Grade 12
Applied Mathematics
Standards Test

Written Test

June 2011

Manitoba

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GRADE 12 APPLIED MATHEMATICS
STANDARDS TEST

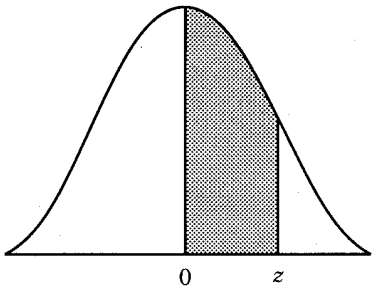
DESCRIPTION

Total Possible Marks: 53 **Time: 2.5 hours**

	Description	Suggested Time	Marks
Section A	9 multiple-choice questions worth 1 mark each	20 minutes	9
Section B	12 constructed-response questions worth from 3 to 5 marks each	130 minutes	44

TEST RESOURCES AND DIRECTIONS

- You are provided with the Standard Normal Distribution table on page 2.
- You may consult your 8½" × 11" individually prepared study sheet during the test.
- You may use a geometry set, a graphing calculator, computer software, and have access to the Internet for tools such as applets or a mortgage payment calculator. **The use of the Internet to access course notes, to find definitions, or to search for conceptual information about the course is prohibited during the test.**
- In Section A, choose the best response and mark it on the *Answer/Scoring Sheet*. Do not fold the *Answer/Scoring Sheet* or the test booklet.



Notes:

1. For values of z above 3.09, use 0.4999 for the area.
2. Use these common values that result from interpolation:

z-score	area
1.645	0.4500
1.960	0.4750
2.575	0.4950

Standard Normal Distribution

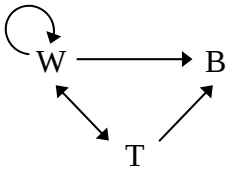
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

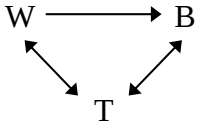
SECTION A: MULTIPLE-CHOICE QUESTIONS

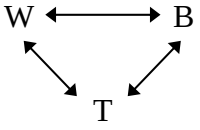
Value: 9 marks **Suggested Time: 20 minutes**

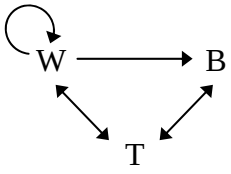
1. Determine the diagram that represents the following matrix.

$$\begin{array}{c} \\ W \\ B \\ T \end{array}\begin{array}{c} \begin{array}{ccc} W & B & T \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

A) 

B) 

C) 

D) 

2. David has purchased a computer valued at $2575.00. The rate at which the computer depreciates in value is 12% per year. What will be the value of David's computer after four years?

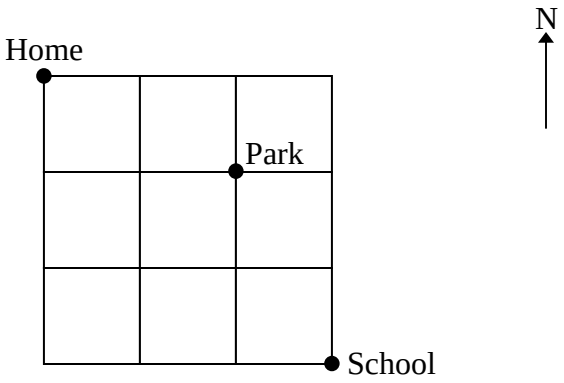
   A) $1339.00

   B) $1544.22

   C) $2266.00

   D) $4051.81

3. Celine travels from home to school every day. The following diagram shows all of the possible routes she can take.



If Celine can only travel south or east, how many routes are possible if she passes by the park on her way to school?

A) 3

B) 6

C) 9

D) 20

4. In how many different ways can the letters of the word "GOLDEYES" be arranged where each of the arrangements begins with the letter "G"?

A) 2520

B) 5040

C) 20 160

D) 40 320

5. A survey shows that 95% of high school graduates are employed. If there are 220 graduates, determine the population standard deviation for the number of graduates who are employed.

