
GENERAL COMMENTS

Grade 12 Pre-Calculus Mathematics Standards Test (January 2011)

Student Performance—Observations

The following observations are based on local marking results and on comments made by markers during the sample marking session. These comments refer to common errors made by students at the provincial level and are not specific to school jurisdictions.

Information regarding how to interpret the provincial test and assessment results is provided in the *Interpreting and Using Results from Provincial Tests and Assessments* document available at www.edu.gov.mb.ca/k12/assess/support/results/index.html.

Changes in performance over time are indicative of various factors: classroom-based, school-based, and home-based contexts, changes to demographics, and student choice of mathematics course. In addition, Grade 12 provincial tests may vary slightly in overall difficulty, though every effort is made to minimize variation throughout the test development and pilot testing processes.

When considering performance relative to specific areas of course content, the level of difficulty of the content and its representation on the provincial test vary over time according to the type of test questions and learning outcomes addressed. Information regarding learning outcomes is provided in the document *Senior 4 Mathematics: Manitoba Curriculum Framework of Outcomes and Senior 4 Standards* (2001).

Total Test

The following chart indicates the provincial test means for the past six test administrations.

June 2008	January 2009	June 2009	January 2010	June 2010	January 2011
65.1%	66.2%	66.6%	62.7%	68.5%	63.4%

Unit A: Circular Functions (provincial mean: 60.8%)

Conceptual Knowledge

When solving trigonometric equations, students found only one solution for the angle within the interval $[0, 2\pi]$ instead of two. Although students sometimes added the general solution when it was not required, others forgot to include it when it was required. When solving multiple-angle trigonometric equations, students did not know that they had to equate the multiple angle to their solutions. When solving a trigonometric equation involving the tangent function, some students thought that the range of the function was only from $[-1, 1]$ and therefore did not find all possible solutions. Students had difficulty with coterminal angles. When asked to find the range of an inverse trigonometric function, many students stated the domain instead.

Procedural Skill

Factoring trigonometric equations was often done incorrectly. Also, students had difficulty simplifying complex fractions. Some students were unaware that when dividing a number by zero, the solution is undefined. When solving multiple-angle trigonometric equations, students encountered algebraic difficulty with the coefficient.

Communication

When asked to solve trigonometric equations, students changed variables without defining them. Also, many used the letter “Q” to represent the quadrant in which the answer was found but also used this as their variable. When determining an undefined value, some students stated that the solution was “impossible”.

Unit B: Transformations (provincial mean: 64.0%)

Conceptual Knowledge

When asked to perform transformations involving reflections and horizontal stretches or compressions, students confused $-f(x)$ with $f(-x)$ and $f\left(\frac{1}{2}x\right)$ with $f(2x)$. When performing multiple transformations on a graph of a given function, some students understood the concept of each individual transformation, but executed them in the incorrect order. When asked to sketch the reciprocal graph of a given function, students graphed the inverse of the function. In a question asking the students to state the zeros of the cotangent function, some students stated the asymptotes. Students did not perform well when asked to transform a logarithmic graph.

Procedural Skill

When sketching the reciprocal graph of a given function, students understood the concept of reciprocal, but plotted the reciprocal of the x -coordinates. When asked to state the parameters of a sinusoidal equation from a given word problem, some students had difficulty determining the parameters from their sketch or sketched the graph incorrectly and therefore arrived at incorrect parameters.

Communication

Many students did not know how to express the zeros of a sinusoidal function as a general solution.

Unit C: Trigonometric Identities (provincial mean: 61.9%)

Conceptual Knowledge

For the sum and difference identity question, many quadrant errors were made while trying to find the exact values of $\sin \beta$ and $\cos \alpha$. After having made an error, students provided solutions which were not within the appropriate range and therefore should have been discarded. Students did not understand how to correctly substitute into the sum/difference identity. Rather than substituting in the exact value, some students attached the trigonometric function, not realizing that they had already used this to find the exact value.

