
Grade 12
Pre-Calculus Mathematics
Standards Test

Marking Guide

January 2012

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General Marking Instructions

Please make no marks in the student test booklets. If the booklets have marks in them, the marks need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that:

- the student test booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers only use a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by test booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education in the envelope provided (for more information see the *Grade 12 Pre-Calculus Mathematics Standards Test: Administration Manual*).

Scoring the Multiple-Choice Questions

Please follow the instructions on the Answer Key for Multiple-Choice Questions section on page 5.

Scoring the Short-Answer and Long-Answer Questions

The remainder of the Grade 12 Pre-Calculus Mathematics Standards Test is composed of short-answer questions and long-answer questions. Short-answer questions are worth 1 mark each and long-answer questions are worth 2 to 5 marks each.

Each question is designed to elicit a well-defined response according to the associated specific learning outcome(s) and relevant mathematical processes. Their purpose is to determine whether a student meets the standards for the course as they relate to the knowledge and skills associated with the question.

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Standards Tests

During the administration of standards tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "0" and/or "NR" only (e.g., student was present but did not attempt any questions) please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

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Scoring Guidelines

Answer Key for Multiple-Choice Questions

If you are using this page by punching out holes corresponding to the correct answers and overlaying it on the *Answer/Scoring Sheet*

- 1) check the student's responses for multiple bubbles first (questions with multiple bubbles are to be scored as "0")
- 2) overlay this page on the *Answer/Scoring Sheet*
- 3) count the number of correct responses, excluding questions with multiple responses, if any

You may also score the multiple-choice questions by making a transparency of this page and overlaying it on the *Answer/Scoring Sheet*.

Remember to write the total score for the multiple-choice questions at the bottom of the *Answer/Scoring Sheet*.

Multiple-Choice Questions / Questions À Choix Multiple

Fill in the best answer for each question. / Choisir la meilleure réponse pour chaque question.

- | | | | | | | | | | |
|----|--|----|--|----|--|----|--|----|--|
| 11 | <input type="radio"/> A <input type="radio"/> B <input checked="" type="radio"/> C <input type="radio"/> D | 14 | <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D | 17 | <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D | 20 | <input type="radio"/> A <input type="radio"/> B <input checked="" type="radio"/> C <input type="radio"/> D | 23 | <input checked="" type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D |
| 12 | <input type="radio"/> A <input type="radio"/> B <input checked="" type="radio"/> C <input type="radio"/> D | 15 | <input type="radio"/> A <input type="radio"/> B <input checked="" type="radio"/> C <input type="radio"/> D | 18 | <input type="radio"/> A <input checked="" type="radio"/> B <input type="radio"/> C <input type="radio"/> D | 21 | <input checked="" type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D | 24 | <input type="radio"/> A <input checked="" type="radio"/> B <input type="radio"/> C <input type="radio"/> D |
| 13 | <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input checked="" type="radio"/> D | 16 | <input checked="" type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D | 19 | <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input checked="" type="radio"/> D | 22 | <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input checked="" type="radio"/> D | 25 | <input type="radio"/> A <input checked="" type="radio"/> B <input type="radio"/> C <input type="radio"/> D |

Marking Key Including Student Exemplars

Part 1: Long-Answer Questions

Question 1

A1

- a) Convert $\frac{\pi}{7}$ radians into degrees. Express your answer correct to 3 decimal places.
- b) A unit circle is divided into equal sections that each have a central angle of $\frac{\pi}{7}$.
How many sections are there?
- c) Give a negative co-terminal angle for $\frac{\pi}{7}$ radians.

Solution

a)

$$\frac{\pi}{7} \left(\frac{180^\circ}{\pi} \right) = \left(\frac{180}{7} \right)^\circ = 25.714\ 286^\circ$$
$$= 25.714^\circ$$

1 mark for multiplying by $\frac{180^\circ}{\pi}$

1 mark

b)

$$\frac{\pi}{7} \cdot \text{number of sections} = 2\pi$$
$$\therefore \text{number of sections} = 14$$

1 mark

c)

$$\frac{\pi}{7} - \frac{14\pi}{7} = -\frac{13\pi}{7}$$

1 mark

or

$$25.714^\circ - 360^\circ = -334.286^\circ$$

Note(s):

→ mark (b) and (c) consistent with (a)

Exemplars

a)

$$\frac{\pi}{7} \times \frac{180^\circ}{\pi} = 25.7^\circ$$

Mark: 1/2 out of 1

Note(s):

→ gave full marks

→ deducted 1/2 mark for incorrect rounding of final solution

b)

$$360 \div 25.7$$
$$= 14 \text{ sections}$$

Mark: 1 out of 1

Note(s):

→ gave full marks

c)

$$\begin{array}{r} 360^\circ \\ - 25.7 \\ \hline 334.3^\circ \end{array}$$

Mark: 0 out of 1

Question 2

A5

Solve the trigonometric equation below given that $\theta \in R$.

$$12 \cos^2 \theta + \cos \theta - 1 = 0$$

Give the general solution in radian measure correct to 3 decimal places.

Solution

$$12 \cos^2 \theta + \cos \theta - 1 = 0$$

$$(4 \cos \theta - 1)(3 \cos \theta + 1) = 0$$

½ mark for factoring

$$\cos \theta = \frac{1}{4} \quad \cos \theta = -\frac{1}{3}$$

½ mark for solving for $\cos \theta$

$$\theta_r = 1.318\ 116 \quad \theta_r = 1.230\ 959$$

$$\theta = 1.318\ 116 \quad \theta = 1.910\ 633$$

$$\theta = 4.965\ 069 \quad \theta = 4.372\ 552$$

2 marks for consistent solutions of trigonometric equations (½ mark for each solution)

$$\left. \begin{aligned} \theta &= 1.318 + 2\pi k, k \in \mathbb{I} \\ \theta &= 1.911 + 2\pi k, k \in \mathbb{I} \\ \theta &= 4.373 + 2\pi k, k \in \mathbb{I} \\ \theta &= 4.965 + 2\pi k, k \in \mathbb{I} \end{aligned} \right\}$$

1 mark for general solution

4 marks

Note(s):

→ give a maximum of 3 marks for answers given in degrees

$$\theta = 75.522^\circ + 360^\circ k, k \in \mathbb{I}$$

$$\theta = 109.471^\circ + 360^\circ k, k \in \mathbb{I}$$

$$\theta = 250.529^\circ + 360^\circ k, k \in \mathbb{I}$$

$$\theta = 284.478^\circ + 360^\circ k, k \in \mathbb{I}$$

Exemplar

$$\begin{aligned}12 \cos^4 \theta + \cos \theta - 1 &= 0 \\12 \cos^3 \theta + \cos \theta &= 1 \\ \cos \theta (12 \cos^2 \theta + 1) &= 1 \\ \cos \theta &= 1 & 12 \cos^2 \theta + 1 &= 1 \\ \theta &= 0 & 12 \cos^2 \theta &= 0 \\ & & \cos^2 \theta &= 0 \\ & & \cos \theta &= 0 \\ \theta &= 90^\circ, 270^\circ & \leftarrow & \\ \theta &= \frac{\pi}{2}, \frac{3\pi}{2} & & \end{aligned}$$

$$\frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi \text{ w/er } k \in \mathbb{Z}$$

Mark: 2 out of 4

Note(s):

- gave 1 mark for consistent solutions of one trigonometric equation
- gave 1 mark for general solution

Question 3

D8

An insect population grows continuously according to the equation:

$$P = P_0 e^{rt}$$

where P = the population after t years

P_0 = the initial population

r = the rate of growth

t = time in years

The initial population is 300 insects. Exactly 4 years later, the population is 560 insects.

If the rate of growth remains the same, how many insects will there be 15 years after the initial insect population?

Express your answer as a whole number.

Solution

$$P = P_0 e^{rt}$$

$$560 = 300e^{r(4)}$$

½ mark for substitution

$$\frac{560}{300} = e^{r(4)}$$

$$\ln(1.866\ 666) = \ln e^{4r}$$

½ mark for applying logarithms

$$\ln(1.866\ 666) = (4r) \ln e$$

1 mark for log theorem

$$\frac{\ln(1.866\ 666)}{4} = r$$

$$r = 0.156\ 038\ 577$$

$$P = P_0 e^{rt}$$

$$P = 300e^{(0.156\ 038\ 577)(15)}$$

½ mark for substitution

$$P = 3116.1736$$

½ mark for evaluating exponential term

$$P = 3116$$

3 marks

Exemplar

$$P = P_0 e^{rt}$$

$$560 = (300)e^{4r}$$

take natural logs of both sides

$$\ln(560) = \ln(300) + \ln e^{4r}$$

$$\ln(560) = \ln(300) + (4r)\ln e$$

$$\ln(560) = \ln(300) + 4r$$

$$\frac{\ln(560)}{\ln(300)} = 4r$$

$$\frac{6.327936784}{5.703782475} = 4r$$

$$\frac{1.10942814}{4} = r$$
$$0.277 = r$$

$$P = (300)e^{(0.277)(15)}$$

$$P = (300)e^{(4.155)}$$

$$P = (300)(63.75196455)$$

$$P = 19126 \text{ insects}$$

Mark: 1½ out of 3

Note(s):

- gave ½ mark for substitution
- gave ½ mark for applying logarithms
- gave ½ mark for substitution
- gave ½ mark for evaluating exponential term
- deducted ½ mark for rounding too early

Question 4

E3

In a particular class, there are 5 girls and 10 boys.

