GENERAL COMMENTS

Grade 12 Pre-Calculus Mathematics Achievement Test (January 2016)

Student Performance—Observations

The following observations are based on local marking results and on comments made by markers during the sample marking session. These comments refer to common errors made by students at the provincial level and are not specific to school jurisdictions.

Information regarding how to interpret the provincial test and assessment results is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at www.edu.gov.mb.ca/k12/assess/support/results/index.html.

Various factors impact changes in performance over time: classroom-based, school-based, and home-based contexts, changes to demographics, and student choice of mathematics course. In addition, Grade 12 provincial tests may vary slightly in overall difficulty although every effort is made to minimize variation throughout the test development and pilot testing processes.

When considering performance relative to specific areas of course content, the level of difficulty of the content and its representation on the provincial test vary over time according to the type of test questions and learning outcomes addressed. Information regarding learning outcomes is provided in the document *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes* (2014).

Summary of Test Results (Province)

January 2016	June 2015	January 2015	June 2014	January 2014
66.0%	67.0%	69.5%	64.5%	59.3%

Unit A: Transformations of Functions (provincial mean: 69.2%)

Conceptual Knowledge

Students generally did well with transformation problems. However, students thought they needed to solve an equation when a description of the domain was required. Some students were not able to distinguish between multiplication of functions and composition of functions. Many students confused inverse functions with reciprocal functions, and absolute value functions with a reflection. In some cases, students had difficulties distinguishing between reflections over the y-axis with reflections over the x-axis. When using mapping to transform a specific point, some students performed the transformation on the wrong coordinate. Students had difficulties recognizing that a linear function multiplied by another linear function results in a quadratic. Students struggled with order of operations in absolute value functions.

Procedural Skill

Students found algebraic manipulation difficult, especially when variables appeared in the denominator. Arithmetic errors were commonly made. Horizontal stretches and compressions were often confused for one another. When graphing from a table of values, students found it difficult to recognize the function the table was describing, which showed their lack of graphing sense.

Communication

When describing functions, many students made notation errors, eliminating *f* or using an incorrect inverse. Students did not use adequately precise language to describe reflections. Some students gave specific functions when a general description of the transformations was asked for. When graphing, students often forgot needed arrowheads.

Unit B: Trigonometric Functions (provincial mean: 65.5%)

Conceptual Knowledge

Most students were able to calculate arc length correctly but some forgot to convert the central angle into radians when it was given in degrees. When solving for exact values, most students knew the correct values but did not consider the quadrants in which each angle terminated, resulting in a sign error in their final answer. Many students had trouble determining a trigonometric function from a word problem. Most students were able to determine amplitude and period but had difficulty determining the vertical and horizontal shifts. Most students were able to correctly graph a trigonometric function from a given equation, but some struggled with the period.

Procedural Skill

When solving for arc length, some students used the diameter instead of the radius in the formula. Some students substituted incorrectly into the Pythagorean Theorem when solving for a variable other than *r*. When graphing trigonometric functions, some students struggled with applying a vertical shift to the graph.

Communication

When solving for arc length, many students missed units of measure or used incorrect units in their final answer. Many students also rounded off too early, leading to a rounding error in their final answer. When using trigonometric functions, some students made notation errors, forgetting to include the variable θ. When graphing trigonometric functions, some students used solid guidelines for the amplitude and did not remove them as part of their graph, and some missed a scale on one or both axes. Students were able to use reciprocal functions but had trouble explaining in words why some had no solutions.

Unit C: Binomial Theorem (provincial mean: 68.3%)

Conceptual Knowledge

Many students mistakenly used permutations instead of combinations when choosing members of committees with combinations. More students struggled when one specific person had to be part of the committee, compared to when members of each gender were being chosen. Some students incorrectly added combinations instead of multiplying to determine the number of committees for each case. When solving permutations with two options, many students were able to correctly find the total number of options. Some students incorrectly solved for $n!$ instead of $n(n-1)$. Others failed to multiply the cases, and instead incorrectly added to find a solution. When solving questions related to binomial theorem expansion, most students were able to substitute correctly into the given formula, but some students were unable to identify the correct term for which they were to solve. Many students chose to guess and check to find the given term and lacked the appropriate strategies needed to solve the problem. Most students were able to correctly substitute into the permutation formula but struggled with expanding factorials. Some students made arithmetic errors that resulted in only one solution of *n* and simplified the question because quadratics were no longer required. Some students chose to guess and check and failed to find the second value of *n*.

