Grade 12
Applied Mathematics
Achievement Test

## Student Booklet

June 2019

## Manitoba Education and Training Cataloguing in Publication Data

Grade 12 applied mathematics achievement test. Student booklet. June 2019

This resource is available in print and electronic formats. ISBN: 978-0-7711-7842-9 (print) ISBN: 978-0-7711-7843-6 (pdf)

1. Educational tests and measurements-Manitoba.
2. Mathematical ability-Testing.
3. Mathematics-Examinations, questions, etc.
4. Mathematics-Study and teaching (Secondary)-Manitoba.
I. Manitoba. Manitoba Education and Training.
510.76

Manitoba Education and Training
Winnipeg, Manitoba, Canada
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This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.
Websites are subject to change without notice.

## Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

# Grade 12 Applied Mathematics <br> ACHIEVEMENT TEST 

## DESCRIPTION

Time Required to Complete the Test: 3 hours
Total Possible Marks: 70 Additional Time Allowed: 30 minutes

| Unit | Marks |
| :--- | :---: |
| Relations and Functions | 16 |
| Probability | 18 |
| Financial Mathematics | 19 |
| Design and Measurement | 9 |
| Logical Reasoning | 8 |

## Directions

Remember to

- indicate your input values by writing them in your booklet or printing a copy if using a technology tool
- state any assumptions you make
- express your answers in decimal and percentage form to at least the nearest hundredth (two decimal places) when rounding, except for monetary values or when otherwise indicated Example: $\frac{15}{29}=0.52$ or $51.72 \%$

Note: Rounding too soon in your solution may result in an inaccurate final answer for which full marks will not be awarded.

A "graphic organizer" is a visual representation of information. Examples include a tree diagram, a chart, a list, a Venn diagram, a truth table, Pascal's triangle, etc.

## A clearly communicated answer

- is easily identified in the response space
— includes the parameters in the equation, and " $y=$ ", "sin", "ln", or " $x$ ", as applicable
- includes the units of measure, where applicable
- includes labels, units, scales for the axes on graphs, and key characteristics of functions (e.g., maximum, minimum, intercepts, and appropriate shape)
- is expressed as an exact value or is appropriately rounded

Marks may be deducted for errors relating to any of the above.

Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited.

## RELATIONS AND FUNCTIONS

## Question 1

Total: 1 mark
Select the best answer.
A stadium field is shaped like a parabola so that rainwater runs off to the sides. The surface of the field can be modelled by the following function:

$$
y=-0.000234(x-80)^{2}+1.5
$$

where $x$ represents the distance from the left side of the field and $y$ represents the height of the field.


Width (ft.)

What is the width of the field?
A) 0.75 ft .
B) 1.5 ft .
C) 80 ft .
D) 160 ft .

In an experiment, a water balloon is dropped from the roof of a school. The height of the water balloon from the ground is a function of time. The height is expressed in metres and the time is expressed in seconds. The domain of this function is $[0,1.43]$.

Give one reason that explains why the domain is restricted.

Taryn cooks a roast and records its internal temperature at specific times throughout the cooking process. Her findings are shown in the table below.

| Time (hours) | 1 | 2 | 5 | 8 |
| :--- | :---: | :---: | :---: | :---: |
| Temperature ( ${ }^{\circ} \mathbf{F}$ ) | 70 | 120 | 150 | 175 |

a) Determine the cubic regression equation that best models the data in this situation. (1 mark)
b) The next time Taryn cooks a roast, she would like its internal temperature to be $160^{\circ} \mathrm{F}$. Determine how long it will take the roast to reach this temperature.
(1 mark)

Scientists have determined that when the weight of an adult electric eel is known, its length can be modelled using the following equation:

$$
y=22.4 \ln (x)-28.7
$$

where $x$ is the eel's weight in pounds and $y$ is the length of the eel in inches.
a) Using the equation, determine the length of an eel that weighs 18 pounds.
(1 mark)
b) Provide one limitation of the equation that models this relationship.
(1 mark)

A boat is propelled by a paddle wheel with a diameter of 14 feet. Each paddle takes 90 seconds to complete one revolution. The logo on one paddle reaches a maximum height of 11 feet above the water after the boat is in motion for 10 seconds.



