GENERAL COMMENTS

Grade 12 Pre-Calculus Mathematics Achievement Test (June 2015)

Student Performance—Observations

The following observations are based on local marking results and on comments made by markers during the sample marking session. These comments refer to common errors made by students at the provincial level and are not specific to school jurisdictions.

Information regarding how to interpret the provincial test and assessment results is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at <www.edu.gov.mb.ca/k12/assess/support/results/index.html>.

Various factors impact changes in performance over time: classroom-based, school-based, and home-based contexts, changes to demographics, and student choice of mathematics course. In addition, Grade 12 provincial tests may vary slightly in overall difficulty although every effort is made to minimize variation throughout the test development and pilot testing processes.

When considering performance relative to specific areas of course content, the level of difficulty of the content and its representation on the provincial test vary over time according to the type of test questions and learning outcomes addressed. Information regarding learning outcomes is provided in the document *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes* (2014).

Summary of Test Results (Province)

June 2015	January 2015	June 2014	January 2014
67.0%	69.5%	64.5%	59.3%

Unit A: Transformations of Functions (provincial mean: 72.3%)

Conceptual Knowledge

Most students were able to correctly apply transformations to a graph. Those who committed errors tended to confuse horizontal reflections with vertical reflections or inverse functions with reciprocal functions. Also, some students did not apply absolute values at all and struggled when trying to graph horizontal compressions. Many students could find the equation of composite functions. Students struggled with restricting the domain of a function to make its inverse a function when given the equation; most did not know what to do or gave random answers that did not pertain to the function.

Procedural Skill

When applying transformations to a graph, some students made an arithmetic error which resulted in one of the points being incorrect, thereby affecting the overall shape of the graph. Most students knew the proper transformations but some were unable to apply the transformations in the correct order.

Communication

Some students were not able to explain transformations in words. Instead, they graphed their solution. Many students had difficulty using proper notation and brackets when creating an equation from words.

Unit B: Trigonometric functions (provincial mean: 69.8%)

Conceptual Knowledge

Many students were able to correctly use the $s = \theta r$ formula to find the value of the arc length. Students struggled when sketching a trigonometric equation in a word problem to determine the correct starting point of the graph. As well, many students were unable to identify the correct period using the information provided in the problem. Some students had difficulty determining the height of their trigonometric graph given the time at which this height occurred. Students who found the height using the graph tended to be more successful. Some students were able to correctly substitute the time into the trigonometric equation but struggled to solve for the height, either leaving the question unfinished or having arithmetic errors in their work. Some students were able to find the values for sine and cosine but did not consider the quadrants in which each angle terminated, therefore ending up with an incorrect sign. Many students struggled with the concept of non-permissible values of trigonometric expressions. A common error was finding a non-permissible value for an expression $(\cos\theta)$ that does not have a reciprocal identity and therefore does not have a non-permissible value. Some students were unable to recognize when to use the Pythagorean identity to find the value of r (radius) when solving for an exact value.

Procedural Skill

Some students did not convert degrees into radians and continued to use degrees when solving for the arc length. Some students confused sine and cosine with the opposite reciprocal identities when working with non-permissible values of trigonometric equations.

Communication

Students demonstrated unit errors when finding the arc length, specifically not knowing that the arc length should also be measured in metres such as the radius. Some students wrote an answer with a different unit of measure such as radians or degrees and some students left out the units all together. Some students did not show their final answer when sketching an angle in standard position as they did not draw the directional arrow to the terminal arm to show whether the angle was positive or negative in direction. Many students were able to correctly graph trigonometric equations but were not communicating their graph with enough details. Many students failed to label the x-axis or y-axis on their trigonometric graphs where axes were not provided for them. The majority of students were able to find the coterminal angles of a radian in a given interval. Some students gave additional angles of measure, most notably when stating a second positive coterminal angle, which was greater than the domain given. Some students gave the general solution when solving for non-permissible values of a trigonometric expression instead of providing solutions in the given domain. Some students demonstrated notation errors when substituting negative values into the Pythagorean identity and included the squared symbol inside of the brackets instead of outside the brackets.

Unit C: Binomial Theorem (provincial mean: 67.2%)

Conceptual Knowledge

When solving questions related to binomial theorem expansion, most students were able to substitute correctly into the given formula, but some students were unable to identify the correct term for which they were to solve.

When attempting a combinations question, many students had difficulty with the concept of different cases. Students struggled to deal with the restrictions, not being able to come up with the proper cases, or tried to use permutations.

Most students were able to correctly substitute into the permutation formula but struggled with expanding factorials. Some chose to guess and check and failed to find the second value. Students were able to correctly explain why a combination formula was undefined using various methods.