Procedural Skill

When using algebra to find a term in a binomial expansion, some students failed to apply exponent laws correctly which led to incorrect terms being stated. Some students also made algebraic errors when trying to simplify their answers. They did not realize that they had to fully simplify the answer after substituting into the formula. When expanding factorials, students were able to correctly substitute into the formula, but many struggled with the simplification process and made many arithmetic errors. Some students did not simplify factorials. Instead, they used cubic functions and polynomial division to simplify their answer, which could still result in a correct answer but required considerably more work. This method also required students to reject two answers.

Communication

Even though students were able to set up their work correctly, some students did not state their final answer at the end with permutation or combination questions. When expanding factorials, some students made notation errors such as forgetting to include the factorial sign, or misplacing it inside a bracket when expanding factorials. Some students forgot to reject a value of n when solving factorial questions or failed to properly communicate their understanding of the rejection.

Unit D: Polynomial Functions (provincial mean: 73.2%)

Conceptual Knowledge

Students did quite well on this unit. Most students knew the relationship between the roots of a polynomial equation and the x-intercepts of a graph. Students were able to identify the missing coefficient in the synthetic division strategy as 0. Some recognized a missing coefficient but incorrectly identified it as 1. Students were able to demonstrate synthetic division successfully. When generating an equation from a graph, students were generally successful in identifying the factors and the multiplicity. Occasionally, they used wrong signs when writing the factors in the equation.

Procedural Skill

Most students were able to solve for the factors when using synthetic division, but some students went further and solved for the x-intercepts. When given a graph, students had difficulty solving for the leading coefficient in the equation.

Communication

Some students were unclear that the roots referred to x-intercepts only. Some students were not clear as to which term needed to be replaced with a zero. Many students thought they also needed to explain the multiplicity of each root. Some students struggled with terminology stating x-intersects instead of x-intercepts. When working with equations some students changed an expression to an equation while working through synthetic division.

Unit E: Trigonometric Equations and Identities (provincial mean: 61.4%)

Conceptual Knowledge

When given a trigonometric equation that was already factored, many students solved for $\sin\theta$, but did not know how to solve for θ. Most of them rejected values that were outside the range of values $[-1, 1]$ for which $\sin\theta$ was defined. Students often failed to give the general solution when it was required. When asked to describe the solution of an equation based on a graph of the left and right sides of the equation, many students did not realize that the solution was the *x*-values of the points of intersection. Some students also described the transformation of the sine function in the equation rather than giving the solution to the equation. When asked to find the exact value of an angle not on the unit circle, students misinterpreted the question thinking that it was a conversion from degrees to radians. They had difficulty using the sum identity. The proof was generally well done.

Procedural Skill

Students made many arithmetic errors when solving using the quadratic formula and when adding fractions. They also had difficulty simplifying and manipulating radicals. The cancellation of portions of numerators with portions of denominators (rather than common factors) was a very common error. On proof questions, some students made algebraic errors and were not able to finish the proof. Many students were not able to distinguish between $\sin\theta$ and θ_r when solving for the final answers of trigonometric equations. Some also forgot to reject values of sine and cosine that do not exist. Some students did not know how to represent the general solution properly.

Communication

In solving trigonometric questions, a common communication error was the incorrect use or lack of use of variables. When solving proof questions, students missed variables after sine or cosine. Some students made transcription errors when copying identities from their formula sheet or in substituting values into formulas. Students also switched variables without defining them, and missed variables in their solution. In addition, some students did not do what was required by the question. For example, they solved the problem instead of using words to describe it for description questions.

Unit F: Exponents and Logarithms (provincial mean: 65.8%)

Conceptual Knowledge

When asked to use laws of logarithms, students generally did well with the power and quotient laws but had more difficulty using the product law. Some students did not understand that they needed to choose an estimated value of a logarithm and only gave a range of values or stated it as an inequality. Most students were able to state the equation of a horizontal asymptote but could not sketch the graph of a logarithmic function. Some students did not recognize the logarithmic function and sketched an incorrect shape, but were able to stretch the graph correctly.

Procedural Skill

Some students did not know how to change into the exponential form and they incorrectly equated the arguments of a logarithmic equation before using laws of logarithms to simplify to a single logarithm. Other students incorrectly crossed out logarithms when equating the arguments. Some students knew the exponential form was needed, but were unable to carry out the process correctly. Others had all the correct work shown but were unable to use their calculator correctly to evaluate a quotient of logarithms. Many students attempted to sketch an exponential function and sketch its inverse in order to sketch a logarithmic function but did so incorrectly. Some students started the graph at the origin rather than continuing the graph with asymptotic behaviour at the vertical asymptote.

