Grade 12
Applied Mathematics
Achievement Test

## Student Booklet

January 2020

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# Grade 12 Applied Mathematics Achievement Test 

## DESCRIPTION

Time Required to Complete the Test: $\mathbf{3}$ hours
Total Possible Marks: 70
Additional Time Allowed: $\mathbf{3 0}$ minutes

| Unit | Marks |
| :--- | :---: |
| Relations and Functions | 17 |
| Probability | 18 |
| Design and Measurement | 7 |
| Financial Mathematics | 20 |
| Logical Reasoning | 8 |

## Directions

Remember to

- indicate your input values by writing them in your booklet or printing a copy if using a technology tool
- state any assumptions you make
- express your answers in decimal and percentage form to at least the nearest hundredth (two decimal places) when rounding, except for monetary values or when otherwise indicated Example: $\frac{15}{29}=0.52$ or $51.72 \%$

Note: Rounding too soon in your solution may result in an inaccurate final answer for which full marks will not be awarded.

## A clearly communicated answer

- is easily identified in the response space
- includes the parameters in the equation, and " $y=$ ", "sin", "ln", or " $x$ ", as applicable
- includes the units of measure, where applicable
- includes labels, units, scales for the axes on graphs, and key characteristics of functions (e.g., maximum, minimum, intercepts, and appropriate shape)
- is expressed as an exact value or is appropriately rounded

Marks may be deducted for errors relating to any of the above.

## Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited.

## Terminology Sheet

Some questions may include directing words such as calculate, determine, and show your work. These directing words are explained below.

| Directing words | The question is asking for... |
| :--- | :--- |
| Calculate/Determine | a mathematical formula, an algebraic equation, or a numerical <br> calculation to solve a problem |
| Describe/Explain | words or symbols, diagrams, charts or graphs, or other methods <br> that clearly show what you are thinking |
| Use a graphic organizer | a visual representation of information such as a tree diagram, a <br> chart, a list, a Venn diagram, a truth table, or Pascal's triangle |
| Identify/Indicate/Select | a stated or circled answer |
| Show your work/Justify | reasons or facts that support a position by using mathematical <br> computations, words, or diagrams |
| State | a word, sentence, or number, without an explanation |

## Formula Sheet: Applied Mathematics

| RELATIONS AND FUNCTIONS |
| :---: |
| $y=a x+b$ |
| $y=a x^{2}+b x+c$ |
| $y=a x^{3}+b x^{2}+c x+d$ |
| $y=a b^{x}$ |
| $y=a+b \ln (x)$ |
| $y=a \sin (b x+c)+d$ |
| $P(A \cup B)=P(A)+P(B)-P(A \cap B)$ |
| $P(A \cap B)=P(A) \times P(B)$ |
| $P(A \cap B)=P(A) \times P(B \mid A)$ |
| ${ }_{n} P_{r}=\frac{n!}{(n-r)!}$ |
| ${ }_{n} C_{r}=\frac{n!}{r!(n-r)!}$ |

Financial Mathematics

$$
\begin{gathered}
t=\frac{72}{i} \\
I=P r t \\
A=P\left(1+\frac{r}{n}\right)^{n t}
\end{gathered}
$$

Net worth $=$ Total assets - Total liabilities
$\begin{aligned} & \text { Debt-to-equity } \\ & \text { ratio (\%) }\end{aligned}=\frac{(\text { Total liabilities }- \text { Mortgage })}{\text { Net worth }} \times 100$
$\left.\begin{array}{l}\text { Gross debt } \\ \begin{array}{l}\text { service } \\ \text { ratio (\%) }\end{array}\end{array}=\frac{\left(\begin{array}{c}\text { Monthly } \\ \text { mortgage }+\begin{array}{c}\text { Monthly } \\ \text { payment }\end{array} \begin{array}{c}\text { Monthly } \\ \text { taxes }\end{array} \\ \text { Gross monthly income } \\ \text { costs }\end{array}\right.}{\text { pating }}\right) ~ \times 100$
$\begin{aligned} & \text { Rate of } \\ & \text { return }(\%)\end{aligned}=\frac{\left(\begin{array}{c}\text { Current value } \\ \text { of portfolio }\end{array} \quad \begin{array}{c}\text { Previous value } \\ \text { of portfolio }\end{array}\right)}{\text { Previous value of portfolio }} \times 100$

## Design and Measurement

Prism: Surface area $=P h+2 B$
Volume $=B h$
Pyramid: Surface area $=B+\frac{1}{2} P s$

$$
\text { Volume }=\frac{1}{3} B h
$$

Cube: Surface area $=6 l^{2}$
Volume $=l^{3}$
Rectangular prism: Surface area $=2 l w+2 l h+2 w h$

$$
\text { Volume }=l w h
$$

Triangular prism: Surface area $=b h+l(a+b+c)$

$$
\text { Volume }=\frac{1}{2} b h l
$$

Square-based pyramid: Surface area $=b^{2}+2 b s$

$$
\text { Volume }=\frac{1}{3} b^{2} h
$$

Sphere: Surface area $=4 \pi r^{2}$

$$
\text { Volume }=\frac{4}{3} \pi r^{3}
$$

Cylinder: Surface area $=2 \pi r^{2}+2 \pi r h$

$$
\text { Volume }=\pi r^{2} h
$$

Cone: Surface area $=\pi r^{2}+\pi r s$

$$
\text { Volume }=\frac{1}{3} \pi r^{2} h
$$

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

## RELATIONS AND FUNCTIONS

## Question 1

Total: 1 mark
Select the best answer.


