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# **Senior 4 Applied Mathematics (40S) Standards Test**

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**Written Test**

**January 2006**

**Manitoba** 

*Building for the Future*

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## SENIOR 4 APPLIED MATHEMATICS (40S) STANDARDS TEST

### DESCRIPTION

**Total Possible Marks: 53**

**Time: 2.5 hours**

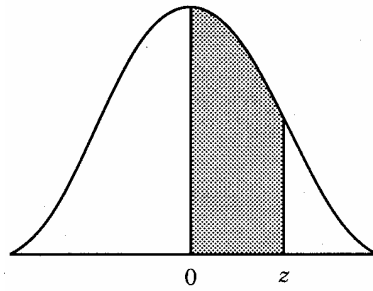
**This test consists of two sections:**

	Description	Suggested Time to Complete	Marks
<b>Section A</b>	7 multiple-choice questions worth 1 mark each	20 minutes	7
<b>Section B</b>	13 constructed-response questions worth from 2 to 5 marks each	130 minutes	46

### TEST RESOURCES AND DIRECTIONS

- ◆ You are provided with the Standard Normal Distribution table (see page 2).
- ◆ You may consult your 8½" × 11" individually prepared study sheet during the test.
- ◆ Use of a graphing calculator, computer software or a Website is allowed.
- ◆ Use of protractor, compass and ruler are allowed.
- ◆ Read all instructions on the test carefully.
- ◆ In Section A, choose the best response and mark it on the *Answer/Scoring Sheet*. Do not fold the *Answer/Scoring Sheet* or the test booklet.
- ◆ If you need more space to answer a question, extra pages may be provided by your teacher. Write your booklet ID number and question number on any extra page(s) used and staple it into the booklet where your answer begins. Indicate in the response space of the question that the answer is on a separate sheet.

## z-Score Chart



Notes:

1. For values of  $z$  above 3.09, use 0.4999 for the area.
2. Use these common values that result from interpolation:

<u>z-score</u>	<u>area</u>
1.645	0.4500
2.575	0.4950

The Standard Normal ( $z$ ) Distribution										
$z$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

## SECTION A: MULTIPLE-CHOICE QUESTIONS

Value: 7 marks

Suggested Time: 20 minutes

1. Given  $R = \begin{bmatrix} 3 & 5 \\ 8 & 7 \end{bmatrix}$ . Which matrix results from the matrix operation  $R + R^2$ ?

A)  $\begin{bmatrix} 9 & 15 \\ 24 & 21 \end{bmatrix}$

B)  $\begin{bmatrix} 12 & 30 \\ 72 & 56 \end{bmatrix}$

C)  $\begin{bmatrix} 52 & 55 \\ 88 & 96 \end{bmatrix}$

D)  $\begin{bmatrix} 547 & 595 \\ 952 & 1023 \end{bmatrix}$

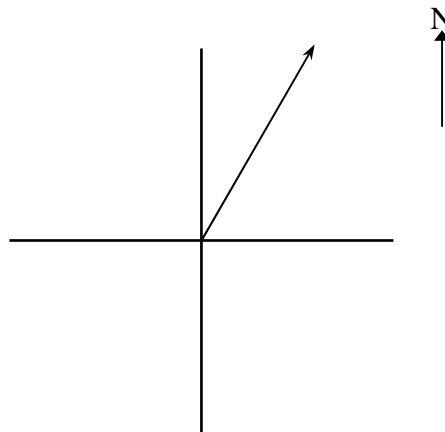
2. Given the scale 1 cm = 30 km/h, the description of this vector is:

A) 30 km/h at E 30° N

B) 30 km/h at N 30° E

C) 90 km/h at E 30° N

D) 90 km/h at N 30° E



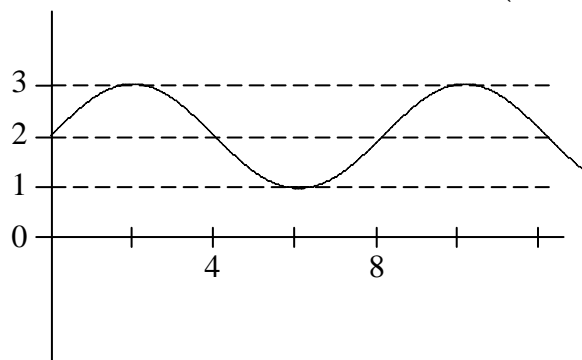
3. Brad has invested \$10 000.00. His annual rate of return is 8.50%. What will the value of his investment be at the end of 5 years if the interest is compounded annually?
- A) \$14 250.00
  - B) \$15 036.57
  - C) \$15 273.01
  - D) \$18 500.00
4. The difference between the value of assets and liabilities is known as:
- A) net worth
  - B) debt equity ratio
  - C) an investment portfolio
  - D) a mutual fund
5. There are 24 students in Jane's Applied Mathematics class. Her teacher wants to break the class into groups of 3 students to work on their matrix project.
- In how many ways can the first group be formed?
- A) 2024
  - B) 4048
  - C) 12 144
  - D) 13 824

6. Tests indicate that the mean life of an electric light bulb in continuous operation is 87 hours with a standard deviation of 7.50 hours.

Assuming a normal distribution in a total of 1000 light bulbs, approximately how many light bulbs will last between 72 and 94.50 hours?

- A) 480
- B) 680
- C) 820
- D) 950

7. The graph below is represented by the equation:  $y = a \sin(bx - c) + d$ .



The value of  $d$  is:

- A) 1
- B) 2
- C) 3
- D) 8



## SECTION B: CONSTRUCTED-RESPONSE QUESTIONS

Value: 46 marks

Suggested Time: 130 minutes

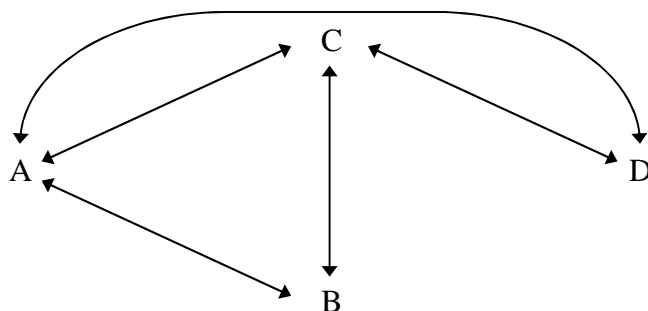
### DIRECTIONS

- ◆ There are 13 constructed-response questions in this section of the test. Each question is worth from 2 to 5 marks.
- ◆ Provide **complete answers** in the spaces provided in the test booklet. You may print out sketches from the computer or your calculator where applicable. Make sure these printouts contain your booklet ID number. Remain seated and your teacher will distribute these printouts to you. Staple the printouts in the booklet where the question begins. Indicate in the response space of the question that the answer is on a printed sheet.
- ◆ Provide clear explanations or justifications where applicable. This could be done through labelled diagrams, in words, by showing mathematical operations to verify your answer, or by referencing a software or a calculator program.
  - If you reference a calculator program, indicate your input values.
  - If you reference a spreadsheet, print a copy of the answers.
  - If you reference a software program or a Website, indicate your input values and print or copy the screen showing answers.
- ◆ Let the mark values for each question guide your time and the amount of detail you use in your answer. Mark values for subsections of a question are indicated in parentheses.
- ◆ Round your final answers to the nearest two decimal places.
- ◆ Unless otherwise specified, it is not necessary to draw sketches to scale. If you draw a scale diagram, make sure you indicate your scale.
- ◆ An answer without any work shown will be considered incomplete.
- ◆ Always state your assumptions.



8. The following diagram represents the routes that a person can use to drive between the towns labelled A, B, C and D.

Total:  
3 marks



- a) Create a matrix  $M$  which corresponds to the network above.  
(1 mark)
- b) A student was asked how to calculate the number of ways to travel between two towns going through exactly one other town. The student answered, " $M + M^2$ ". Explain their error and show the correct answer.  
(2 marks)

9. A paddle wheel boat operator charges the following rates for a two-hour trip: adults—\$12.50, students—\$8.50 and children under 6 years of age—\$2.50. The number of each type of ticket that was sold during the weekend is given in the following matrix:

Total:  
2 marks

	Saturday	Sunday
Adults	108	75
Students	84	122
Children	53	72

Determine the boat operator's revenue for each day. Show your work using the appropriate matrix operations.

