GENERAL COMMENTS

Grade 12 Pre-Calculus Mathematics Achievement Test (June 2016)

Student Performance—Observations

The following observations are based on local marking results and on comments made by markers during the sample marking session. These comments refer to common errors made by students at the provincial level and are not specific to school jurisdictions.

Information regarding how to interpret the provincial test and assessment results is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at www.edu.gov.mb.ca/k12/assess/support/results/index.html.

Various factors impact changes in performance over time: classroom-based, school-based, and home-based contexts, changes to demographics, and student choice of mathematics course. In addition, Grade 12 provincial tests may vary slightly in overall difficulty although every effort is made to minimize variation throughout the test development and pilot testing processes.

When considering performance relative to specific areas of course content, the level of difficulty of the content and its representation on the provincial test vary over time according to the type of test questions and learning outcomes addressed. Information regarding learning outcomes is provided in the document *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes* (2014).

Summary of Test Results (Province)

June 2016	January 2016	June 2015	January 2015	June 2014
66.6%	66.0%	67.0%	69.5%	64.5%

Unit A: Transformations of Functions (provincial mean: 65.2%)

Conceptual Knowledge

Students sometimes did the inverse or the reflection over the incorrect axis when reflecting a coordinate point. Many students had difficulty in sketching the absolute value transformation of a given function. Some experienced difficulty with restricting the domain when combining functions or adding both x and y values of the original functions, instead of only the y values. Students were generally able to determine equations when required to multiply functions; however, they had difficulty determining the equation of a composition of functions. Most students struggled with graphing a reciprocal graph when the parent graph was given. Many confused reciprocal and inverse, or drew points of discontinuity in the place of a vertical asymptote. Students had difficulty when required to describe a characteristic of an inverse graph. Many answers lacked explanation or confused inverse with reflections.

Procedural Skill

Students generally did well in this area. Many students did not use all the key points in the original functions to sketch a graph for the combined function. Arithmetic errors were common, especially when expanding binomials.

Communication

When writing a new coordinate on a graph after transformations, some students did not write their answers as a coordinate. When students were asked to sketch a graph after multiple transformations, the general shape of the graph was not done well. When determining equations involving combining and composition of functions, many students made notation errors when identifying their solutions. When graphing, students sometimes forgot needed arrowheads or added arrowheads when not required.

Unit B: Trigonometric Functions (provincial mean: 69.6%)

Conceptual Knowledge

Most students were able to calculate arc length correctly but some forgot to convert the central angle into radians when it was given in degrees. When finding a coterminal angle, some students thought they needed to convert the angle to degrees, which led to simply restating the same answer as was given in the question. A few students also made arithmetic errors that led to answers that were not coterminal. When graphing a trigonometric function from a given equation, most students were able to apply a vertical translation. However, many had difficulty finding the period as well as applying a horizontal translation. Some students forgot to apply reflection to the function when sketching, and instead added a vertical stretch that was not part of the original equation. Most students were able to use Pythagorean Theorem to find a missing variable, but some did not know what to do with their answer to determine the exact value. Many students stated an answer for sine that was less than -1 or greater than 1, not recognizing that their answers were non-permissible. Some students mistakenly thought the value given was on the unit circle, and tried to use exact values from the unit circle to solve the problem. Most students were able to find the amplitude and vertical translation when determining the equation of a sinusoidal function, but some struggled to determine the period of the function and did not know where to insert the period into the equation formula.

Procedural Skill

When solving for arc length, some students used the diameter instead of the radius in the formula. Some students included the general solution as part of their answers when stating one coterminal angle. When graphing trigonometric functions, many students plotted points which led to inaccurate shapes. Some students used the wrong trigonometric ratio which led to incorrect substitution into the Pythagorean Theorem, and made arithmetic errors when solving for r.

Communication

When solving for arc length, many students missed units of measure, or used incorrect units, and made rounding errors in their final answers. Some students drew diagrams when asked to find a coterminal angle, and did not state a final answer. When graphing, some students forgot to label the *x*-axis and/or *y*-axis. Many students did not consider the domain of the graph and sketched less or more than was required. When using Pythagorean Theorem, some students made bracket errors, forgetting to include brackets around negative values when substituting into the formula. When stating the range of a trigonometric function, many students gave the correct answer, but stated the values in the wrong order when using interval notation. Some students used inequality symbols incorrectly when answering in set notation.