Diagram is not drawn to scale.
a) Determine a possible sinusoidal equation that models the height of the logo over time. Show your work.

(2 marks) | Time (s) | Height (ft.) |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

b) Determine how long the logo is underwater during one revolution. Show your work.
(2 marks)

An Australian farmer records the rabbit population on his farm over the course of one year. The table below represents his data.

| Time (months) | 1 | 4 | 7 | 9 | 11 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Number of rabbits | 25 | 50 | 100 | 150 | 240 |

a) Create a clearly labelled graph by plotting the given data. Draw a curve of best fit. (3 marks)

b) Determine the exponential regression equation that best models the data in this situation. (1 mark)
c) Using your equation in (b), determine how long it will take for the rabbit population to reach 400.
(l mark)
d) At a different farm, there are 300 rabbits at the beginning of the following year. This rabbit population increases, but less quickly than on the first farm. Write an exponential equation that models this situation.
(1 mark)

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## Probability

## Question 7

Total: 1 mark

Select the best answer.
Cintra scores a goal on $11 \%$ of the shots she takes.
Identify the odds against her scoring a goal.
A) $11: 89$
B) $89: 11$
C) 11:100
D) $89: 100$

## Question 8

Corbin wants to create an image of himself on his cellphone. There are 7 choices for hair colour, 3 choices for hair length, and 9 choices for a hat.

Using these options, how many different images in total can he create with or without a hat? Show your work.

## Question 9

Total: 2 marks

There are 21 students in an applied math class. On a test, 10 students used an app, 14 students used a graphing calculator, and 4 students used both.

What is the probability that a randomly selected student used only an app on the test? Show your work.

## Question 10

Pedro is walking from the train station to the convention centre and must withdraw money at the bank on his way. He can only walk south and east. How many different ways can he get to the convention centre? Show your work.


There are 5 biology books, 4 math books, and 7 history books randomly placed on a shelf.
a) Luis selects 2 books, one after the other. Determine the probability that both books are on the same subject. Show your work.
b) Are the events in (a) independent or dependent? Explain.
(1 mark)

Construction work has slowed travel near Dauphin, Manitoba. Harry must drive through the construction zone to get to work. The probability that he will be delayed because of the construction is $45 \%$. If he is delayed, the probability he will get to work on time is $70 \%$. If he is not delayed, the probability he will get to work on time is $85 \%$.
a) Use a graphic organizer to show all possible outcomes for this situation.
(1 mark)
b) Determine the probability that Harry will get to work on time. Show your work.
(2 marks)

A group of friends is ordering a meal of 3 pizzas and 2 salads from a restaurant. The restaurant offers 6 types of pizzas and 4 types of salads.

If all pizzas and salads chosen must be different from one another, how many meal options do the friends have? Show your work.

A dealership has 6 cars, 2 vans, and 4 trucks for sale.
a) An employee is asked to park all of these vehicles in a row. How many different ways can this be done if all of the cars must be together, all of the vans must be together, and all of the trucks must be together? Show your work.
(2 marks)

## Financial Mathematics

b) Diane visits the dealership and decides to buy one of the trucks. The price of this truck is $\$ 36500.00$, taxes included. She has $\$ 4000.00$ for a down payment. The balance will be financed at an interest rate of $2.99 \%$, compounded monthly, for 7 years. Calculate her monthly payment. Show your work.
(2 marks)
c) What is the total amount Diane will pay to buy the truck?
(1 mark)

## Question 15

Select the best answer.
Renasha deposits $\$ 1200.00$ into a savings account that earns simple interest at a rate of $1.72 \%$ annually.

What is the total value of Renasha's account at the end of one year?
A) $\$ 20.64$
B) $\$ 1220.64$
C) $\$ 1447.68$
D) $\$ 3264.00$

Mr. Chen is moving to Manitoba and is looking for a place to live.
State two reasons why Mr. Chen would prefer renting a house instead of buying a similar house.