A) 1.64

B) 3.23

C) 14.46

D) 209

6. Diane scored 81% on a test. The class average was 78% with a standard deviation of 1.5%.

What is Diane's *z*-score?

A) –2

B) 2

C) 24

D) 29

7. Which of the following sinusoidal functions has a period of 10?

A) $y = 6.28\sin(1.59x)$

B) $y = 1.59\sin(6.28x)$

C) $y = 0.628\sin(10x)$

D) $y = 10\sin(0.628x)$

8. The owners of an electronics firm had sales of $100 000.00 in their first year of business. If their sales increased by $15 000.00 each year for the 8 years that followed, what were the sales in their fifth year of business?

A) $160 000.00

B) $175 000.00

C) $205 000.00

D) $220 000.00

9. A ball is dropped from a height of 2 metres. After each bounce, it rises to 60% of its previous height. After how many bounces does the ball reach a height of less than 0.1 metres for the first time?

A) 3

B) 5

C) 6

D) 7

SECTION B: CONSTRUCTED-RESPONSE QUESTIONS

Value: 44 marks **Suggested Time: 130 minutes**

DIRECTIONS

- There are 12 constructed-response questions in this section of the test. Each question is worth from 3 to 5 marks.

- Provide **complete answers** in the spaces provided in the test booklet. You may print out diagrams from the computer or your calculator where applicable. Indicate your booklet number and question number on the printouts. Remain seated and your teacher will distribute these printouts to you. Indicate in the response space of the question that the answer is on a printed sheet and staple it to the page.

- If you need more space to answer a question in Section B, extra paper may be provided by your teacher. Write your booklet number and question number on any extra paper used and staple it into the booklet where your answer begins. Indicate in the response space of the question that the answer is on a separate sheet.

- Provide clear explanations or justifications where applicable. This can be done through labelled diagrams, in words, by showing mathematical operations to verify your answer, or by referring to a calculator or software program.

  — If you refer to a calculator program, indicate your input values.
  — If you refer to a software program or a website, indicate your input values and print or copy the screen showing the answers.
  — If you refer to a spreadsheet, print a copy of the answers.

- Let the mark values for each question guide your time and the amount of detail you use in your answer.

- Round your final answers to the nearest two decimal places.

- Unless otherwise indicated, it is not necessary to draw diagrams to scale. If you draw a diagram to scale, make sure to indicate the scale you used.

- An answer without any work shown will be considered incomplete.

- Always state your assumptions.

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

10. The following information was gathered for the number of books sold, by category (mystery, health, and science), at two different bookstores during the week of July 1 to 7.

Total: 3 marks

Store A: 82 mystery, 65 health, 90 science
Store B: 31 mystery, 19 health, 45 science

a) Create a matrix *S* to show the number of books sold at both stores during the week of July 1 to 7.

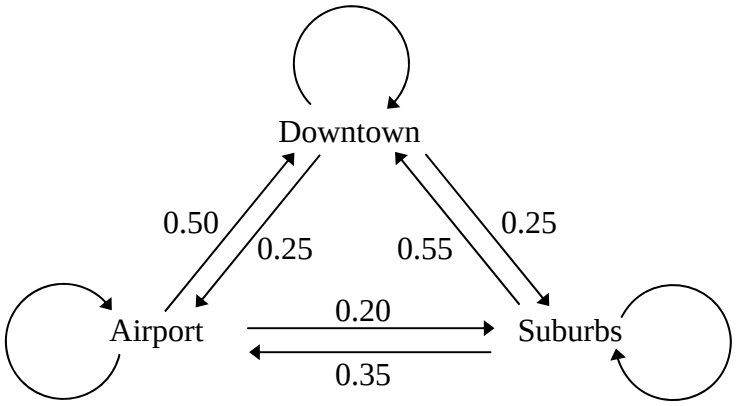
(1 mark)

b) If mystery books sell for $9.00 each, health books sell for $25.00 each, and science books sell for $37.00 each, determine the total value of the books sold at each store. Show your work using matrix operations.

(2 marks)

11. A study on the taxi industry in Winnipeg reports the following daily movements of taxis between three locations: downtown, airport, and suburbs.