- a) How many committees of 7 people can be formed if there must be 3 girls and 4 boys on the committee?
Express your answer as a whole number.
- b) How many committees of 7 people can be formed if there is at least one girl on the committee?
Briefly explain your calculations.
Express your answer as a whole number.

Solution

a)

$$\begin{aligned} 3 \text{ girls and 4 boys} &= {}_5C_3 \cdot {}_{10}C_4 \\ &= (10)(210) \\ &= 2100 \end{aligned}$$

 $\frac{1}{2}$ mark for ${}_5C_3$ $\frac{1}{2}$ mark for ${}_{10}C_4$

1 mark for product of combinations

2 marks

b)

Method 1

At least one girl = All committees - committees with no girls

$$\begin{aligned} &= {}_{15}C_7 - {}_{10}C_7 \\ &= 6435 - 120 \\ &= 6315 \end{aligned}$$

 $\frac{1}{2}$ mark for ${}_{15}C_7$ $\frac{1}{2}$ mark for ${}_{10}C_7$

1 mark for subtracting combinations

2 marks**Method 2**

Case 1: 1 girl, 6 boys ${}_5C_1 \cdot {}_{10}C_6 = 1050$

Case 2: 2 girls, 5 boys ${}_5C_2 \cdot {}_{10}C_5 = 2520$

Case 3: 3 girls, 4 boys ${}_5C_3 \cdot {}_{10}C_4 = 2100$

Case 4: 4 girls, 3 boys ${}_5C_4 \cdot {}_{10}C_3 = 600$

Case 5: 5 girls, 2 boys ${}_5C_5 \cdot {}_{10}C_2 = 45$

1 mark for all cases

$$1050 + 2520 + 2100 + 600 + 45 = 6315$$

1 mark for addition of cases

2 marks

Exemplars

a)

$$\frac{\text{Girls}}{5C_3} + \frac{\text{Boys}}{10C_4} = 220$$

Mark: 1 out of 2

Note(s):

→ gave ½ mark for $5C_3$

→ gave ½ mark for $10C_4$

b)

$$1 \text{ girl: } 5C_1 + 10C_6 = 215$$

$$2 \text{ girls: } 5C_2 + 10C_5 = 262$$

$$3 \text{ girls: } 5C_3 + 10C_4 = 220$$

$$4 \text{ girls: } 5C_4 + 10C_3 = 125$$

$$5 \text{ girls: } 5C_5 + 10C_2 = 46$$

Total 868

Mark: 2 out of 2

Note(s):

→ gave full marks [marked consistent with (a)]

Question 5

E4

Find the 14th term in the binomial expansion of $\left(\frac{3}{2x^2} - x^3\right)^{16}$.

Simplify your answer completely.

Solution

$$t_{14} = {}_{16}C_{13} \left(\frac{3}{2x^2}\right)^3 (-x^3)^{13}$$

$$= 560 \left(\frac{27}{8x^6}\right) (-x^{39})$$

$$= -1890x^{33}$$

2 marks (1 mark for ${}_{16}C_{13}$, $\frac{1}{2}$ mark for each consistent factor)

1 mark for simplification ($\frac{1}{2}$ mark for coefficient, $\frac{1}{2}$ mark for exponent laws)

3 marks

Exemplar

$$T_{k+1} = n C_k a^{n-k} b^k$$

$$T_{13+1} = 16 C_{13} \left(\frac{3}{2x^2}\right)^{16-13} (-x^3)^{13}$$

$$T_{14} = 560 \left(\frac{3^3}{2^3 x^6}\right) (-x^{39})$$

$$T_{14} = \frac{5040 - x^{39}}{8x^6}$$

$$T_{14} = \frac{630 - x^{39}}{x^6}$$

$$T_{14} = 630 - x^{33}$$

Mark: 1½ out of 3

Note(s):

- gave 1 mark for ${}_{16}C_{13}$
- gave 1 mark for consistent factors
- gave ½ mark for exponent laws
- deducted 1 mark for concept error (subtracting instead of multiplying)

Question 6a) G2 b) G3 c) G4

The probability that Sam wears a hat is 0.24.

If Sam wears a hat, the probability he wears jeans that day is 0.71.

If Sam does not wear a hat, the probability he wears jeans is 0.34.

- Find the probability that Sam wears jeans.
- Find the probability that Sam does not wear jeans.
- Given that Sam wears jeans, find the probability that he wears a hat.
Express your answer correct to 3 decimal places.

Solution

a)

$$\begin{aligned} P(\text{jeans}) &= P(\text{hat and jeans}) + P(\text{no hat and jeans}) \\ &= (0.24)(0.71) + (0.76)(0.34) \\ &= 0.1704 + 0.2584 \\ &= 0.4288 \quad \text{or} \quad 0.429 \end{aligned}$$

½ mark for P(hat and jeans)
½ mark for P(no hat and jeans)
1 mark for adding probabilities

2 marks

b)

$$\begin{aligned} P(\text{no jeans}) &= 1 - P(\text{jeans}) \\ &= 1 - 0.4288 \\ &= 0.5712 \quad \text{or} \quad 0.571 \end{aligned}$$

1 mark for complement

1 mark

c)

$$\begin{aligned} P(\text{hat} | \text{jeans}) &= \frac{P(\text{hat and jeans})}{P(\text{jeans})} \\ &= \frac{(0.24)(0.71)}{(0.24)(0.71) + (0.76)(0.34)} \\ &= 0.397388 \\ &= 0.397 \end{aligned}$$

½ mark for P(hat and jeans)
½ mark for P(jeans)
1 mark for conditional probability fraction (including addition)

2 marks

Exemplars

a)

$$P = \frac{(0.24)(0.71) + (0.34)(0.76)}{(0.24)(0.29) + (0.76)(0.60)}$$
$$= \frac{0.4288}{0.9712} = 0.7507$$

Mark: 1 out of 2

Note(s):

→ gave full marks

→ deducted 1 mark for concept error

b)

$$(0.24)(0.29) + (0.76)(0.66)$$
$$= 0.9712$$

Mark: 1 out of 1

Note(s):

→ gave full marks

c)

$$P = \frac{(0.24)(0.71)}{(0.7507)}$$
$$= 0.227$$

Mark: 2 out of 2

Note(s):

→ gave full marks [marked consistent with (a)]

Question 7a) H1 b) H2

The 1st term in a geometric sequence is 5 and the 6th term is 12.4416.

- a) Find the 30th term of this sequence.

Express your answer correct to 3 decimal places.

- b) Find the sum of the first 30 terms.

Express your answer correct to 3 decimal places.

Solution

a) $t_6 = t_1 r^5$

$$12.4416 = 5r^5$$

$$\frac{12.4416}{5} = r^5$$

$$r^5 = 2.48832$$

$$r = 1.2$$

1 mark for solving for r

$$t_{30} = t_1 r^{29}$$

$$= 5(1.2)^{29}$$

$$= 989.067974$$

$$= 989.068$$

1 mark for solving for t_{30}

2 marks

- b) **Method 1**

$$S_{30} = \frac{t_1(1 - r^{30})}{1 - r}$$

$$= \frac{5(1 - 1.2^{30})}{1 - 1.2}$$

$$= 5909.407845$$

$$= 5909.408$$

1 mark for substitution
into correct formula

1 mark

- Method 2**

$$S_{30} = \frac{t_1 - t_{30}r}{1 - r}$$

$$= \frac{5 - 989.067974(1.2)}{1 - 1.2}$$

$$= 5909.407845$$

$$= 5909.408$$

Note(s):

→ mark (b) consistent with (a)

Exemplars

a)

$$5, \dots, 12.4416$$
$$t_n = t_1 r^{n-1}$$
$$t_{30} = t_1 r^{29}$$
$$t_{30} = 5(1.164084139)^{29}$$
$$t_{30} = 409.748$$
$$n = \sqrt[6]{\frac{12.4416}{5}}$$
$$r = \sqrt[6]{2.48832}$$
$$r = 1.164084139$$

Mark: 1 out of 2

Note(s):

→ gave 1 mark for solving for t_{30}

b)

$$S_{30} = \frac{5 - t_{30}r}{1 - r}$$
$$S_{30} = \frac{5 - 409.748(1.164084139)}{1 - 1.164084139}$$
$$= \frac{5 - 477.85279}{-0.164084139}$$
$$S_{30} = 6670.269$$

Mark: ½ out of 1

Note(s):

→ gave full marks

→ deducted ½ mark for changing multiplication ($t_{30}r$) to exponent (t_{30}^r)

Question 8

D6

Solve algebraically:

$$3^{4x} = 5^{x+2}$$

Express your answer correct to 3 decimal places.

Solution

$$3^{4x} = 5^{x+2}$$

$$\log 3^{4x} = \log 5^{x+2}$$

 $\frac{1}{2}$ mark for applying logarithms

$$(4x)\log 3 = (x+2)\log 5$$

1 mark for power rule

$$4x\log 3 = x\log 5 + 2\log 5$$

$$4x\log 3 - x\log 5 = 2\log 5$$

 $\frac{1}{2}$ mark for collecting terms with x

$$x(4\log 3 - \log 5) = 2\log 5$$

$$x = \frac{2\log 5}{4\log 3 - \log 5}$$

 $\frac{1}{2}$ mark for solving for x

$$x = 1.155\ 786$$

 $\frac{1}{2}$ mark for evaluating quotient of logarithms

$$x = 1.156$$

3 marks

Exemplar

$$\log 3^{4x} = \log 5^{x+2}$$

$$4x \log 3 = x + 2 \log 5$$

$$4x \log 3 - x = 2 \log 5$$

$$4x - x = \frac{2 \log 5}{\log 3}$$

$$3x = \frac{2 \log 5}{\log 3}$$

$$3x = 2.929947$$

$$x = 0.976649$$

Mark: 1½ out of 3

Note(s):

- gave ½ mark for applying logarithms
- gave 1 mark for power rule
- gave ½ mark for collecting terms with x
- gave ½ mark for evaluating quotient of logarithms
- deducted 1 mark for concept error of missing brackets

Question 9a) F1 b) F2

The equation of a conic section is $5x^2 + 5y^2 - 30x + 20y - 115 = 0$.