## Reason 1:

## Reason 2:

Nadia hopes to save enough money over a 6-year period to go on a trip estimated to cost $\$ 11600.00$. To start saving, she deposits $\$ 1250.00$ into an account that earns an interest rate of $4.21 \%$, compounded monthly.
a) If she makes monthly payments of $\$ 110.00$ into this account, how much money will she have at the end of the 6 -year period? Show your work.
(2 marks)
b) What is the minimum number of additional monthly payments Nadia needs to make in order to save enough money for the trip?
(1 mark)

Marshall and Kim are renting a house that they hope to purchase. They pay $\$ 1800.00$ in annual heating costs and know that the annual property taxes are $\$ 2500.00$. Their combined gross income is $\$ 5200.00$ per month.
a) Based on the gross debt service ratio (GDSR), what is the maximum monthly mortgage payment they can afford? Show your work.
(2 marks)
b) If they have saved $\$ 30000.00$ for a down payment, what is the maximum house price they can afford based on a 25 -year amortization period at an interest rate of $4.64 \%$, compounded semi-annually? Show your work.
(3 marks)

Amor and Angélique each want to retire at age 65. They start investing in tax-free savings accounts (TFSA) at different ages in their lives.

- At age 28, Amor starts investing $\$ 240.00$ per month into a TFSA that earns an interest rate of $2.95 \%$, compounded monthly.
- At age 42, Angélique makes an initial deposit of $\$ 5000.00$ and starts investing $\$ 350.00$ per month into a TFSA that earns an interest rate of $3.50 \%$, compounded monthly.
a) What is the value of each of their TFSAs at age 65? Show your work.

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## DESIGN AND MEASUREMENT

## Question 20

## Total: 1 mark

Select the best answer.
A wheelbarrow is able to transport 6 cubic feet of soil per load.


What is the minimum number of loads required to fill the following garden with soil?

A) 3
B) 5
C) 14
D) 504

You are in charge of purchasing enough sports drink for the participants of a 5-kilometre race. There are 300 participants and you assume they will each drink the equivalent of 2 cups of sports drink. The cups are cone shaped with the following dimensions:

a) What volume of sports drink must you purchase? Show your work.
b) Each container of sports drink costs $\$ 5.00$ and contains $8 \mathrm{~L}\left(1 \mathrm{~L}=1000 \mathrm{~cm}^{3}\right)$ of drink.

How much will you spend on sports drink, before taxes?
(1 mark)

You are to design a bedroom. The bedroom design must satisfy the following conditions:

- The bedroom must be rectangular.
- The bedroom must include one dresser, one desk, and one bed chosen from the furniture options below.

|  | Furniture Options |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Dresser |  | Desk |  | Bed |  |
|  | Tall | Wide | Small | Large | Queen | King |
| Dimensions (ft.) <br> (width $\times$ length) | $1.5 \times 4$ | $1.5 \times 6$ | $2 \times 4$ | $2 \times 6$ | $4 \times 6$ | $5 \times 6$ |
| Area (ft ${ }^{\mathbf{2}}$ ) | 6 | 9 | 8 | 12 | 24 | 30 |
| Cost (\$) | 559.00 | 729.00 | 379.00 | 449.00 | 1669.00 | 1779.00 |

a) Draw or describe the bedroom design, including dimensions, where

- one of the dimensions of the bedroom floor must be at least 10 feet
- the area of the bedroom must be between double and triple the area occupied by the total of all the furniture
(2 marks)

Dimensions of bedroom: $\qquad$ $\mathrm{ft} . \times$ $\qquad$ ft .
b) You must also install flooring in the bedroom. Flooring must be purchased in whole units. You can buy carpet or hardwood.

- You must include an additional $10 \%$ to the area for installation and waste.
- The carpet costs $\$ 14.40$ per yd ${ }^{2}$.
- The hardwood costs $\$ 19.80$ per $\mathrm{yd}^{2}$.

Calculate the cost of the flooring before taxes.
(2 marks)
c) Calculate the total cost of the furniture and the flooring, plus GST and PST. (Note: $\mathrm{GST}=5 \%, \mathrm{PST}=8 \%$ )
(1 mark)

## LOGICAL REASONING

## Question 23

a) Create a true conditional statement using two of the statements given below.

