________________________________________________

Problem(s) noted: ____________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

Question(s) affected: _________________________________________________

________________________________________________________________________

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________________________________________________________________________

Action taken or rationale for assigning marks: ____________________________

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________________________________________________________________________
Follow-up: ______________________________________________________________

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Decision: ________________________________________________________________

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________________________________________________________________________

Marker’s Signature: ______________________________________________________

Principal’s Signature: ____________________________________________________

For Department Use Only—After Marking Complete

Consultant: ______________________________________________________________

Date: ________________________________________________________________
## Table of Questions by Unit and Learning Outcome

### Unit A: Transformations of Functions

<table>
<thead>
<tr>
<th>Question</th>
<th>Learning Outcome</th>
<th>Mark</th>
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<td>15</td>
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<td>42 b)</td>
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### Unit B: Trigonometric Functions

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### Unit C: Binomial Theorem

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### Unit D: Polynomial Functions

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### Unit E: Trigonometric Equations and Identities

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### Unit F: Exponents and Logarithms

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### Unit G: Radicals and Rationals

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