Procedural Skill

When calculating the length of the missing side of the alpha/beta right-angled triangles, students often misused the Pythagorean Theorem. When asked to prove an identity, students encountered difficulties with simplifying and therefore could not properly equate the two sides. When multiplying by a certain expression to simplify a complex fraction, many students did not multiply every term of the complex fraction by the chosen expression.

Communication

In general, students did not show all their work when answering questions involving trigonometric identities. Some students changed $\cos 2\theta$ to $\cos^2 \theta$. Although a T-chart was provided, students equated the two sides of the given equation throughout the identity proof.

Unit D: Exponents and Logarithms (provincial mean: 65.4%)

Conceptual Knowledge

Many students had difficulty knowing where to start when solving logarithmic equations or exponential equations. When solving the logarithmic equation, they dropped the logs instead of applying either the product or quotient rule. When solving the exponential equation, they did not apply logs or, if they did, they did not recognize that the following step involved the power rule. Instead of applying the product rule, many students multiplied the bases, which then made the rest of the question easier. Some students had difficulty with the general shape of the graph of a logarithmic equation.

Procedural Skill

While solving the logarithmic equation, some students cancelled the base of the logs and then solved the remaining incorrect equation. Some students distributed the logs through the argument and then simplified from there. Some students did not know how to solve for x^2 . They produced only the positive solution, thereby losing the opportunity to reject the negative root. Also, some students used the power rule incorrectly, either by not dropping the exponent in front of the correct base or, when the exponent was a binomial, by forgetting to include brackets. Some students tried to reduce fractions when it was not appropriate. For example, some tried to reduce the $\log 14$ s in the equation $x = \frac{-\log 5 - \log 14}{\log 7 - 3\log 14}$.

Communication

When indicating the operation of division while solving for x , a common error was writing a line under the entire equation rather than each side individually. When simplifying equations involving logs, some students wrote down each calculator result instead of leaving all the values in the calculator and reporting one final answer. This resulted in incorrect rounding of decimals.

Unit E: Permutations, Combinations, and Binomial Theorem (provincial mean: 57.8%)

Conceptual Knowledge

When using the given formula to expand binomials, some students incorrectly substituted the term they were asked to find as the value of k . Students struggled with the various conditions in both questions regarding permutations (one using a given set of letters and the other using given digits). A large portion of students did not understand that different cases were required and therefore attempted to solve these questions using only one case. In the question requiring students to choose letters from a larger group and then arrange them, students that did have the correct cases often did not understand that selecting a combination of letters was first required before attempting to arrange them. Many students understood that cases were needed to solve the permutation question with digits, but did not separate the cases properly, as they struggled with the restriction that two of the digits (0 and 8) needed to be separated from the cases involving the other even digits. When solving an equation with factorials, some students solved incorrectly and did not recognize they needed to make a correction after finding a value for n that was not a whole number.

Procedural Skill

The binomial expansion was well done although some students made errors in applying the exponent laws. When solving the factorial equation, the main error occurred when reducing the factorials. Students also had difficulty dividing fractions.

Communication

A few students did not simplify, thus leaving their answers in factorial form.

Unit F: Conic Sections (provincial mean: 61.7%)

Conceptual Knowledge

Many students had difficulties with the equation of a parabola. They did not understand the concept of putting it into standard form. Students confused the equation of a hyperbola with that of an ellipse when giving the range. Also, they confused the range with the domain.

Procedural Skill

When converting the general equation of a conic section into standard form, students knew to complete the square, but made errors such as squaring the length of the axis. Some students did not write the equation of the parabola in standard form before stating the vertex, resulting in an x -value of zero. When finding the slope of the asymptotes of a hyperbola, students mixed up the a and b values, which resulted in finding the reciprocal of the slope.

Communication

Some errors of omission were made such as forgetting the squares or not equating the left side of the equation of an ellipse in standard form to 1. Students did not know how to show the standard form of a parabola and divided by the a -value.