Communication

Students made some notation errors when solving a logarithmic equation and some students changed an equation to an expression. When graphing a logarithmic function, some students correctly drew the graph with correct asymptotic behaviour but failed to include the vertical asymptote and/or identify their final answer. Students made many notation errors when stating the equation of a horizontal asymptote. Some students introduced variables when expressing a logarithm in exponential form. Many students also introduced a variable without defining it when estimating the value of a logarithm. Some students did not include the base when expanding a single logarithm using the laws of logarithms. Also, students made bracket errors when solving an exponential function using logarithms.

Unit G: Radicals and Rationals (provincial mean: 69.7%)

Conceptual Knowledge

When asked to create a rational function without any vertical asymptotes, students created a radical function instead. Students successfully identified a point of discontinuity of a rational function, but many only gave the x-coordinate correctly. Students knew the shape of a rational graph when asked to sketch a function. When asked to sketch the radical graph of a given function, students had difficulty restricting the domain properly and frequently mixed up vertical and horizontal stretches. Some did not know that the radical represented a fractional exponent.

Procedural Skill

When graphing a rational function, students had difficulty finding the horizontal asymptote. Other graphing errors included not identifying correct points on the graph, and not including one point in each section of the rational or radical graphs. Many students stated that one could not take the square root of zero when restricting the domain of a radical function.

Communication

When asked to create a rational function, students stated it as an expression. Unnecessary dotted lines were displayed when students sketched a radical function and some represented them as an asymptote. Bracket errors or inequality errors were made by some students when stating domain or range of a radical function. Students also made notation errors by missing a union symbol when stating a domain with multiple parts.

Communication Errors

Errors that are not related to the concepts or procedures are called "Communication Errors" and these were tracked on the *Answer/Scoring Sheet* in a separate section. There was a maximum ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type did not further affect a student's mark).

The following table indicates the percentage of students who had at least one error for each type.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated	18.5%
E2 equation/expression	▪ changing an equation to an expression ▪ equating the two sides when proving an identity	28.1%
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined	36.7%
E4 brackets	▪ "$\sin x^2$" written instead of "$\sin^2 x$" ▪ missing brackets but still implied	16.9%
E5 units	▪ missing units of measure ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa	17.2%
E6 rounding	▪ rounding error ▪ rounding too early	19.6%
E7 notation/transcription	▪ notation error ▪ transcription error	36.5%
E8 domain/range	▪ answer given outside the domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order	6.7%
E9 graphing	▪ incorrect or missing endpoints or arrowheads ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly	25.8%
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes missing but still implied ▪ graph crosses or curls away from asymptotes	9.4%

Marking Accuracy and Consistency

Information regarding how to interpret the marking accuracy and consistency reports is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at www.edu.gov.mb.ca/k12/assess/support/results/index.html.

These reports include a chart comparing the local marking results to the results from the departmental re-marking of sample test booklets. Provincially, 38.5% of the test booklets sampled resulted in a higher score locally than those given at the department; in 10.8% of the cases, local marking resulted in a lower score. Overall, the accuracy of local versus central marking for the test was consistent. To highlight this consistency, 50.8% of the booklets sampled and marked by the department received a central mark within $\pm 2\%$ of the local mark and 95.1% of the sampled booklets were within $\pm 6\%$. Scores awarded at the local level were, on average, 1.2% higher than the scores given at the department.

Survey Results

Teachers who supervised the Grade 12 Pre-Calculus Mathematics Achievement Test in January 2016 were invited to provide comments regarding the test and its administration. A total of 119 teachers responded to the survey. A summary of their comments is provided below.

After adjusting for non-responses:

- 98.2% of the teachers indicated that all of the topics in the test were taught by the time the test was written.
- 98.3% of the teachers indicated that the test content was consistent with the learning outcomes as outlined in the curriculum document. 99.2% of teachers indicated that the reading level of the test was appropriate and 96.7% of them thought the test questions were clear.
- 98.3% and 91.8% of the teachers, respectively, indicated that students were able to complete the questions requiring a calculator and the entire test in the allotted time.
- 98.3% of the teachers indicated that their students used a formula sheet throughout the semester and 99.1% of teachers indicated that their students used the formula sheet during the test.
- 48.3% of the teachers indicated that graphing calculators were incorporated during the instruction of the course and 91.4% of teachers indicated that the use of a scientific calculator was sufficient for the test.