Total: 5 marks



a) Create a transition matrix T to represent this situation. *(2 marks)*

$$T = \begin{array}{c} \\ \text{Downtown} \\ \text{Airport} \\ \text{Suburbs} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Downtown} & \text{Airport} & \text{Suburbs} \end{array} \\ \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{array}$$

b) The following matrix *A* represents the current number of taxis present at each location.

$$A = \begin{array}{c} \begin{matrix} \text{Downtown} & \text{Airport} & \text{Suburbs} \end{matrix} \\ \begin{bmatrix} 180 & 160 & 90 \end{bmatrix} \end{array}$$

The mayor of Winnipeg wants a daily movement pattern where 150 to 170 taxis would always be present at the airport. Using matrix operations, show that the current daily movements do not accomplish this.

(1 mark)

c) Change your transition matrix in (a) to model a movement pattern that would allow for the desired number of taxis at the airport. Using matrix operations, show that the number of taxis will stabilize between 150 and 170.

(2 marks)

$$T = \begin{array}{c} \\ \text{Downtown} \\ \text{Airport} \\ \text{Suburbs} \end{array} \begin{array}{c} \begin{matrix} \text{Downtown} & \text{Airport} & \text{Suburbs} \end{matrix} \\ \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{array}$$

12. Julian wonders if the bank will lend him money to go on a holiday. His financial situation is as follows:

Total: 5 marks

- He has invested $8000.00 in mutual funds.
- He has $1500.00 in a chequing account and $2500.00 in a savings account.
- He has invested $5000.00 in an RRSP.
- He has a registered pension plan valued at $9000.00.
- He has purchased a house valued at $250 000.00 and his mortgage is $150 000.00.
- He has purchased a car valued at $35 000.00 and has an outstanding balance of $30 000.00 on the loan.
- He owes $4500.00 on a stereo system and has a debt of $5200.00 on his credit card.
- He owes $20 000.00 on his line of credit.

a) Complete a statement of net worth for Julian, indicating the total assets, total liabilities, and net worth.

(3 marks)

b) Calculate Julian's debt equity ratio. Based on his debt equity ratio, do you think the bank will lend him money? Explain your answer.

(2 marks)

Statement of Net Worth

ASSETS (WHAT YOU OWN)

1. Liquid/Current Assets
   - i. Bank Accounts _____
   - ii. Cash _____

   Total Liquid Assets _____

2. Semi-Liquid Assets
   - i. Mutual Funds _____
   - ii. Stocks/Bonds _____
   - iii. RRSPs _____
   - iv. Registered Pension Plan _____
   - v. Life Insurance/Cash Value _____

   Total Semi-Liquid Assets _____

3. Non-Liquid Assets
   - i. Principal Residence _____
   - ii. Vehicles _____
   - iii. Other _____

   Total Non-Liquid Assets _____

TOTAL ASSETS _____

LIABILITIES (WHAT YOU OWE)

4. Short-Term Debt
   - i. Credit Card _____
   - ii. Short-Term Loans _____

   Total Short-Term Debt _____

5. Long-Term Debt
   - i. Mortgage _____
   - ii. Line of Credit _____
   - iii. Other _____

   Total Long-Term Debt _____

TOTAL LIABILITIES _____

NET WORTH _____

DEBT EQUITY RATIO _____

Note: To calculate the Debt Equity Ratio, subtract the mortgage from the total liabilities, then divide by the net worth.

13. Paulette and Roger purchase a house for $210 000.00 and have saved $25 000.00 for a down payment. They obtain a mortgage amortized over 20 years at an interest rate of 5.25% compounded semi-annually.

Total: 3 marks

a) Determine Paulette and Roger's monthly mortgage payment. Show your work.
(2 marks)

b) How much will they have paid in interest after the first 10 years?
(1 mark)

14. Catherine, Suzanne, and Donald are on the last hole of their game of miniature golf. They each have a chance to win a free game if they can get a hole in one. The probability of each getting a hole in one is 20%, 25%, and 30% respectively.

Total: 3 marks

a) What is the probability that no one will win a free game?

(1 mark)

b) What is the probability that exactly two of them will win a free game? Show your work.