- a) Identify this conic section.
b) Write the above equation in standard form.

Solution

- a) circle

1 mark

b)
$$5x^2 - 30x + 5y^2 + 20y = 115$$

$$5(x^2 - 6x) + 5(y^2 + 4y) = 115$$

$$5(x^2 - 6x + 9) + 5(y^2 + 4y + 4) = 115 + 45 + 20$$

$$5(x - 3)^2 + 5(y + 2)^2 = 180$$

$$(x - 3)^2 + (y + 2)^2 = 36$$

{ 1 mark for completing the square for x
($\frac{1}{2}$ mark for left side, $\frac{1}{2}$ mark for right side)
1 mark for completing the square for y
($\frac{1}{2}$ mark for left side, $\frac{1}{2}$ mark for right side)

1 mark for standard form
($\frac{1}{2}$ mark for left side, $\frac{1}{2}$ mark for right side)

3 marks

Note(s):

→ mark (b) consistent with (a)

Exemplars

a)

Circle

Mark: 1 out of 1

→ gave full marks

b)

$$5x^2 + 5y^2 - 30x + 20y - 115 = 0$$

$$5x^2 - 30x + 5y^2 + 20y = 115$$

$$5(x^2 - 6x + 9) + 5(y^2 + 4y + 4) = 115 + 20 + 45$$

$$\frac{5(x-3)^2}{180} + \frac{5(y+2)^2}{180} = \frac{180}{180}$$

$$\boxed{\frac{(x-3)^2}{36} + \frac{(y+2)^2}{36} = 1}$$

Mark: 3 out of 3

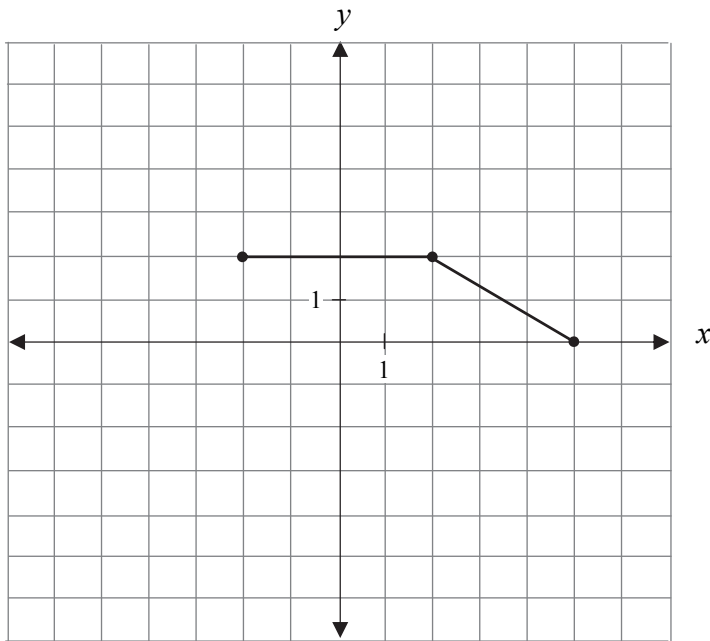
Note(s):

→ gave full marks

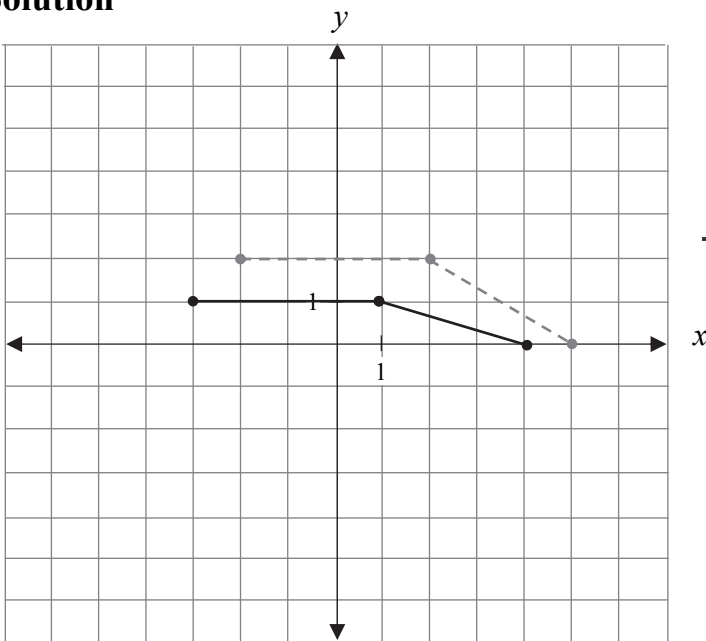
Question 10

B1, B2

The graph of $y = 2f(x - 1)$ is sketched below.



Sketch a clearly labelled graph of $y = f(x)$.

Solution

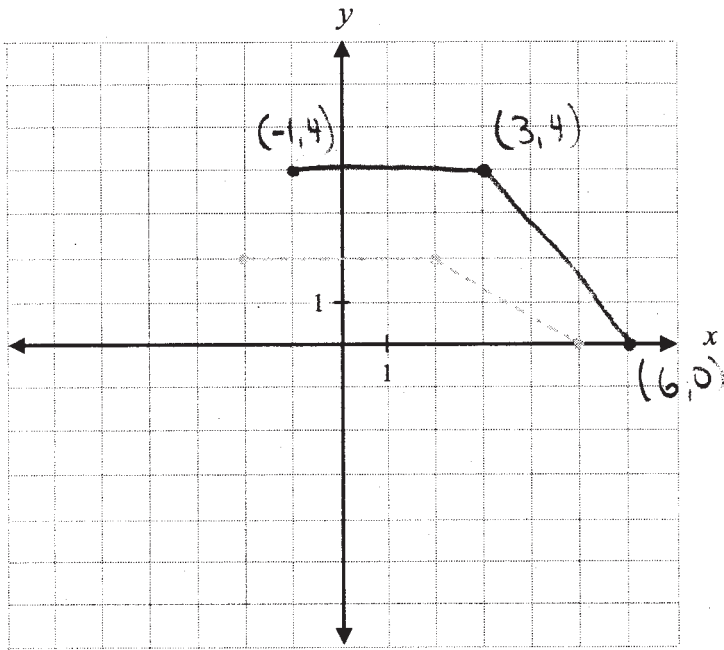
1 mark for vertical compression by a factor of 2
1 mark for horizontal shift of 1 unit to the left

2 marks

Note(s):

→ give a maximum of 1 mark for mistaking the given graph as $y = f(x)$ and then sketching the graph of $y = 2f(x - 1)$

Exemplar



X	Y
-2	2
2	2
5	0

↓

X	Y
-1	4
3	4
6	0

Mark: 1 out of 2

Note(s):

→ see note on previous page

Part 2: Multiple-Choice Questions

B1 11. If the graph of $f(x) = |x - 1|$ is shifted 2 units down, the equation for the resulting graph is:

- a) $y = |x + 1|$
- b) $y = |x - 3|$
- c) $y = |x - 1| - 2$
- d) $y = |x - 1| + 2$

A4 12. How many solutions are there to the equation $\cos x = \frac{1}{2}$ over the interval $[0, 3\pi]$?

- a) 1
- b) 2
- c) 3
- d) 4

D2 13. Solve:

$$3^{x-10} = \left(\frac{1}{9}\right)^{2x}$$

- a) -10
- b) $-\frac{10}{3}$
- c) $\frac{1}{2}$
- d) 2

D1 14. Find the y -intercept of $f(x) = -3^x - 2$.

a) $y = -5$

b) $y = -3$

c) $y = -2$

d) $y = 0$

A2 15. Find the value of $\sec^2 \theta$ if $\tan \theta = -1$.

a) 0

b) 1

c) 2

d) undefined

A3 16. What is the exact value of $\cos(-17\pi)$?

a) -1

b) 0

c) 1

d) undefined

B7 17. What is the period of the function $f(x) = \tan(4x)$?

a) 2π

b) π

c) $\frac{\pi}{2}$

d) $\frac{\pi}{4}$

D4 18. State the equation of an asymptote for the graph of $f(x) = \ln x + 2$.

a) $y = 0$

b) $x = 0$

c) $y = 2$

d) $x = -2$

A2 19. The point $P(\theta) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ is on the unit circle. What are the coordinates of $P\left(\theta + \frac{\pi}{2}\right)$?

a) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

b) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

c) $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

d) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

E2 20. How many ways can 6 people be seated at a round table?

a) ${}_6C_6$

b) ${}_6C_5$

c) $5!$

d) $6!$

F3 21. A conic section is described by the equation $\frac{(y)^2}{9} - \frac{(x-1)^2}{16} = 1$.