- the number is positive
- the number is negative
- the number is less than zero
- the number is prime
- the number is odd
- the number is even
- the number is 5
(1 mark)
b) Write the inverse of the statement created in (a).
(1 mark)

The universal set $B$ represents the breakfast items in Pierre's backpack:

$$
B=\{\text { apple }, \text { yogurt, granola bar }\}
$$

Pierre must choose at least one item to eat for breakfast.
Write all of Pierre's breakfast options as subsets.

Students attend a field trip to the Canadian Museum of Human Rights. They have the option of visiting Gallery A, Gallery B, Gallery C, or any combination of these.

Their visits can be represented by the Venn diagram below.
Shade the Venn diagram to illustrate the students who visited either Gallery B or Gallery C $(B \cup C)$.


A magic square is an array that has the same sum in each row, column, and diagonal. This sum is called the magic number.

| 7 | 20 | 11 | 12 |
| :---: | :---: | :---: | :---: |
| 14 | 8 | 23 | 15 |
| 20 | 6 | 13 | 11 |
| 9 | 16 | 3 | 22 |

a) Determine the number in the array above that prevents it from being a magic square. (1 mark)
b) What number should replace the number found in (a) to make the array a magic square? (1 mark)
c) What is the magic number of the resulting magic square?
(1 mark)

## NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

## Formula Sheet: Applied Mathematics

| RELATIONS AND FUNCTIONS |
| :---: |
| $y=a x+b$ |
| $y=a x^{2}+b x+c$ |
| $y=a x^{3}+b x^{2}+c x+d$ |
| $y=a b^{x}$ |
| $y=a+b \ln (x)$ |
| $y=a \sin (b x+c)+d$ |
| $P(A \cup B)=P(A)+P(B)-P(A \cap B)$ |
| $P(A \cap B)=P(A) \times P(B)$ |
| $P(A \cap B)=P(A) \times P(B \mid A)$ |
| ${ }_{n} P_{r}=\frac{n!}{(n-r)!}$ |
| ${ }_{n} C_{r}=\frac{n!}{r!(n-r)!}$ |

Financial Mathematics

$$
\begin{gathered}
t=\frac{72}{i} \\
I=P r t \\
A=P\left(1+\frac{r}{n}\right)^{n t}
\end{gathered}
$$

Net worth $=$ Total assets - Total liabilities
$\begin{aligned} & \text { Debt-to-equity } \\ & \text { ratio (\%) }\end{aligned}=\frac{(\text { Total liabilities }- \text { Mortgage })}{\text { Net worth }} \times 100$
$\left.\begin{array}{l}\begin{array}{l}\text { Gross debt } \\ \text { service } \\ \text { ratio (\%) }\end{array}\end{array}=\frac{\left(\begin{array}{c}\text { Monthly } \\ \text { mortgage }+\begin{array}{c}\text { Monthly } \\ \text { payment }\end{array} \begin{array}{c}\text { Monthly } \\ \text { taxes }\end{array} \\ \text { Gross monthly income } \\ \text { costs }\end{array}\right.}{\text { pating }}\right) ~ \times 100$
$\begin{aligned} & \text { Rate of } \\ & \text { return }(\%)\end{aligned}=\frac{\left(\begin{array}{c}\text { Current value } \\ \text { of portfolio }\end{array} \quad \begin{array}{c}\text { Previous value } \\ \text { of portfolio }\end{array}\right)}{\text { Previous value of portfolio }} \times 100$

## Design and Measurement

Prism: Surface area $=P h+2 B$
Volume $=B h$
Pyramid: Surface area $=B+\frac{1}{2} P s$

$$
\text { Volume }=\frac{1}{3} B h
$$

Sphere: Surface area $=4 \pi r^{2}$

$$
\text { Volume }=\frac{4}{3} \pi r^{3}
$$

Cylinder: Surface area $=2 \pi r^{2}+2 \pi r h$

$$
\text { Volume }=\pi r^{2} h
$$

Cone: Surface area $=\pi r^{2}+\pi r s$

$$
\text { Volume }=\frac{1}{3} \pi r^{2} h
$$

Square-based pyramid: Surface area $=b^{2}+2 b s$

$$
\text { Volume }=\frac{1}{3} b^{2} h
$$


[^0]:    b) Explain why a small long-term investment may be worth more than a large short-term investment.
    (l mark)