(2 marks)

15. A survey of 500 students showed that:

Total: 3 marks

- 93 are enrolled in a French course
- 78 are enrolled in a Spanish course
- 21 are enrolled in both a French course and a Spanish course

a) How many students are enrolled in only a Spanish course?

(1 mark)

b) If a student is selected at random, what is the probability that he is enrolled in a French course or a Spanish course? Show your work.

(2 marks)

16. A baker randomly recorded the weights of different loaves of bread and noted the following information:

Total: 4 marks

Weight of a loaf of bread (g)	772.5	777.5	782.5	787.5	792.5	797.5	802.5	807.5	812.5	817.5	822.5	827.5
Frequency	3	17	44	100	141	192	191	150	90	42	14	9

For the data above, the mean is 800 grams and the standard deviation is 10 grams.

a) The baker thinks that the weights of the loaves of bread are normally distributed. Indicate two reasons that prove she is correct. Show your work using statistics.

(2 marks)

b) What is the probability of selecting a loaf of bread that weighs more than 815 grams? Show your work using statistics.

(2 marks)

17. A math student recently surveyed 300 students from his high school and found that 36 were left handed. He then calculated the 95% confidence interval for the number of left-handed students and made the following statement.

Total: 3 marks

"The number of left-handed students is between 24.97 and 47.03."

a) What information should be added to his statement to make it complete?

(1 mark)

b) Are the values indicated in the student's statement correct? Show your work using statistics.

(2 marks)

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

18. The volume of air inside the lungs varies with time as people breathe in and out. The volume of air inside the lungs of a sleeping person is modelled by the following sinusoidal equation:

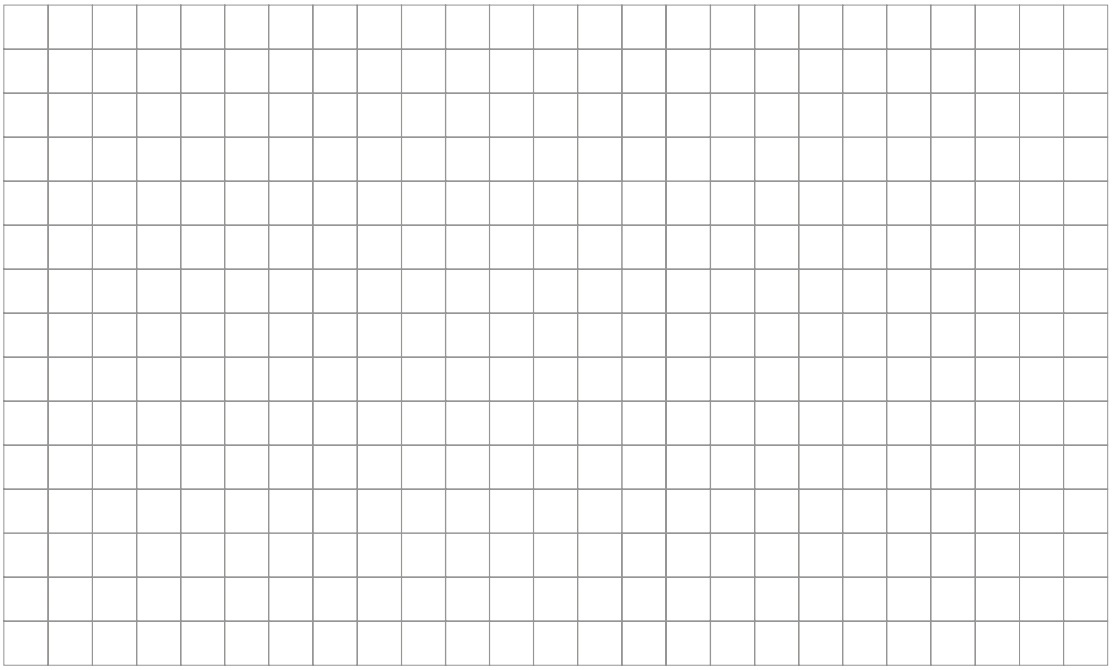
Total: 4 marks

$$y = 1.5\sin(1.57x - 1.57) + 2.5$$

where x is the time in seconds and
y is the volume of air in litres.

a) Draw a clearly labelled graph which represents the volume of air in a sleeping person's lungs over a period of at least 10 seconds. Indicate the maximum and minimum values.