What is the domain of this conic section?

a) $(-\infty, \infty)$

b) $(-\infty, -3] \cup [5, \infty)$

c) $[-3, 5]$

d) $[-2, 4]$

D5 22. Given that $\log_a 2 = y$ and $\log_a 3 = x$, determine the value of $\log_a 18$ in terms of x and y .

a) $2xy$

b) x^2y

c) $x^2 + y$

d) $2x + y$

H3 23. Find the sum of the infinite series:

$$8 + 4 + 2 + 1 + \dots$$

a) 16

b) 15

c) 8

d) $\frac{1}{2}$

A6 24. What is the range of the function $y = -2 \sin x + 1$?

a) $[-2, 2]$

b) $[-1, 3]$

c) $[-1, 1]$

d) $[0, 2]$

E4 25. The 4th term in the binomial expansion of $(x - 2)^n$ is $-80x^2$.

Find the value of n .

a) 4

b) 5

c) 6

d) 7

Part 2: Short-Answer Questions

Award half marks for incorrect answers that have resulted from arithmetic or notation errors as indicated in the marking guidelines.

Do not award any other half marks unless indicated in this marking guide.

Question 26

G2

1 mark

The probability that Jake rides his bike to school is 0.7.

The probability that Paul walks to school is 0.9.

What is the probability that Jake rides his bike to school and Paul does not walk to school?

Solution

$$\begin{aligned} P(\text{rides and does not walk}) &= (0.7)(0.1) \\ &= 0.07 \end{aligned}$$

Note(s):

→ give ½ mark for $(0.7)(0.1)$

Question 27

A2

1 mark

The point $P(\theta) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ is on the unit circle. What is a possible value of θ ?

Solution

$$\theta = \frac{3\pi}{4} \quad \text{or} \quad 135^\circ$$

Note(s):

→ other possible solutions: $\frac{3\pi}{4} + 2\pi k$ or $135^\circ + 360^\circ k$, $k \in \mathbb{I}$

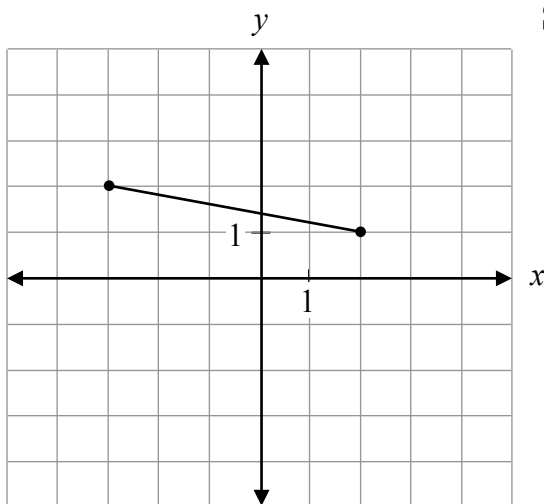
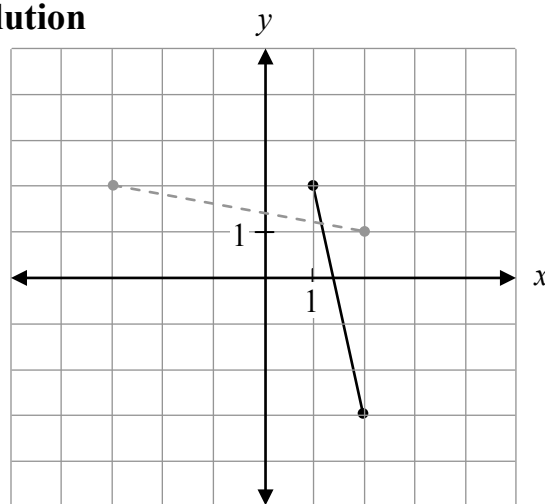
Question 28

B3

1 mark

The graph of $y = f(x)$ is sketched below to the left.

Sketch the graph of $y = f^{-1}(x)$ on the axes provided to the right.

**Solution**

Question 29

C2

1 mark

Evaluate the expression:

$$\sin 45^\circ \cos 15^\circ - \cos 45^\circ \sin 15^\circ$$

Solution

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\sin(45^\circ - 15^\circ) = \sin 30^\circ$$

$$= \frac{1}{2}$$

Note(s):

→ give $\frac{1}{2}$ mark for $\sin(45^\circ - 15^\circ)$

Question 30

G1

1 mark

Two six-sided cubes numbered 1 to 6 are tossed at the same time.

What is the probability that the sum of the numbers on the two cubes is 7?

Solution

$$P(\text{sum of 7}) = \frac{6}{36} \text{ or } \frac{1}{6}$$

Question 31

A5

1 mark

The general solution to a trigonometric equation is $\theta = \frac{\pi}{2} + 2\pi k$, $k \in \mathbb{I}$.

Give one possible equation that would have this general solution.

Solution

$$\sin \theta = 1 \text{ or } \csc \theta = 1$$

Question 32

A3

1 mark

What is the exact value of $\sec\left(\frac{5\pi}{3}\right)$?

Solution

2

Question 33

E2

1 mark

How many ways can 2 boys and 2 girls be seated in a row if they cannot sit beside someone of the same sex?

Solution

8

Question 34

A6

1 mark

Write the equation of an asymptote for $f(x) = \csc x$.

Solution $x = 0$ or $x = \pi k, k \in \mathbb{I}$

Note(s):

→ give $\frac{1}{2}$ mark for $x \neq 0$ or $x \neq \pi k, k \in \mathbb{I}$

Question 35

G2

1 mark

A bag contains 4 red marbles and 1 blue marble. A marble is drawn from the bag and not replaced. A second marble is drawn from the bag.

Find the probability that both marbles drawn from the bag are red.

Solution

$$\begin{aligned} P(RR) &= \left(\frac{4}{5}\right)\left(\frac{3}{4}\right) \\ &= \frac{12}{20} \quad \text{or} \quad \frac{3}{5} \end{aligned}$$

Note(s):

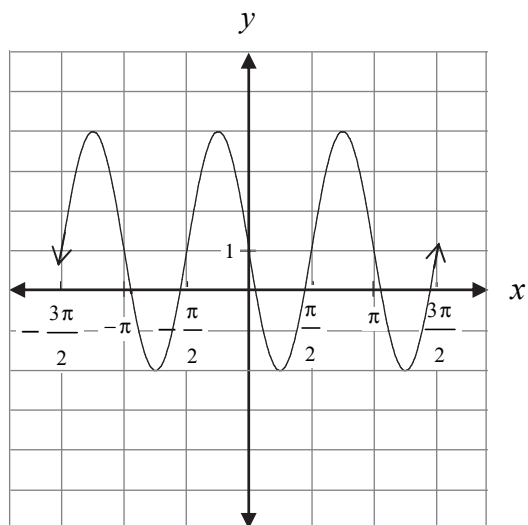
→ give ½ mark for $\left(\frac{4}{5}\right)\left(\frac{3}{4}\right)$

Question 36

B7

1 mark

What is the amplitude of the following graph?

**Solution**

3

Question 37

E3

1 mark

Determine the value of n .

$${}_n C_9 = {}_n C_3$$

Solution $n = 12$

Question 38

C1

1 mark

Express $\frac{\csc \theta}{\cot \theta}$ as a single trigonometric function.

Solution

$$\begin{aligned}\frac{\csc \theta}{\cot \theta} &= \frac{\frac{1}{\sin \theta}}{\frac{\cos \theta}{\sin \theta}} \\ &= \frac{1}{\cos \theta} \quad \text{or} \quad \sec \theta\end{aligned}$$

Note(s):

→ give ½ mark for $\frac{\frac{1}{\sin \theta}}{\frac{\cos \theta}{\sin \theta}}$

Question 39

E1

1 mark

Solve:

$$\frac{14!}{12!} = 14n$$

Solution

$$n = 13 \quad \text{or} \quad \frac{182}{14}$$

Question 40

E2

1 mark

How many different 4-letter arrangements can be made from the letters D A D A?

Solution

$$\frac{4!}{2!2!} \quad \text{or} \quad 6$$

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Part 2: Long-Answer Questions

Question 41

D6

Solve the following equation:

$$\log_2(x - 2) - \log_2(x) = 3$$

Solution

$$\log_2(x - 2) - \log_2(x) = 3$$

$$\log_2 \frac{(x - 2)}{(x)} = 3$$

1 mark for log theorem

$$2^3 = \frac{(x - 2)}{(x)}$$

1 mark for exponential form

$$8x = x - 2$$

$$7x = -2$$

$$x = -\frac{2}{7}$$

½ mark for simplification

$$\cancel{x = -\frac{2}{7}}$$

½ mark for rejecting extraneous root

3 marks

∴ no solution

Exemplar

$$\log_2 \left(\frac{x-2}{x} \right) = 3$$

$$3^2 = \left(\frac{x-2}{x} \right)$$

$$9 = \frac{x-2}{x}$$

$$\uparrow x = x-2$$

$$\ominus x = -2$$

$$x = \frac{-2}{8}$$

$$x = -\frac{1}{4}$$

Mark: 1½ out of 3

Note(s):

- gave 1 mark for log theorem
- gave ½ mark for simplification

Question 42C1

Prove the identity:

$$\sin x + \cot x \cos x = \csc x$$

Solution

$$\text{LHS} = \sin x + \cot x \cos x$$

$$= \sin x + \frac{\cos x}{\sin x} \cdot \cos x$$

 $\frac{1}{2}$ mark for identity

$$= \sin x + \frac{\cos^2 x}{\sin x}$$

$$= \frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x}$$

1 mark for common denominator

$$= \frac{\sin^2 x + \cos^2 x}{\sin x}$$

$$= \frac{1}{\sin x}$$

1 mark for identity

$$= \csc x$$

 $\frac{1}{2}$ mark for identity

$$= \text{RHS}$$

3 marks

Exemplar

Left-Hand Side	Right-Hand Side
$\sin x + \cot x \cos x$ $\frac{\sin x}{\sin x} + \frac{\cos x}{\sin x} \frac{\cos x}{\sin x}$ $\frac{\sin x + \cos x^2}{\sin x}$ $\cos x^2 = \text{RHS}$	$\cos x = \cos x^2$

Mark: 1/2 out of 3

Note(s):

→ gave 1/2 mark for identity on left-hand side

Question 43

E2

- a) Using the digits 0, 1, 3, 4, 5, and 8, how many 4-digit numbers are possible if repetition of digits is not allowed?
- b) Using the digits 0, 1, 3, 4, 5, and 8, how many 4-digit numbers greater than 4000 and divisible by 5 are possible if repetition of digits is not allowed?
Briefly explain your calculations.