Unit C: Binomial Theorem (provincial mean: 63.1%)

Conceptual Knowledge

When justifying questions involving binomial theorem expansion, many students had difficulty understanding how to determine a pattern using both terms of the binomial. When attempting a question involving factorials with restrictions, students had difficulty with the concept of grouping and subtracting arrangements. Many students incorrectly used the fundamental counting principle and/or cases. The wrong number of groups was a common error. Most students were able to correctly substitute into the combination formula when solving an equation, but struggled with expanding factorials. Some chose to guess and check but failed to determine the second value. Some students did not reject positive values greater than *n* that were the result of algebraic errors.

Procedural Skill

When solving questions related to binomial theorem expansion, some students did not provide a proper justification. Some students made algebraic errors or made mistakes with the exponent laws when trying to simplify their answers. Some students did not realize that they had to fully simplify the answer after substituting into the formula. When expanding factorials, even though students were able to correctly substitute into the formula, many struggled with the simplification process and made many arithmetic errors in the process. When asked to explain how to solve a problem involving the fundamental counting principle, students solved the question, rather than using words to make the explanation.

Communication

When explaining their answers, some responses lacked clarity. When expanding factorials, some students made notational errors such as forgetting to include the factorial sign and/or misplacing it inside a bracket. Many students changed an equation to an expression when solving a problem involving the combination formula.

Unit D: Polynomial Functions (provincial mean: 75.3%)

Conceptual Knowledge

Most students were able to correctly use the process of synthetic division, but some forgot to use a placeholder when there was a missing term. When solving word problems, most students were

able to solve for the factors by using synthetic division, but some went further and solved for the x -intercepts. When using the remainder theorem, some students used zero instead of the value of the remainder. When graphing a polynomial function, some students did not know how to correctly graph a multiplicity of 2.

Procedural Skill

Even though most students were able to use the remainder theorem and synthetic division correctly, some had difficulty with the procedures and made sign errors when solving for the coefficient k. When graphing a polynomial function, some students had trouble graphing the x-intercepts correctly (correct value but opposite sign) while other students had the correct intercepts but had trouble connecting them with a proper curved shape.

Communication

When graphing polynomial functions, sometimes scales were not indicated and arrowheads were missing. When working with equations, some students changed the equation into an expression while working through synthetic division.

Unit E: Trigonometric Equations and Identities (provincial mean: 64.3%)

Conceptual Knowledge

When solving trigonometric equations, students were able to correctly substitute identities. Some students struggled to determine which answers were part of the solution, and gave answers outside the required domain, or reported angles in all four quadrants or the wrong quadrants.

Procedural Skill

Students struggled to demonstrate correct algebraic procedures when solving a proof question. They had difficulty with the strategy of using common denominators. When solving a trigonometric equation, students omitted the second branch as they forgot the plus/minus sign when taking the square root. Students struggled with factoring trigonometric equations. They had difficulty solving for a trigonometric value such as $\sin \theta$.

Communication

When solving proof questions, students missed variables after sine or cosine. Some students made transcription errors when copying identities from their formula sheet or in substituting values into formulas. Some students equated both sides of the proof by adding equal signs throughout. When solving a trigonometric equation, students changed the equation to an expression by omitting the equal sign. Some students made arithmetic errors when solving trigonometric equations. Students struggled with simplifying fractions and radicals. When asked to explain an error made in solving a trigonometric equation, many responses lacked clarity.

Unit F: Exponents and Logarithms (provincial mean: 69.3%)

Conceptual Knowledge

When asked to use the laws of logarithms, students struggled with the product law. They also had difficulty handling a binomial argument. When logs were not necessary with a common base, many students still used logs even though they did not have access to the calculator. When asked to find the number of necessary periodic payments for a loan, many students did not substitute correctly into the given equation, or did not use logs to solve the question. Some used logs, but were unable to solve correctly due to the negatives. When graphing the exponential graph, many students did not attempt the vertical translation of the horizontal asymptote, and some confused the exponential graph with a logarithmic graph.

Procedural Skill

Students incorrectly moved terms from one side of the equation to the other. There were also numerous bracket errors which threw off the necessary distributive law application. Students also omitted brackets on the binomial argument. When solving the log problem, some students forgot exponential form while others tried to break up a log with a binomial argument. When graphing, some students did not graph to the left of the *y*-axis.