Procedural Skill

When solving a combination question with restrictions, most students knew that they had to add or subtract cases. When solving a permutations equation, students failed to solve for both values of n, and as a result could not show that one of the values would have been rejected. When solving questions related to binomial theorem expansion, some students forgot to include negative signs for terms from the question into the formula. Some students made algebraic errors or made mistakes with the exponent laws when trying to simplify their answers. Some students did not realize that they had to fully simplify the answer after substituting into the formula. When expanding factorials, even though students were able to correctly substitute into the formula, many struggled with the simplification process and made many arithmetic errors in the process.

Communication

Students struggled with labelling the cases when solving a combinations question. Even though students were able to set up their work correctly, some did not state their final answer at the end with simple permutation and fundamental counting principle questions. When explaining their answers, some students' responses lacked clarity. When expanding factorials, some students had notational errors such as forgetting to include the factorial sign, or misplacing it inside a bracket when expanding factorials.

Unit D: Polynomial Functions (provincial mean: 74.4%)

Conceptual Knowledge

Many students knew that an exponent on a factor affected the multiplicity, but struggled to explain thoroughly its effect at a specific point along the graph. A few students confused the effect of the exponent with the leading coefficient. Most students knew how to use synthetic division to find the factors of an expanded polynomial. They also knew how to find the factors given a graph of a polynomial.

Procedural skill

This unit was done quite well by most students. Common errors made in synthetic division were using the wrong signed value as the divisor and arithmetic errors. With graphing, students struggled to find the a value and multiplicity when given a graph. When given the equation, they could not graph an appropriate y-intercept. Students did well in finding the factors from an expanded polynomial and graphing the polynomial once they had the factors. They also could derive the factors from a graph very well.

Communication

When graphing polynomials, students had difficulty restricting the graph to the given domain. They also did not label the axes. When asked to explain concepts relating to polynomials, students often were too general in their explanation and only gave part of the solution. For example, they discussed odd, but not even degreed functions. The other communication error that was very common was the use of a different variable without defining it.

Unit E: Trigonometric Equations and Identities (provincial mean: 68.7%)

Conceptual Knowledge

Many students factored a trigonometric equation and solved for values of $\tan\theta$ but did not know where to go from there as they rejected values of $\tan\theta$ that were greater than 1. Some students did not know how to use the double angle identity to solve a trigonometric equation, and often only identified the reference angle and did not solve for the second angle. Students knew unit circle values of trigonometric functions well but many only found one correct value when there were two possible values. When determining non-permissible values, students often set everything equal to zero rather than just the denominator. Many students determined a correct sum of a combination of angles but used an incorrect combination and/or did not use the sum identity. Some students used the incorrect reciprocal function when proving an identity. Other students simply flipped the right hand side and called it “flipping” or “inverse”.

Procedural Skill

When solving a trigonometric equation, some students did not include the general solution or stated it incorrectly. Factoring was difficult for some students and arithmetic errors were common. While students were able to use the correct substitution of appropriate identities to prove an identity, they were not able to complete the proof using the necessary algebraic strategies (e.g., determining a common denominator and simplifying complex fractions). Many just cancelled common factors. Most students knew unit circle values of trigonometric functions well but did not know how to use these fractional values within a trigonometric question. There were various arithmetic errors made when working with these fractional values. Some students used incorrect values, or an incorrect quadrant, or used both an incorrect value and an incorrect quadrant.

Communication

The most common error was omitting a variable (i.e., writing sine, cosine, or tangent without a variable following it). Some students kept sin and cos in the sum identity with the exact values but then continued correctly to determine a correct value. Some students also failed to consider the domain, expressing answers in degrees rather than radians, and vice versa. A number of students changed equations to expressions while solving a trigonometric equation.

Unit F: Exponents and Logarithms (provincial mean: 59.2%)

Conceptual Knowledge

When asked to use laws of logarithms, students generally did well with the quotient rule but had more difficulty using the power law. Some students were able to solve a logarithmic equation but did not reject the extraneous root. Students had difficulty sketching the graph of a logarithmic function. Most students did not apply transformations to a base graph which resulted in an omission of asymptotes and graphs that only included quadrant I. Some students were able to solve an application of a logarithmic function question but didn’t know how to substitute the values into the formula correctly.

Procedural Skill

When solving exponential equations, students knew they had to apply logarithms but had trouble applying all rules correctly. They also used incorrect order of operations. When changing into an exponential form, some students did so incorrectly. Some students were unable to correctly substitute values into the formula. Most students set up the ratio with the greater number over the smaller number, even if the values were not correct. Students incorrectly equated the arguments of a logarithmic equation before using laws of logarithms to simplify to single logarithms. After solving a logarithmic equation, some students did not reject the extraneous root.

Communication

Communication errors were minimal in the questions from this unit. Students made some notation errors while solving a logarithmic equation and some students changed an equation to an expression. Most students understood the concept of rejecting an extraneous root but they lacked the correct terminology to explain the argument. When graphing a logarithmic function, some students correctly drew the graph with correct asymptotic behaviour but failed to include the vertical asymptote. A few students did not state the ratio as a whole number even though this was stated in the question.