Solution

a)

$$\frac{5}{\text{not } 0} \cdot 5 \cdot 4 \cdot 3 = 300$$

1 mark

b)

Method 1

$$\text{Case 1: } \frac{3}{4, 5, \text{ or } 8} \cdot 4 \cdot \frac{3}{0} \cdot \frac{1}{0} = 36$$

1 mark for Case 1

$$\text{Case 2: } \frac{2}{4 \text{ or } 8} \cdot 4 \cdot \frac{3}{5} \cdot \frac{1}{5} = 24$$

1 mark for Case 2

$$36 + 24 = 60$$

1 mark for addition of cases

3 marks**Method 2**

$$\text{Case 1: } \frac{2}{4 \text{ or } 8} \cdot 4 \cdot \frac{3}{0 \text{ or } 5} \cdot \frac{2}{0 \text{ or } 5} = 48$$

1 mark for Case 1

$$\text{Case 2: } \frac{1}{5} \cdot 4 \cdot \frac{3}{0} \cdot \frac{1}{0} = 12$$

1 mark for Case 2

$$48 + 12 = 60$$

1 mark for addition of cases

3 marks

Note(s):

→ in (b), give a maximum of 1 mark if student attempted to solve without using cases

Exemplars

a)

$$\underline{6} \quad \underline{5} \quad \underline{4} \quad \underline{3} = \boxed{360 \text{ numbers}}$$

Mark: 0 out of 1

b)

$$\begin{array}{cccc} \underline{3} & \underline{4} & \underline{3} & \underline{2} & = \\ 4, 5 \text{ or } 8 & & & 5 \text{ or } 0 & \end{array}$$

$$\boxed{72 \text{ numbers}}$$

Mark: 1 out of 3

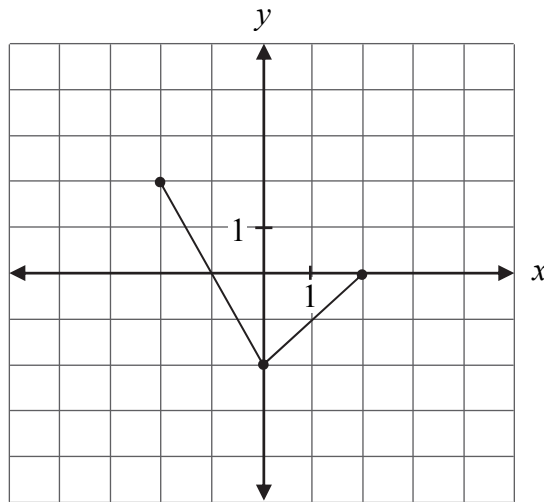
Note(s):

→ see note on previous page

Question 44

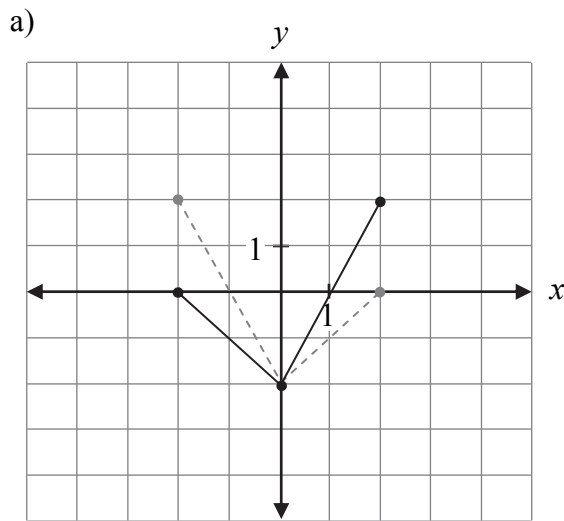
a) B3 b) B5 c) B1 d) B4

The graph of $y = f(x)$ is sketched below.



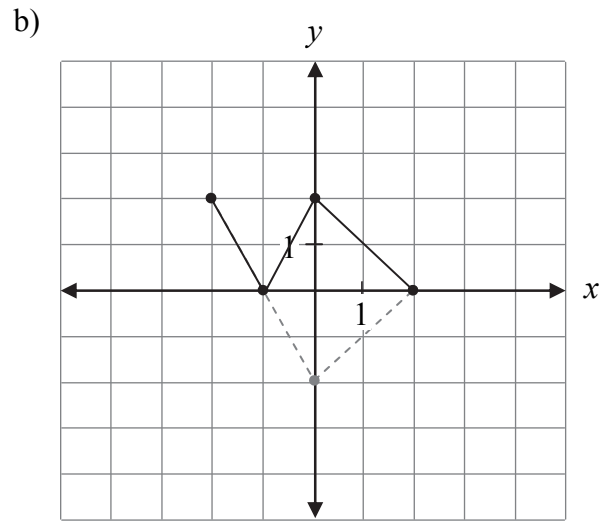
- a) Sketch a clearly labelled graph of $y = f(-x)$.
- b) Sketch a clearly labelled graph of $y = |f(x)|$.

Solution



1 mark for reflection in y-axis

1 mark



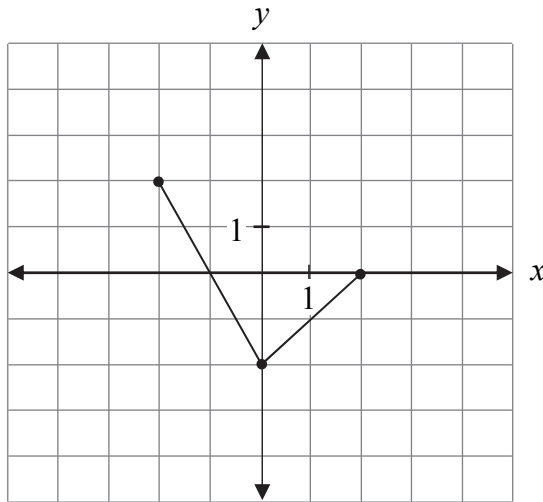
1 mark for absolute value

1 mark

Question 44

a) B3 b) B5 c) B1 d) B4

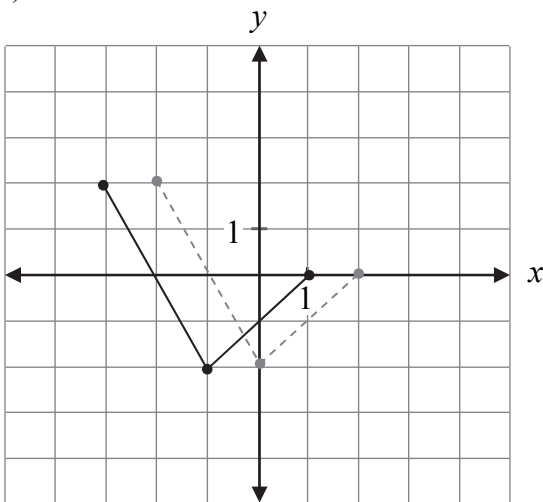
The graph of $y = f(x)$ is sketched below.



- c) Sketch a clearly labelled graph of $y = f(x+1)$.
- d) Sketch a clearly labelled graph of $y = \frac{1}{f(x)}$.

Solution

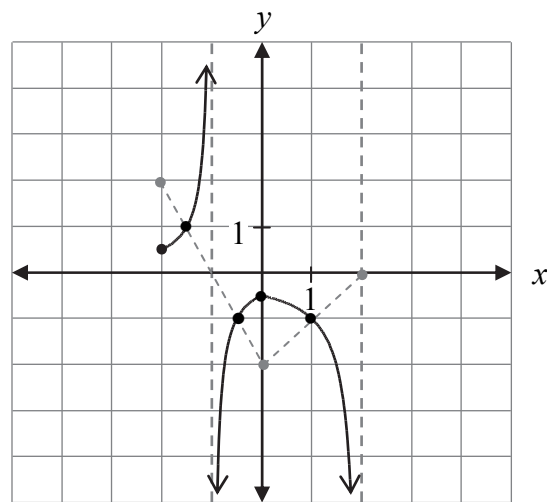
c)



1 mark for horizontal shift

1 mark

d)



1 mark for asymptotes (½ mark for each correctly placed asymptote)

