

---

---

# GENERAL COMMENTS

---

---

## Grade 12 Pre-Calculus Mathematics Standards Test (June 2011)

### Student Performance—Observations

The following observations are based on local marking results and on comments made by markers during the sample marking session. These comments refer to common errors made by students at the provincial level and are not specific to school jurisdictions.

Information regarding how to interpret the provincial test and assessment results is provided in the *Interpreting and Using Results from Provincial Tests and Assessments* document available at [www.edu.gov.mb.ca/k12/assess/support/results/index.html](http://www.edu.gov.mb.ca/k12/assess/support/results/index.html).

Changes in performance over time are indicative of various factors: classroom-based, school-based, and home-based contexts, changes to demographics, and student choice of mathematics course. In addition, Grade 12 provincial tests may vary slightly in overall difficulty, though every effort is made to minimize variation throughout the test development and pilot testing processes.

When considering performance relative to specific areas of course content, the level of difficulty of the content and its representation on the provincial test vary over time according to the type of test questions and learning outcomes addressed. Information regarding learning outcomes is provided in the document *Senior 4 Mathematics: Manitoba Curriculum Framework of Outcomes and Senior 4 Standards* (2001).

### Total Test

The following chart indicates the provincial test means for the past six test administrations.

January 2009	June 2009	January 2010	June 2010	January 2011	June 2011
66.2%	66.6%	62.7%	68.5%	63.4%	<b>68.1%</b>

### Unit A: Circular Functions (provincial mean: 62.3%)

#### Conceptual Knowledge

Many students had difficulty solving a trigonometric equation with a double-angle identity. They knew it was possible to treat the double angle as an identity, but did not know how to use this information to find the solution. When solving trigonometric equations, some students had problems using the reference angle correctly.

When graphing the function  $y = \tan x$ , students commonly sketched the general shape of  $y = \cot x$ . They also incorrectly placed the asymptotes.

When asked to find the arc length, many students did not convert the angle from degrees to radians. When given the coordinates of a point  $P(\theta)$  on the unit circle, students did not know how to find the coordinates of  $P(\theta - \pi)$ . Students did not know how to locate the quadrant of an angle (in  $\pi$  radians) when given the numeric value for  $\theta$ . Many had trouble locating the quadrant for an angle greater than  $2\pi$ .

Students were unclear as to where to substitute a given angle in a squared trigonometric equation. Some students did not know which trigonometric functions went with which reciprocal trigonometric functions.

### **Procedural Skill**

When solving a trigonometric equation with a double-angle identity, some students divided the original expression or equation by two, whereas some multiplied by two. After substituting a given angle into a squared trigonometric expression, some students forgot to square the exact value.

Many students had difficulty simplifying and reducing fractions. Some thought that the denominator had to be the larger number when reducing fractions.

### **Communication**

Expressing the general solution of a trigonometric equation was poorly done. Some students did not define the variable  $k$ . Although students often changed equations to expressions when solving equations, many of them only did this once per question.

When sketching trigonometric functions, some students sketched the asymptotes as solid lines while others indicated the horizontal scale in degrees.

## **Unit B: Transformations (provincial mean: 62.4%)**

### **Conceptual Knowledge**

Students exhibited poor mastery of the inverse and reciprocal functions. For reciprocal functions, students struggled with demonstrating the correct asymptotic behaviour. When graphing or modeling a sinusoidal function, students were more effective in working with the vertical components than with the horizontal components. Students had more difficulty than usual with modeling a sinusoidal equation; some struggled with the contextual nature of the question.

### **Procedural Skill**

Although some students understood the concepts, many still had difficulty completing the procedure correctly. This was especially evident in graphing and modeling sinusoidal functions. When asked to sketch a given sinusoidal equation, some students stated the correct amplitude, period, and horizontal and vertical shifts, but were unable to produce the correct graph. Students also made errors when trying to combine a vertical shift with a vertical stretch, resulting in an incorrect range. Some students correctly identified the period, but incorrectly used it.

When analyzing a given sinusoidal scenario, some students understood how to obtain the value of  $B$ , but misunderstood the value of the period. They knew that the variable  $C$  represented a horizontal shift, but chose a value of  $C$  for cosine more often than the required value for sine.

### **Communication**

Students lost part marks for the incorrect use of arrowheads and endpoints. In the transformation of the exponential function, students that graphed two points on the original graph were more effective in communicating their knowledge of the subsequent transformations.

## **Unit C: Trigonometric Identities (provincial mean: 62.2%)**

### **Conceptual Knowledge**

The identity was completed with more success than usual. However, the simplification of a given trigonometric expression presented more of a challenge for students, even though most knew which identities were required. Many students did not consider quadrants when determining exact values. Some students selected the incorrect reciprocal trigonometric function.

### **Procedural Skill**

Students were usually able to substitute the correct identities, but could not simplify the expressions. Students had major difficulties with adding and dividing rational expressions. This was a concern whether students were proving an identity, simplifying an expression, or finding an exact value.

### **Communication**

Many students lost part marks due to poor communication in all types of questions. Errors included changing an equation to an expression, missing  $\theta$ 's, and introducing a new variable without defining it.

## **Unit D: Exponents and Logarithms (provincial mean: 73.4%)**

### **Conceptual Knowledge**

Many students had difficulty with the logarithmic laws. The quotient rule was often improperly applied by simply subtracting the arguments instead of dividing them. When using the power rule, students often treated the exponent as an exponent of the logarithm instead of the argument. When solving a logarithmic equation, students knew to apply logs, but were confused about how to deal with the denominator.

Students did not know how to use a given value of a logarithmic expression to find another value. The asymptote of a logarithmic graph was often included in the domain. When sketching the exponential graph, students often did not know the general shape.