(2 marks)



maximum value = _____

minimum value = _____

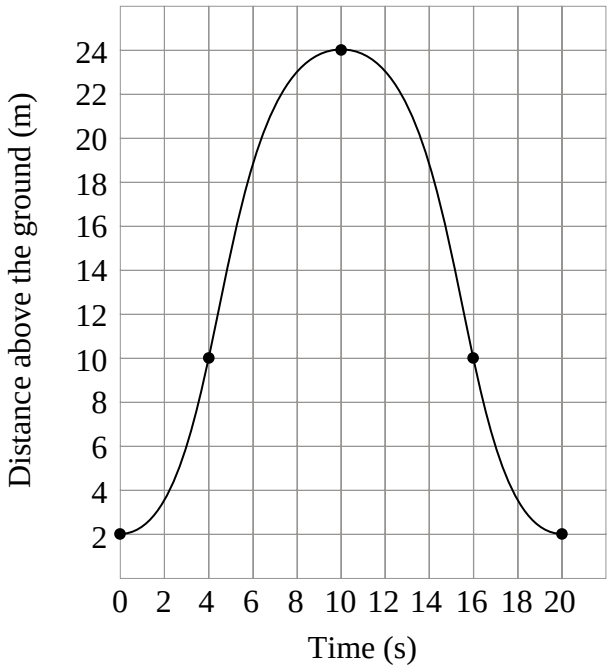
b) When a sleeping person breathes out, the volume reaches its minimum value. When a person breathes in, they snore until the volume reaches 3 litres.

Using the sinusoidal equation, determine the total amount of time a person will be snoring during the first 10 seconds. Show your work and indicate your answer to 2 decimal places.

(2 marks)

19. The graph below represents one complete revolution of a Ferris wheel.

Total: 4 marks



a) Determine the diameter of this Ferris wheel.

(1 mark)

b) Determine the sinusoidal equation which best represents the graph. Explain how you arrived at your answer. Indicate the input values if you use technology.

(2 marks)

c) In this situation, explain why the minimum value must be greater than 0.

(1 mark)

20. Roland has a field where 1000 Christmas trees have been planted. Every year, he decides to sell 20% of his trees and plant 120 new trees.

Total:
3 marks

a) How many trees will Roland have after 5 years? Show your work.

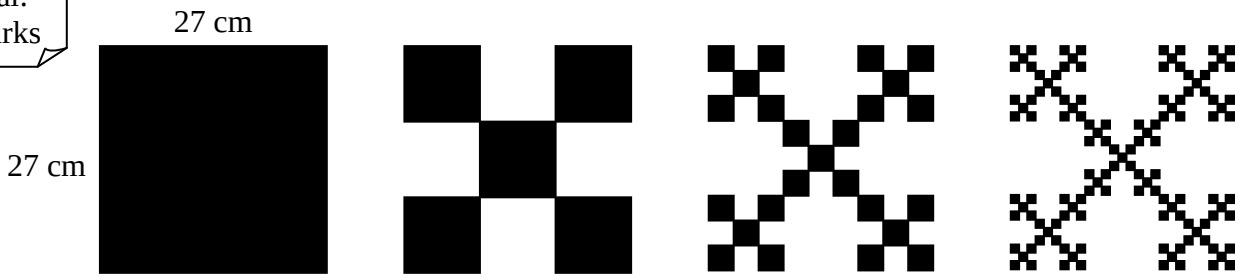
(2 marks)

b) How many trees will there be when this pattern stabilizes?

(1 mark)

21. Given the fractal below showing the original square and the first three iterations:

Total: 4 marks



a) Complete the following table.

(3 marks)

	Area of One Black Square (cm^2)	**Number of Black Squares**	**Total Area of Black Squares (cm^2)**
Original			
1st iteration			
2nd iteration			
⋮	⋮	⋮	⋮
6th iteration			

b) Will the area of black squares eventually disappear from view? Explain your answer.

(1 mark)

END OF TEST