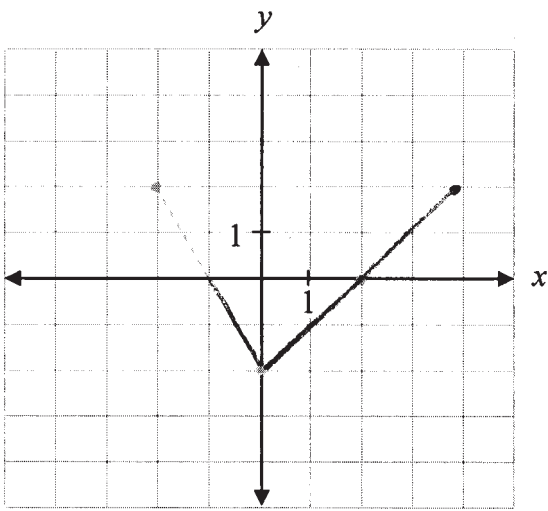
½ mark for the graph on the interval $[-2, -1)$

½ mark for the graph on the interval $(-1, 2)$

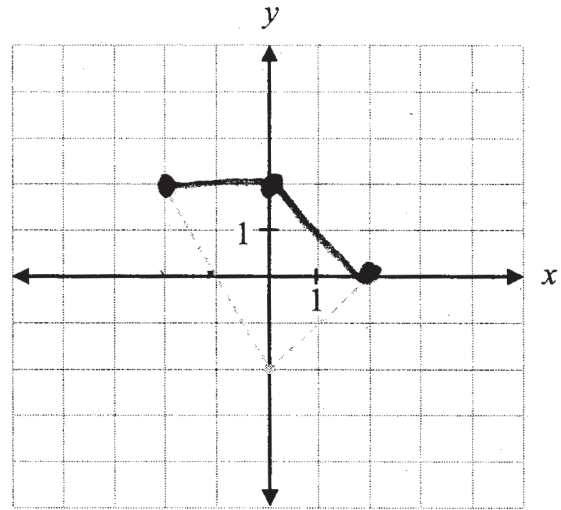
2 marks

Exemplars

a)



b)

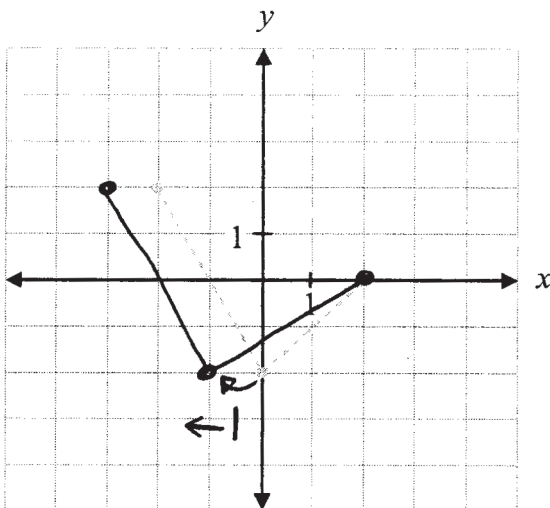


$(-2, 2) \rightarrow (2, 2)$
 $(0, 2) \rightarrow (2, 0)$
 $(2, 0) \rightarrow (0, 2)$
 $(-2, 2) \rightarrow (-2, 2)$
 $(0, -2) \rightarrow (0, 2)$
 $(2, 0) \rightarrow (2, 0)$

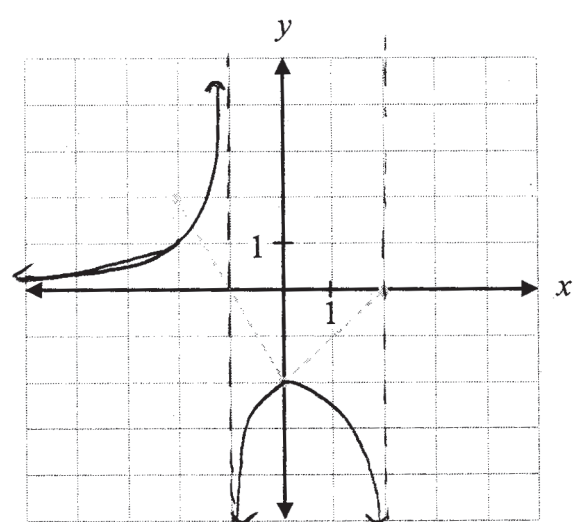
Mark: 0 out of 1

Mark: 0 out of 1

c)



d)



Mark: 1/2 out of 1

Note(s):

- gave full marks
- deducted 1/2 mark for one inaccurate endpoint

Mark: 1 out of 2

Note(s):

- gave 1 mark for asymptotes

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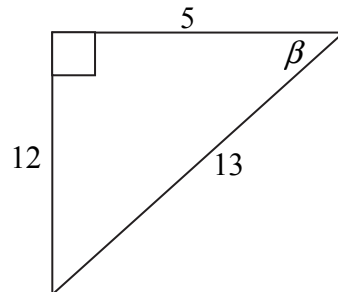
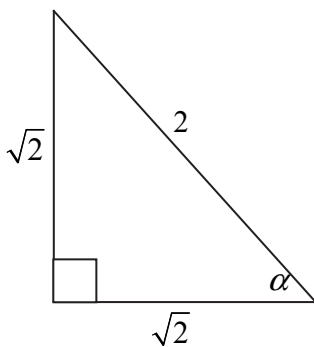
Question 45a) C2 b) A2

Given that $\sin \alpha = \frac{\sqrt{2}}{2}$, where α is in quadrant II, and $\cos \beta = -\frac{5}{13}$, where β is in quadrant III,

- a) find the exact value of $\tan(\alpha - \beta)$.
b) find the exact value of $\cot(\alpha - \beta)$.

Solution

a)



1 mark for triangle values
($\frac{1}{2}$ mark for $\sqrt{2}$, $\frac{1}{2}$ mark for 12)

$$\begin{aligned}\tan(\alpha - \beta) &= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \\ &= \frac{(-1) - \left(\frac{12}{5}\right)}{1 + (-1)\left(\frac{12}{5}\right)} \\ &= \frac{-\frac{17}{5}}{-\frac{5}{5}} \\ &= \frac{17}{7}\end{aligned}$$

1 mark ($\frac{1}{2}$ mark for $\tan \alpha$, $\frac{1}{2}$ mark for $\tan \beta$)
 $\frac{1}{2}$ mark for substitution into correct formula

$\frac{1}{2}$ mark for simplifying a complex fraction

3 marks

b)

$$\cot(\alpha - \beta) = \frac{7}{17}$$

1 mark

Exemplars

a)

$$\frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\frac{-1 - (5/12)}{1 + (-1)(5/12)}$$

$$\frac{-12/12 - 5/12}{-12/12 - 5/12}$$

$$\frac{-17/12}{-17/12}$$

$$\frac{-17/12}{12/12 + (-5/12)}$$

$$\frac{17}{12} \div \frac{7}{12}$$

$$\frac{17}{7}$$

Mark: 2 out of 3

Note(s):

- gave 1 mark for triangle values
- gave ½ mark for $\tan \alpha$
- gave ½ mark for substitution into correct formula

b)

$$\frac{7}{17}$$

Mark: 1 out of 1

Note(s):

- gave full marks

Question 46F3

- a) Sketch a clearly labelled graph of the conic section described by the equation:

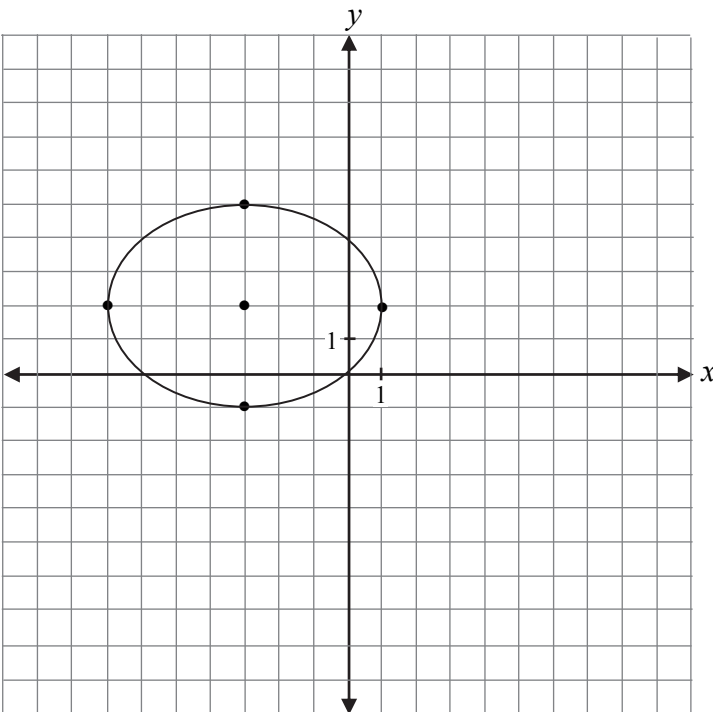
$$\frac{(x+3)^2}{16} + \frac{(y-2)^2}{9} = 1$$

- b) Determine the length of the major axis for the conic section described by the equation

$$\frac{(x+3)^2}{16} + \frac{(y-2)^2}{9} = 1.$$

Solution

a)

 $\frac{1}{2}$ mark for ellipse $\frac{1}{2}$ mark for centre $\frac{1}{2}$ mark for endpoints of major axis $\frac{1}{2}$ mark for endpoints of minor axis