Unit G: Radicals and Rationals (provincial mean: 71.9%)

Conceptual Knowledge

Some students found it difficult to distinguish between an algebraic solution and a graphical solution. Students had some difficulty in distinguishing between a y-intercept and an x-intercept. Identifying the correct function to graph was sometimes difficult, namely distinguishing between radical, rational, logarithmic, and exponential graphs. Some students had difficulties in determining whether a non-permissible value produced a point of discontinuity (hole) or a vertical asymptote. They also sometimes thought that there was a point of discontinuity (hole) and a horizontal asymptote at the same point. Other occasional difficulties occurred when graphing—students' graphs incorrectly crossed the asymptote and the graphs of rational functions were only drawn between the vertical asymptotes.

Procedural Skill

When providing an equation of a radical function, students attempted to put all transformations under the radical sign. Arithmetic errors were common when solving for radical equations, specifically when squaring the radical. When finding points of discontinuity (holes), arithmetic errors produced incorrect y values. Students often forgot to show their horizontal asymptotes when graphing rationals, especially when the asymptotes were on the x-axis. Other rational graphing errors included not identifying correct points on the graph, and not including one point in each section of the rational graphs.

Communication

Terminology was used incorrectly when referring to "roots", by which the students meant "x-intercepts". When a specific equation was required, students often would express the transformations in the more general function notation. Students often had difficulties applying brackets properly, especially when dealing with horizontal compressions and horizontal translations. In rational graphs, students failed to label their scales and their graphs curled away from the asymptotes. When explaining rational graphs, students' responses lacked clarity, especially surrounding asymptotic behaviours. They did not identify that graphs approached negative infinity as well as positive infinity.

Communication Errors

Errors that are not related to the concepts within a question are called "Communication Errors" and these were indicated on the *Answer/Scoring Sheet* in a separate section. There was a maximum ½ mark deduction for each type of communication error committed, regardless of the number of errors committed for a certain type (i.e., committing a second error for any type did not further affect a student's mark).

The following table indicates the percentage of students who had at least one error for each type.

E1	▪ answer given as a complex fraction ▪ final answer not stated	25.1%
E2	▪ changing an equation to an expression ▪ equating the two sides when proving an identity	28.5%
E3	▪ variable omitted in an equation or identity ▪ variables introduced without being defined	21.1%
E4	▪ "$\sin x^2$" written instead of "$\sin^2 x$" ▪ missing brackets but still implied	15.4%
E5	▪ missing units of measure ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa	30.5%
E6	▪ rounding error ▪ rounding too early	30.1%
E7	▪ notation error ▪ transcription error	38.3%
E8	▪ answer included outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order	36.8%
E9	▪ incorrect or missing endpoints or arrowheads ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly	24.0%
E10	▪ asymptotes drawn as solid lines ▪ asymptotes missing but still implied ▪ graph crosses or curls away from asymptotes	8.5%

Marking Accuracy and Consistency

Information regarding how to interpret the marking accuracy and consistency reports is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at <www.edu.gov.mb.ca/k12/assess/support/results/index.html>.

These reports include a chart comparing the local marking results to the results from the departmental re-marking of sample test booklets. Provincially, 41.3% of the test booklets sampled resulted in a higher score locally than those given at the department; in 9.9% of the cases, local marking resulted in a lower score. Overall, the accuracy of local versus central marking for the test was consistent. To highlight this consistency, 48.9% of the booklets sampled and marked by the department received a central mark within $\pm 2\%$ of the local mark and 94.2% of the sampled booklets were within $\pm 6\%$. Scores awarded at the local level were, on average, 1.5% higher than the scores given at the department.

Survey Results

Teachers who supervised the Grade 12 Pre-Calculus Mathematics Achievement Test in June 2015 were invited to provide comments regarding the test and its administration. A total of 102 teachers responded to the survey. A summary of their comments is provided below.

After adjusting for non-responses:

- 92.0% of the teachers indicated that all of the topics in the test were taught by the time the test was written.
- 97.0% of the teachers indicated that the test content was consistent with the learning outcomes as outlined in the curriculum document. 98.0% of teachers indicated that the reading level of the test was appropriate and 97.0% of them thought the test questions were clear.
- 92.8% and 87.8% of the teachers, respectively, indicated that students were able to complete the questions requiring a calculator and the entire test in the allotted time.
- 96.1% of the teachers indicated that their students used a formula sheet throughout the semester and 99.0% of teachers indicated that their students used the formula sheet during the test.
- 56.9% of the teachers indicated that graphing calculators were incorporated during the instruction of the course and 86.9% of teachers indicated that the use of a scientific calculator was sufficient for the test.