Communication

When graphing, students forgot the horizontal asymptote and the scale. When solving for the number of monthly payments, students did not round their answers correctly. For complex problems, students forgot to carry down both sides of the equation, changing an equation into an expression, and vice versa. Some did not reject extraneous values or made transcription or notation errors. Bracket errors were also very common.

Unit G: Radicals and Rationals (provincial mean: 70.1%)

Conceptual Knowledge

When asked to sketch a rational function containing a point of discontinuity, students often mistakenly placed an asymptote in its position. When asked to compare equations, one with an asymptote and one with a point of discontinuity, many students did not distinguish the major differences between these two kinds of graphs. When asked to sketch the radical graph of a given function, students generally were able to sketch the graph and its transformations. However, when asked to sketch a rational function from an existing graph, many students were not able to restrict the domain or to properly sketch a resulting rational function.

Procedural Skill

Common graphing errors included not identifying correct points on the graph, and not including one point in each section of the rational or radical graphs. When sketching a radical function, students did not clearly draw the graph above the original function when $0<y<1$.

Communication

Students often did not draw the horizontal asymptote at y=0 when sketching a rational function. Generally, rational graphs were not drawn very well. Errors included crossing asymptote, incorrect shape, and not indicating at least one correct point on each branch. Students had difficulty describing how to properly sketch a radical graph with invariant points and the behaviour between and above these points.

Communication Errors

Errors that are not related to the concepts or procedures are called "Communication Errors" and these were tracked on the *Answer/Scoring Sheet* in a separate section. There was a maximum ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type did not further affect a student's mark).

The following table indicates the percentage of students who had at least one error for each type.

E1 final answer	§ answer given as a complex fraction § final answer not stated	10.6%
E2 equation/expression	§ changing an equation to an expression § equating the two sides when proving an identity	28.1%
E3 variables	§ variable omitted in an equation or identity § variables introduced without being defined	9.0%
E4 brackets	§ "$\sin x^2$" written instead of "$\sin^2 x$" § missing brackets but still implied	16.5%
E5 units	§ missing units of measure § incorrect units of measure § answer stated in degrees instead of radians or vice versa	11.7%
E6 rounding	§ rounding error § rounding too early	27.3%
E7 notation/transcription	§ notation error § transcription error	36.1%
E8 domain/range	§ answer given outside the domain § bracket error made when stating domain or range § domain or range written in incorrect order	25.5%
E9 graphing	§ incorrect or missing endpoints or arrowheads § scale values on axes not indicated § coordinate points labelled incorrectly	24.0%
E10 asymptotes	§ asymptotes drawn as solid lines § asymptotes missing but still implied § graph crosses or curls away from asymptotes	14.4%

Marking Accuracy and Consistency

Information regarding how to interpret the marking accuracy and consistency reports is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at www.edu.gov.mb.ca/k12/assess/support/results/index.html.

These reports include a chart comparing the local marking results to the results from the departmental re-marking of sample test booklets. Provincially, 34.6% of the test booklets sampled resulted in a higher score locally than those given at the department; in 13.8% of the cases, local marking resulted in a lower score. Overall, the accuracy of local versus central marking for the test was consistent. To highlight this consistency, 51.6% of the booklets sampled and marked by the department received a central mark within $\pm 2\%$ of the local mark and 91.9% of the sampled booklets were within $\pm 6\%$. Scores awarded at the local level were, on average, 1.2% higher than the scores given at the department.

Survey Results

Teachers who supervised the Grade 12 Pre-Calculus Mathematics Achievement Test in June 2016 were invited to provide comments regarding the test and its administration. A total of 115 teachers responded to the survey. A summary of their comments is provided below.

After adjusting for non-responses:

- 94.7% of the teachers indicated that all of the topics in the test were taught by the time the test was written.
- 98.2% of the teachers indicated that the test content was consistent with the learning outcomes as outlined in the curriculum document. 97.4% of teachers indicated that the reading level of the test was appropriate and 99.1% of them thought the test questions were clear.
- 98.3% and 85.2% of the teachers, respectively, indicated that students were able to complete the questions requiring a calculator and the entire test in the allotted time.
- 99.1% of the teachers indicated that their students used a formula sheet throughout the semester and 98.3% of teachers indicated that their students used the formula sheet during the test.
- 57.4% of the teachers indicated that graphing calculators were incorporated during the instruction of the course and 91.3% of teachers indicated that the use of a scientific calculator was sufficient for the test.