2 marks

- b)
- $2(4) = 8$

1 mark

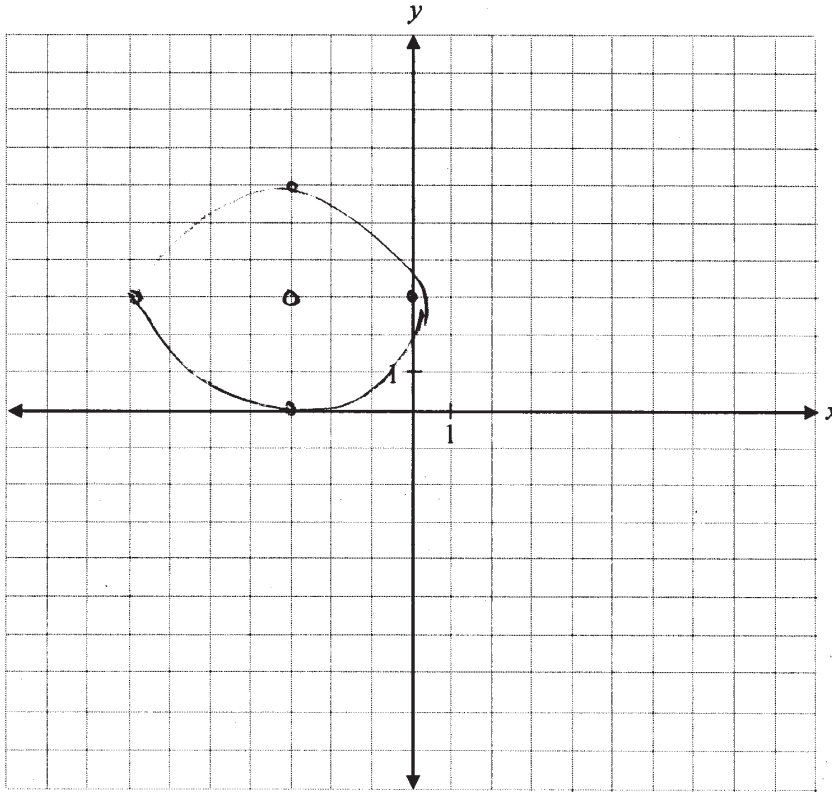
Note(s):

→ in (a), give a maximum of 1 mark for a correctly sketched graph of $\frac{(x+3)^2}{16} + \frac{(y-2)^2}{9} = 1$

→ in (b), give full marks for an answer of 8 or an answer consistent with the graph sketched in (a)

Exemplars

a)



Mark: 1 out of 2

Note(s):

→ gave $\frac{1}{2}$ mark for ellipse

→ gave $\frac{1}{2}$ mark for endpoints of minor axis

b)

$$3 \times 2 = 6$$

Mark: 0 out of 1

Question 47F2, F3

Determine the coordinates of the vertex of the parabola described by the equation:

$$2y^2 - x + 4y + 3 = 0$$

Solution

$$2y^2 + 4y - x + 3 = 0$$

$$2(y^2 + 2y) = x - 3$$

$$2(y^2 + 2y + 1) = x - 3 + 2$$

$$2(y + 1)^2 = (x - 1)$$

1 mark for completing the square for y
($\frac{1}{2}$ mark for left side, $\frac{1}{2}$ mark for right side)

Vertex: $(1, -1)$

1 mark for vertex ($\frac{1}{2}$ mark for each coordinate)

2 marks

Exemplar

$$\begin{aligned}2y^2 + 4y - x + 3 \\2y^2 + 4y \text{ --- } -x = -3 \\2(y^2 + 2y + 1) - x = -3 + 2 \\ \frac{2(y+1)^2 - x}{2} = \frac{-1}{2} \\(y+1)^2 - \frac{x}{2} = -\frac{1}{2} \\ \left(+\frac{1}{2}, -1\right)\end{aligned}$$

Mark: 1½ out of 2

Note(s):

- gave 1 mark for completing the square for y
- gave ½ mark for y coordinate of vertex

Question 48A4, C1

Solve the following equation over the interval $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

$$\sec^2 \theta - \tan \theta = 1$$

Solution**Method 1**

$$(\tan^2 \theta + 1) - \tan \theta = 1$$

1 mark for identity

$$\tan^2 \theta - \tan \theta = 0$$

$$\tan \theta(\tan \theta - 1) = 0$$

 $\frac{1}{2}$ mark for simplifying and factoring

$$\tan \theta = 0 \quad \tan \theta = 1$$

 $\frac{1}{2}$ mark for solving for $\tan \theta$

$$\theta = 0, \pi, 2\pi \quad \theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$\theta = \pi \quad \theta = \frac{5\pi}{4}$$

1 mark ($\frac{1}{2}$ mark for each value of θ)**3 marks****Method 2**

$$\frac{1}{\cos^2 \theta} - \frac{\sin \theta}{\cos \theta} = 1$$

1 mark for identities

$$1 - \sin \theta \cos \theta = \cos^2 \theta$$

$$1 - \sin \theta \cos \theta = 1 - \sin^2 \theta$$

 $\frac{1}{2}$ mark for identity

$$\sin \theta \cos \theta = \sin^2 \theta$$

$$\sin^2 \theta - \sin \theta \cos \theta = 0$$

$$\sin \theta(\sin \theta - \cos \theta) = 0$$

$$\sin \theta = 0 \quad \sin \theta = \cos \theta$$

 $\frac{1}{2}$ mark for solving for $\sin \theta$

$$\theta = 0, \pi, 2\pi \quad \theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$\theta = \pi \quad \theta = \frac{5\pi}{4}$$

1 mark ($\frac{1}{2}$ mark for each value of θ)**3 marks**

Note(s):

→ deduct $\frac{1}{2}$ mark if any value of θ is given outside the domain $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

Exemplar

$$\frac{1}{\cos^2\theta} - \frac{\sin\theta}{\cos\theta} = 1$$

$$\frac{1}{\cos^2\theta} - \frac{\sin\theta\cos\theta}{\cos^2\theta} = 1$$

$$\frac{1 - \sin\theta\cos\theta}{\cos^2\theta} = 1$$

$$1 - \sin\theta\cos\theta = \cos^2\theta$$

$$1 - \cos^2\theta = \sin\theta\cos\theta$$

$$\frac{\sin^2\theta}{\sin\theta} = \frac{\sin\theta\cos\theta}{\sin\theta}$$

$$\sin\theta = \cos\theta$$

$$\theta = \pi/4$$

$$\begin{aligned} \pi + \pi/4 &= 5\pi/4 \\ \pi - \pi/4 &= 3\pi/4 \end{aligned} = \theta$$

Mark: 1½ out of 3

Note(s):

- gave 1 mark for identities in line 1
- gave ½ mark for identity in line 6
- gave ½ mark for one value of θ
- deducted ½ mark for incorrect value of θ

Question 49B7

A sinusoidal curve has a maximum at (3, 6). The next maximum on the curve is at (11, 6).

The range of this function is $[-4, 6]$.

Find the values of A, B, C, and D if the sinusoidal equation for this curve is $y = A \sin[B(x - C)] + D$.

Solution**Method 1**

$$A = 5$$

1 mark for A

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{8} = \frac{\pi}{4}$$

1 mark (½ mark for period, ½ mark for consistent B value)

$$C = 1 \text{ or } 9$$

1 mark for C

$$D = 1$$

1 mark for D

4 marks**Method 2**

$$A = -5$$

1 mark for A

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{8} = \frac{\pi}{4}$$

1 mark (½ mark for period, ½ mark for consistent B value)

$$C = -3 \text{ or } 5 \text{ or } 13$$

1 mark for C

$$D = 1$$

1 mark for D

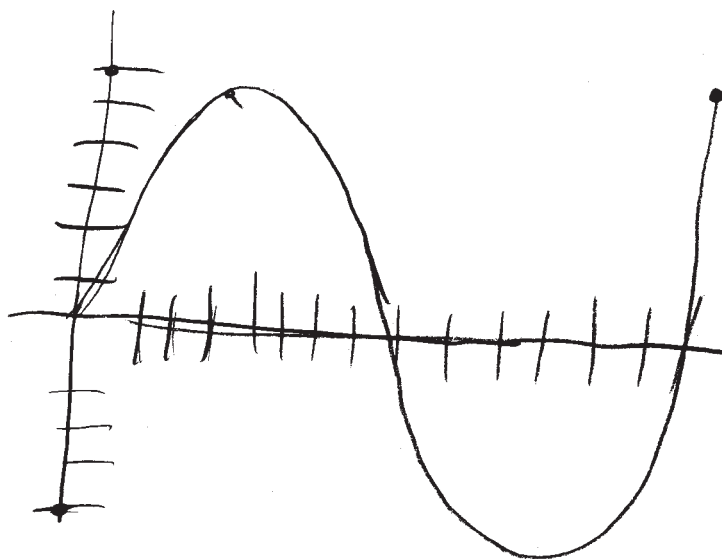
4 marks

Exemplar

$$A = \underline{5}$$
$$B = \underline{\frac{4}{\pi}}$$
$$C = \underline{0}$$
$$D = \underline{1}$$

$$\frac{\text{max} - \text{min}}{2} = \frac{6 - (-4)}{2} = \frac{10}{2} = 5$$
$$b = \frac{\text{per}}{2\pi} = \frac{8}{2\pi}$$

$$\frac{\text{max} + \text{min}}{2} = \frac{6 + (-4)}{2} = \frac{2}{2} = 1$$
$$11 - 3 = 8$$



Mark: 2½ out of 4

Note(s):

- gave 1 mark for A
- gave ½ mark for period
- gave 1 mark for D

Question 50

H2

Evaluate:

$$\sum_{n=2}^5 2(2^{n+1})$$

Solution

$$\begin{aligned}\sum_{n=2}^5 2(2^{n+1}) &= 2(2^3) + 2(2^4) + 2(2^5) + 2(2^6) \\ &= 16 + 32 + 64 + 128 \\ &= 240\end{aligned}$$