### **Procedural Skill**

When solving the logarithmic equation, some students cancelled the logs and then solved the equation. They also treated the logarithm as a variable rather than a function.

### **Communication**

When using the power rule, brackets were often missed when dropping the exponent. There were calculation errors in the final evaluation of the quotient of logarithms. When transforming the exponential graph, some students did not draw the asymptote even though it was no longer on the  $x$ -axis.

## **Unit E: Permutations, Combinations, and Binomial Theorem (provincial mean: 70.0%)**

### **Conceptual Knowledge**

When answering the binomial expansion question, some students had the correct combination but incorrect factors, and vice versa.

When finding the number of possible seating arrangements, some students forgot to apply the restriction that a certain person must sit in the middle. For the permutation question that involved two people not sitting together, many students eliminated these two students rather than counting them as one group. When selecting a committee of boys and girls, some students added the factors instead of multiplying them.

### **Procedural Skill**

When expanding a binomial, many students forgot to include the negative sign with the second factor. Some students did not use parentheses around the factors, which resulted in a binomial rather than a monomial.

When finding the number of arrangements for two people not sitting together, most students knew that a subtraction statement must be used, but they did not correctly set up this statement.

### **Communication**

Many students forgot to pay attention to the instruction “Briefly explain your calculations.” Some students left their answers as combinations, not values, while others multiplied factorials incorrectly (e.g.,  $6!2! = 12!$ ).

## **Unit F: Conic Sections (provincial mean: 74.6%)**

### **Conceptual Knowledge**

When asked to identify the type of conic section, many students manipulated the general form of the equation into the standard form, which was not necessary. Students confused the equation of a hyperbola with that of an ellipse when sketching the conic.

### **Procedural Skill**

When converting the general equation of a conic section into standard form, students knew to complete the square, but made errors when factoring out the coefficient of the “ $y^2$ ” term and moving the constant to the other side. Some students had difficulty balancing the equation. Most students switched the  $x$  and  $y$  values of the coordinates of the centre when sketching the hyperbola.

### **Communication**

Some errors of omission were made such as forgetting the squares or not equating the left side of the equation of the ellipse in standard form to 1. The asymptotes of the hyperbola were drawn as solid lines.

## **Unit G: Probability (provincial mean: 74.9%)**

### **Conceptual Knowledge**

In a particular probability question, the vast majority of students were not able to recognize that the given event was dependent. They were able to find one factor of each term correctly, but the second factor was incorrect since they treated it as an independent event. Some students multiplied instead of added when calculating a certain probability.

### **Procedural Skill**

Students had difficulty multiplying decimals and made arithmetic errors when multiplying and adding fractions.

### **Communication**

Students expressed the sample space using tree diagrams rather than stating the individual outcomes.

## **Unit H: Geometric Sequences (provincial mean: 71.8%)**

### **Conceptual Knowledge**

When calculating the value of  $r$  in a geometric sequence, many students used a compound interest formula. Some students treated the geometric sequence as an arithmetic sequence, even though the sequence was identified as geometric in the question. When asked to find the total amount earned for one year, some students found the twelfth term of the sequence rather than finding the sum of the first twelve terms.

### **Procedural Skill**

Some students, who made a new sequence by taking the given second term and using it as the first term, forgot to go back to the original sequence to find the correct first term. When finding the value of  $r$ , many students found the fourth root of  $r$  rather than the third root of  $r$ . Some students used a rounded value of  $r$  in order to find the first term.

## Communication

A notation error of writing a line under the entire equation when dividing (rather than each side individually) occurred when some students found the value of  $r$  given two non-consecutive terms.

## Summary of Test Results (province)

Unit A: Circular Functions	62.3%
Unit B: Transformations	62.4%
Unit C: Trigonometric Identities	62.2%
Unit D: Exponents and Logarithms	73.4%
Unit E: Permutations, Combinations, and Binomial Theorem	70.0%
Unit F: Conic Sections	74.6%
Unit G: Probability	74.9%
Unit H: Geometric Sequences	71.8%
Total Test	68.1%

## Marking Accuracy and Consistency

Information regarding how to interpret the marking accuracy and consistency reports is provided in the *Interpreting and Using Results from Provincial Tests and Assessments* document available at [www.edu.gov.mb.ca/k12/assess/support/results/index.html](http://www.edu.gov.mb.ca/k12/assess/support/results/index.html).

These reports include a chart comparing the local marking results to the results from the central re-marking of sample test booklets. Overall, the accuracy of local versus central marking for the Pre-Calculus Mathematics Standards Test was very consistent. To highlight this consistency, 65.1% of the booklets sampled by the Department received a central mark within  $\pm 2\%$  of the local mark and 97.6% of the sampled booklets were within  $\pm 6\%$ . Scores awarded at the local level were, on average, 0.59% higher than the scores given at the Department.

## Survey Results

Teachers who supervised the Grade 12 Pre-Calculus Mathematics Standards Test in June 2011 were invited to provide comments regarding the test and its administration. A total of 108 teachers responded to the survey. A summary of their comments is provided below.

- 93% of the teachers indicated that all of the topics in the test were taught by the time the test was written. Additional time (in excess of the regular class time) was spent by 46% of the teachers in order to prepare students for the test.
- 97% of the teachers indicated that the test content was consistent with the standards and learning outcomes as outlined in the curriculum document. Approximately 96% of teachers indicated that the reading level of the test was appropriate and that the test questions were clear and fair to students.
- 85% and 95% of the teachers, respectively, indicated that students were able to complete *Booklet 1* and *Booklet 2* in the allotted time.
- 68% and 78% of the teachers, respectively, indicated that their students used graphing and scientific calculators during Part 1 of the test.