10. Each year at Manitoba High School, the teachers sit on a committee.

Total:  
4 marks

Currently:

- 30% are on the Sports committee
- 20% are on the Arts committee
- 50% are on the Graduation committee

At the end of the school year, the teachers can change committees.

Here are the changes that took place last June:

- 10% of the teachers from each committee stayed on their committee
- 15% of the members on the Sports committee joined the Graduation committee
- 5% of the members on the Arts committee joined the Sports committee
- 30% of the members of the Graduation committee joined the Arts committee

a) Create a transition matrix which shows the changes in committees.

(2 marks)

$$T = \begin{matrix} & & \text{(To)} \\ & & \text{S} & \text{A} & \text{G} \\ \text{(From)} & \text{S} & & & \\ & \text{A} & & & \\ & \text{G} & & & \end{matrix} \left[ \begin{array}{ccc} & & \\ & & \\ & & \end{array} \right]$$

b) If this trend continues, determine the percentage of teachers on each committee once it stabilizes. Provide proof of stabilization.

(2 marks)

11. Marina and Phillip are pulling on a rock with equal force. They are at an angle of  $40^\circ$  to each other.

Total:  
4 marks

- a) If the resultant vector has a force of 50 newtons, determine the magnitude of the force with which each is pulling. Show your work.

(2 marks)

- b) If the resultant force remains at 50 newtons, at what angle could Marina and Phillip pull in order for Marina to pull with double the force of Phillip? Show your work.

(2 marks)

12. An airplane travels at a speed of 250 km/h in a direction of  $30^\circ$  north of west for one hour. It then travels at a speed of 300 km/h for 40 minutes in a direction of  $42^\circ$  east of north.

To return directly to its starting point, in what direction and what distance must the airplane travel? Show your work.

Total:  
4 marks

13. Derek is considering a \$7500.00 loan amortized over 5 years at an interest rate of 6.50%, compounded monthly.

Total:  
3 marks

- a) What will his monthly payment be?

(1 mark)

- b) How much interest will he pay in total over the 5 years? Show your work.

(2 marks)

14. The Smith family can purchase a house valued at \$100 000.00 with a down payment of \$10 000.00, or they could rent a similar house.

Total:  
5 marks

- a) The monthly payment for a mortgage, amortized over 15 years at an interest rate of 6.50% compounded monthly, is \$784.00. If the Smiths purchase the house, calculate the total amount paid, including the down payment, by the end of the first 5 years.

(1 mark)

- b) Real estate appreciates at a rate of 3% per year. Determine the equity in the purchased house after 5 years. Show your work.

(3 marks)

- c) Given that it is more expensive to rent, give one reason why the Smiths would choose to rent.

(1 mark)

15. A ship is equipped with 6 flags which are identical except in colour. Of the 6 flags, 3 are red, 2 are blue and 1 is green. Depending on the order of the colour and the number of flags flown, different signals can be sent.

Total:  
3 marks

- a) Using all 6 flags, how many different signals can be sent?  
(1 mark)

- b) Using only 3 flags, create a sample space showing the different signals that can be made if at least 2 of the flags must be red. State the number of different signals.  
(2 marks)

16. A group of 480 students at the Manitoba Collegiate Institute are classified by year and mathematics course taken. The following chart shows the result of this classification:

Total:  
3 marks

	<b>Consumer Mathematics</b>	<b>Applied Mathematics</b>	<b>Pre-Calculus Mathematics</b>
<b>Senior 2</b>	50	60	40
<b>Senior 3</b>	60	70	30
<b>Senior 4</b>	70	70	30

- a) What is the probability that a randomly selected student is taking a Consumer Mathematics course?

(1 mark)

- b) Consider only Senior 3 and Senior 4 students. What is the probability that a randomly selected student is taking an Applied Mathematics or Pre-Calculus Mathematics course? Show your work.

(2 marks)

17. The marks for a first year biology course at Zenith University are normally distributed. The professors decided to award final grades using the following system:

Total:  
5 marks

- the top 5% of the students will receive an A
- the next 15% of the students will receive a B
- 60% of the students will receive a C
- 15% of the students will receive a D
- 5% of the students will receive an F

Brigitte has a final grade of 77% in the course. The mean is 64% and the standard deviation is 7%.

- a) What is Brigitte's  $z$ -score?

(1 mark)

- b) Find the letter grade that Brigitte will get in the course. Show your work.