1 mark for $n = 2$ 1 mark for $n = 3, 4, \text{ and } 5$ 1 mark for addition**3 marks**

Note(s):

→ deduct 1 mark for having too many terms

Exemplar

$$z^4 \cdot z^3 \cdot z^2 \cdot z^1 \cdot z^0$$

$$z(z^5+1) + z(z^4+1) + z(z^3+1) + z(z^2+1) + z(z+1)$$
$$128 + 64 + 32 + 16 + 8$$

$$\begin{array}{r} 12 \\ 116 \\ 128 \\ \hline 256 \end{array} = 248$$

Mark: 2 out of 3

Note(s):

→ gave full marks

→ deducted 1 mark (see note on previous page)

Question 51A3

Give a possible value of θ that will satisfy the equation below.

$$\cos \theta = \sin\left(-\frac{7\pi}{6}\right) - 1$$

Solution

$$\cos \theta = \sin\left(-\frac{7\pi}{6}\right) - 1$$

$$\cos \theta = \frac{1}{2} - 1$$

1 mark for evaluating $\sin\left(-\frac{7\pi}{6}\right)$

$$\cos \theta = -\frac{1}{2}$$

$$\theta = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3} \text{ or } \dots$$

1 mark for one correct value of θ

or

$$\theta = 120^\circ \text{ or } 240^\circ \text{ or } \dots$$

2 marks

Exemplar

$$-\frac{\sqrt{3}}{2} \neq \frac{1}{2} - 1$$

$$-\frac{1}{2} - \frac{1}{2} = -1$$

$$\cos\left(\frac{2\pi}{3}\right) - \sin\left(-\frac{\pi}{6}\right) = -1$$



Mark: 1½ out of 2

Note(s):

- gave full marks (trial and error method)
- deducted ½ mark for not stating “ $\theta =$ ” in final answer

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Appendices

Appendix A

MARKING GUIDELINES

- Arithmetic error, deduct ½ mark
- Concept error, deduct 1 mark
- Notation error, deduct ½ mark
- For each different type of notation error, deduct ½ mark to a maximum of 1 mark per question
- Incorrect precision or rounding, deduct ½ mark
- Final probability answer greater than 1, deduct ½ mark
- Intercept stated as an ordered pair, deduct ½ mark
- Changing an equation to an expression or vice versa more than once, deduct ½ mark
- Variable introduced more than once without being defined, deduct ½ mark

Example: $2 \sin x = 1$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \text{ over } [0, 2\pi]$$

$$x = \frac{\pi}{6} + 2k\pi \text{ or } \frac{5\pi}{6} + 2k\pi, "k" \text{ not defined, deduct } \frac{1}{2} \text{ mark}$$

(Note: Student can write " $k \in I$ " or " k is an integer".)

- LHS and RHS equated throughout the proof of an identity, deduct 1 mark
- Variable omitted more than once in a trig identity or equation, deduct ½ mark
- Variable changed without being redefined more than once, in either an equation or an identity, deduct ½ mark

Example: $2 \sin \theta = 1$

$$\cos^2 \theta = 1$$

$$\sin x = \frac{1}{2}$$

or

$$x^2 = 1$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \pm 1$$

- Parentheses omitted such as:
 - a) $\log 3^{x+1} = x + 1 \log 3 = x \log 3 + \log 3$, deduct ½ mark
 - b) $\log 3^{x+1} = x + 1 \log 3 = x + \log 3$, deduct 1 mark
 - c) $\frac{2^2}{3} = \frac{4}{9}$, deduct ½ mark

- Unsimplified fractions such as $\frac{\frac{1}{2} + \frac{1}{3}}{2}$, deduct ½ mark

(Note: Unreduced fractions such as $\frac{6}{8}$ or unrationalized fractions such as $\frac{\sqrt{3}}{\sqrt{2}-1}$ are acceptable.)

- Cases in permutations, combinations, or probability problems not briefly described, deduct ½ mark
- If an r value greater than 1 is used to calculate the sum of an infinite geometric series, deduct ½ mark
- Endpoints or arrowheads incorrectly shown, deduct ½ mark
- Asymptotes shown as solid lines or not shown, deduct ½ mark
- Graphs of functions drawn to cross or curl away from an asymptote, deduct ½ mark
- Domain or range written in incorrect order, deduct ½ mark

Example: $(-\infty, 0]$ written as $[0, -\infty)$

- Bracket error(s) made when stating the domain or range, deduct ½ mark

Appendix B

IRREGULARITIES IN STANDARDS TESTS

A GUIDE FOR LOCAL MARKING

During the marking of standards tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the Department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all "NR") or only incorrect responses ("0")

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the Department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a standards test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the Department along with the test materials.

Irregular Test Booklet Report

Test: _____

Date marked: _____

Booklet No.: _____

Problem(s) noted: _____

Question(s) affected: _____

Action taken or rationale for assigning marks: _____

Follow-up: _____

Decision: _____

Marker's Signature: _____

Principal's Signature: _____

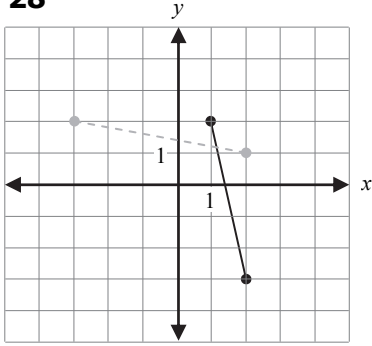
For Department Use Only—After Marking Complete

Consultant: _____

Date: _____

Appendix C

SUMMARY OF MARKS FOR SHORT-ANSWER QUESTIONS

<p>26*</p> $P(\text{rides and does not walk}) = (0.7)(0.1)$ $= 0.07$	<p>33</p> <p>8</p>
<p>27*</p> $\theta = \frac{3\pi}{4} \quad \text{or} \quad 135^\circ$	<p>34*</p> $x = 0 \quad \text{or} \quad x = \pi k, k \in \mathbb{I}$
<p>28</p> 	<p>35*</p> $P(\text{RR}) = \left(\frac{4}{5}\right)\left(\frac{3}{4}\right)$ $= \frac{12}{20} \quad \text{or} \quad \frac{3}{5}$
<p>29*</p> $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$ $\sin(45^\circ - 15^\circ) = \sin 30^\circ$ $= \frac{1}{2}$	<p>36</p> <p>3</p>
<p>30</p> $P(\text{sum of 7}) = \frac{6}{36} \quad \text{or} \quad \frac{1}{6}$	<p>37</p> $n = 12$
<p>31</p> $\sin \theta = 1 \quad \text{or} \quad \csc \theta = 1$	<p>38*</p> $\frac{\csc \theta}{\cot \theta} = \frac{\frac{1}{\sin \theta}}{\frac{\cos \theta}{\sin \theta}}$ $= \frac{1}{\cos \theta} \quad \text{or} \quad \sec \theta$
<p>32</p> <p>2</p>	<p>39</p> $n = 13 \quad \text{or} \quad \frac{182}{14}$
	<p>40</p> $\frac{4!}{2!2!} \quad \text{or} \quad 6$

* refer to marking notes in the short-answer section of this guide

Appendix D

TABLE OF QUESTIONS BY UNIT AND LEARNING OUTCOME

Unit A: Circular Functions			
Question	Type*	Learning Outcome	Mark
1a)	LA	A1	1
1b)	LA	A1	1
1c)	LA	A1	1
2	LA	A5	4
12	MC	A4	1
15	MC	A2	1
16	MC	A3	1
19	MC	A2	1
24	MC	A6	1
27	SA	A2	1
31	SA	A5	1
32	SA	A3	1
34	SA	A6	1
45b)	LA	A2	1
48	LA	A4	2
51	LA	A3	2
Unit B: Transformations			
10	LA	B1, B2	2
11	MC	B1	1
17	MC	B7	1
28	SA	B3	1
36	SA	B7	1
44a)	LA	B3	1
44b)	LA	B5	1
44c)	LA	B1	1
44d)	LA	B4	2
49	LA	B7	4
Unit C: Trigonometric Identities			
29	SA	C2	1
38	SA	C1	1
42	LA	C1	3
45a)	LA	C2	3
48	LA	C1	1

*MC = Multiple Choice, LA = Long Answer, SA = Short Answer

Unit D: Exponents and Logarithms			
Question	Type*	Learning Outcome	Mark
3	LA	D8	3
8	LA	D6	3
13	MC	D2	1
14	MC	D1	1
18	MC	D4	1
22	MC	D5	1
41	LA	D6	3
Unit E: Permutations, Combinations, and Binomial Theorem			
4a)	LA	E3	2
4b)	LA	E3	2
5	LA	E4	3
20	MC	E2	1
25	MC	E4	1
33	SA	E2	1
37	SA	E3	1
39	SA	E1	1
40	SA	E2	1
43a)	LA	E2	1
43b)	LA	E2	3
Unit F: Conic Sections			
9a)	LA	F1	1
9b)	LA	F2	3
21	MC	F3	1
46a)	LA	F3	2
46b)	LA	F3	1
47	LA	F2, F3	2
Unit G: Probability			
6a)	LA	G2	2
6b)	LA	G3	1
6c)	LA	G4	2
26	SA	G2	1
30	SA	G1	1
35	SA	G2	1
Unit H: Geometric Sequences			
7a)	LA	H1	2
7b)	LA	H2	1
23	MC	H3	1
50	LA	H2	3

*MC = Multiple Choice, LA = Long Answer, SA = Short Answer