Unit G: Probability (provincial mean: 70.7%)

Conceptual Knowledge

The vast majority of students were able to recognize conditional probability questions, but some struggled with setting up the proper ratio. When asked to find the probability of a question involving permutations, some students misunderstood or did not read that determining the probability was required. A few students multiplied instead of added when calculating a certain probability, resulting in a probability greater than 1. Many of these students did not attempt to correct their error. Another common error was using conditional probability when it was not required or not using conditional probability when it was required. In determining the probability of two non-mutually exclusive events, the most common error was adding the individual probabilities without subtracting the probability of them occurring at the same time.

Procedural Skill

Students had difficulty setting up the conditional probability ratio. Many students calculated the various branches correctly, but did not select or combine the branches properly. Some students misread part of a probability question and mistakenly calculated the probability of the complement.

Communication

There was a lack of communication in the question regarding a permutation probability when a few students obtained the correct answer with no explanation and no work shown.

Unit H: Geometric Sequences (provincial mean: 76.0%)

Conceptual Knowledge

Numerous mistakes were made during the set-up stage when finding the value of r given a geometric sequence with two non-consecutive terms. Some students used the formula for finding the value of a term incorrectly and some substituted in incorrect terms. When calculating the value of r , many students solved an even root which required two possible solutions, one positive and one negative, which they disregarded. When finding the sum of an infinite geometric series, some students did not recognize that they needed to use the infinite sum formula. Other students simply added the given first three terms rather than considering the series as a whole.

Procedural Skill

Some students, who made a new sequence by taking the given third term and using it as their first term, forgot to go back to the original sequence and find the correct first term. When finding the sum of an infinite geometric series, errors occurred while working with fractions.

Communication

When using the formula for finding the sum of a geometric series, some students forgot brackets, while others made rounding errors. When finding various consecutive terms of a sequence, some students rounded their individual values, which resulted in rounding errors. A notation error of writing a line under the entire equation when dividing (rather than each side individually) occurred when some students found the value of r given two non-consecutive terms.

Summary of Test Results (province)

Unit A: Circular Functions	60.8%
Unit B: Transformations	64.0%
Unit C: Trigonometric Identities	61.9%
Unit D: Exponents and Logarithms	65.4%
Unit E: Permutations, Combinations, and Binomial Theorem	57.8%
Unit F: Conic Sections	61.7%
Unit G: Probability	70.7%
Unit H: Geometric Sequences	76.0%
Total Test	63.4%

Marking Accuracy and Consistency

Information regarding how to interpret the marking accuracy and consistency reports is provided in the *Interpreting and Using Results from Provincial Tests and Assessments* document available at www.edu.gov.mb.ca/k12/assess/support/results/index.html.

These reports include a chart comparing the local marking results to the results from the central re-marking of sample test booklets. Overall, the accuracy of local versus central marking for the Pre-Calculus Mathematics Standards Test was consistent. To highlight this consistency, 54.0% of the booklets sampled by the Department received a central mark within $\pm 2\%$ of the local mark and 96.3% of the sampled booklets were within $\pm 6\%$. Scores awarded at the local level were, on average, 0.72% higher than the scores given at the Department.

Survey Results

Teachers who supervised the Grade 12 Pre-Calculus Mathematics Standards Test in January 2011 were invited to provide comments regarding the test and its administration. A total of 101 teachers responded to the survey. A summary of their comments is provided below.

- 93% of the teachers indicated that all of the topics in the test were taught by the time the test was written. Additional time (in excess of the regular class time) was spent by 49% of the teachers in order to prepare students for the test.
- 99% of the teachers indicated that the test content was consistent with the standards and learning outcomes as outlined in the curriculum document. Approximately 92% of teachers indicated that the reading level of the test was appropriate and that the test questions were clear and fair to students.
- 85% of the teachers indicated that students were able to complete *Booklet 1* and *Booklet 2* in the allotted time.
- 58% and 89% of the teachers, respectively, indicated that their students used graphing and scientific calculators during Part 1 of the test.