(2 marks)

- c) With a minimum of a B letter grade, Robert will maintain his basketball scholarship. What is the minimum grade, as a percent, that Robert must obtain in the biology course to earn a B? Show your work.

(2 marks)

18. A research study on fruit flies found that an average of 50% of them stay alive after 32 hours.

Total:  
3 marks

- a) Determine the 95% confidence interval for the percentage of a sample of 1000 fruit flies still alive after 32 hours. Show your work.

(2 marks)

- b) Explain the meaning of the confidence interval in a).

(1 mark)

19. The following data represents the sunrise times in Brandon, Manitoba on the first day of each month during the year 2003. Sunrise times are expressed in hours after midnight.

Total:  
4 marks

Month/Day	Day of year	Sunrise time (hours)
January 1	1	8.62
February 1	32	8.20
March 1	60	7.38
April 1	91	6.27
May 1	121	5.90
June 1	152	5.60
July 1	182	5.58
August 1	213	6.15
September 1	244	6.92
October 1	274	7.65
November 1	305	7.98
December 1	335	8.25

- a) Determine the equation of the sinusoidal curve of best fit for this data.  
(2 marks)
- b) According to your equation, what is the median sunrise time for the year 2003?  
(1 mark)
- c) What does one complete cycle represent?  
(1 mark)

20. A stone has become stuck in Justin’s bicycle tire. The following equation describes the height of the stone, as compared to the ground, as the tire rotates.

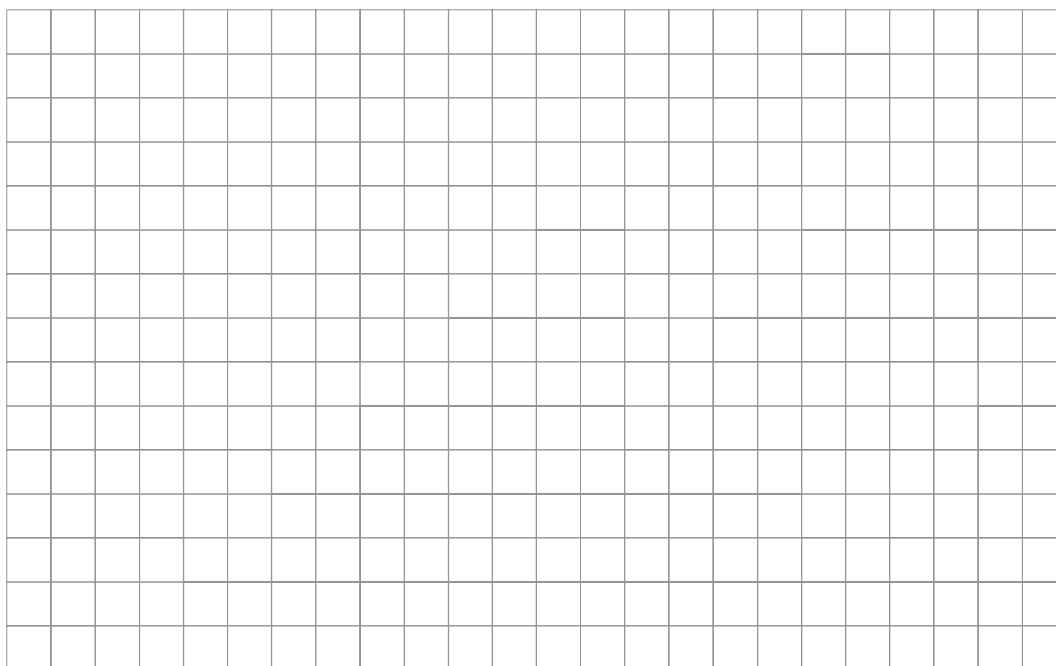
Total:  
3 marks

$$h = 30.79 \sin(14.47t - 1.87) + 30.79$$

where  $t$  represents time in seconds  
and  $h$  represents the height of the stone above the ground in centimetres.

- a) Sketch a graph which represents the height of the stone as a function of time. The graph must start at  $t = 0$  and show one second. Label the maximum and minimum values of your graph.

(2 marks)



- b) What will the height of the stone be at  $t = 1.00$  second?

(1 mark)

**END OF TEST**

**NO MARKS WILL BE AWARDED FOR WORK DONE  
ON THIS PAGE.**

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