Identify which equation the graph represents.
A) $y=2 x^{3}-x^{2}+5 x+3$
B) $y=-2 x^{3}-x^{2}+5 x+3$
C) $y=2 x^{3}-x^{2}+5 x-3$
D) $y=-2 x^{3}-x^{2}+5 x-3$

## Question 2

Select the best answer.
Identify the graph of the following function:

$$
y=3.5 \sin (0.8 x+1.57)+4
$$

A) $y$

C)

B)

D)


## Question 3

Total: 3 marks

At the Manitoba provincial track and field meet, an athlete is competing in the javelin event. On the athlete's first attempt

- the javelin was thrown at a starting height of 1.6 m
- the javelin reached a height of 4 m at a horizontal distance of 7.2 m from the athlete
- the javelin hit the ground 38 m away from the athlete

a) Determine a quadratic regression equation that models the height of the javelin as a function of the horizontal distance from the athlete. Show your work.
(2 marks)

| Horizontal <br> Distance (m) | Height (m) |
| :---: | :---: |
|  |  |
|  |  |
|  |  |

b) Determine the maximum height reached by the javelin.
(1 mark)

In 2020, the elk population in Riding Mountain National Park can be predicted by the following exponential equation:

$$
P=3500(1.03)^{t}
$$

where $P$ represents the elk population and $t$ represents the time (in years) starting in January 2020.
a) Create a clearly labelled graph of the predicted elk population over the next 50 years. (3 marks)

b) Assume that Riding Mountain National Park can support a maximum population of 16000 elk. Using the exponential equation, determine in what year the population will reach 16000.
(1 mark)

An observer collects data for the sea level in Churchill, Manitoba. The sea level rises and falls twice daily. The data is shown below:

| Time (h) | Sea Level (m) |
| :---: | :---: |
| 0 | 4.31 |
| 3 | 2.41 |
| 6 | 0.51 |
| 9 | 2.41 |
| 12 | 4.31 |

a) Determine a sinusoidal regression equation that models this data.
(l mark)
b) Determine the sea level at 5.5 hours.
(1 mark)
c) State the range and explain its meaning in this situation. (2 marks)

A patient has his blood pressure monitored for 16 hours. During this period, his blood pressure can be modelled by the following cubic function:

$$
\begin{aligned}
& \qquad P=-0.05 t^{3}+1.28 t^{2}-7.46 t+101 \\
& \text { where } P \text { represents the blood pressure (in mm of mercury) } \\
& \text { and } t \text { represents the amount of time his blood pressure is monitored (in hours). }
\end{aligned}
$$

a) Determine his lowest blood pressure during this period, in mm of mercury.
(1 mark)
b) Determine how long his blood pressure is at 99 mm of mercury or below. Show your work. (2 marks)
c) Identify the domain of the equation in this situation.
(1 mark)
Select the best answer.
A) $\{0 \leq t \leq 16\}$
B) $\{t \geq 0\}$
C) $\{t \in \mathbb{R}\}$
D) $\{t \leq 16\}$

## Probability

## Question 7

Total: 1 mark

## Select the best answer.

At a marathon, there is a table with 27 cups filled with either a sports drink or water.
The probability that you grab a cup of sports drink as you run by the table is $\frac{19}{27}$.

Identify the odds in favour of grabbing a cup of sports drink.
A) $8: 27$
B) $27: 19$
C) $8: 19$
D) $19: 8$

## Question 8

A student enters a maze and needs to make 4 turns. She must turn left or right at each intersection.

Determine how many different paths are possible if she makes either 1 or 2 right turns. Use a graphic organizer to show your work.

## Question 9

Students at a high school were surveyed about their use of online television services.
The survey results showed the following:

- $48 \%$ of students use Service A
- $40 \%$ of students use Service B
- the remaining students do not use any service
- no student uses both services
a) According to the survey results, is the use of online television services mutually exclusive? Justify your reasoning.
(1 mark)
b) According to the survey results, determine the odds against a student using an online television service.
(1 mark)


## Question 10

Twenty cards numbered 11 to 30 are placed in a box.
Determine the probability of selecting one card from the box that is a multiple of 3 or a multiple of 4 . Show your work.

## Question 11

Your school requires a group of 4 actors for a play.
a) Determine how many ways the group of 4 actors can be chosen from 23 interested students. (1 mark)
b) You and your best friend are 2 of the 23 interested students. Determine the probability that you both are chosen. Show your work.

## Question 12

Total: 4 marks

A dance studio has 9 students: 4 students are ballet dancers and 5 students are hip-hop dancers. They are arranging themselves in a row for a year-end photo.
a) Determine how many ways the dancers can be arranged for the photo if they must alternate between their type of dance. Show your work.
(2 marks)
b) Determine how many ways the dancers can be arranged for the photo if the ballet dancers must all stand together. Show your work.
(2 marks)

## Question 13

Kyla wants to buy a cup of tea for $\$ 2$. She has the following coins in her pocket:

- 2 identical toonies ( $\$ 2$ coin)
- 6 identical loonies ( $\$ 1$ coin)
- 3 identical quarters ( $25 ¢$ coin)
a) Determine the probability of randomly drawing 2 loonies, one after the other, if the first coin is not replaced in her pocket before drawing the second coin. Show your work.
(2 marks)
b) Once she has paid for her tea using the 2 loonies, Kyla decides to stack all of the remaining coins in a tower. Determine the number of different ways she can stack the coins. Show your work.
(2 marks)



## Design and Measurement

## Question 14

## Total: 1 mark

## Select the best answer.

A frozen dessert is made of ice and syrup.
Shauna wants to estimate the amount of ice and syrup in her frozen dessert. Identify the expression that will help her with her estimation.
A) $\frac{\pi r^{2} h}{3}+\frac{4 \pi r^{3}}{3}$
B) $\frac{\pi r^{2} h}{3}+\frac{2 \pi r^{3}}{3}$
C) $\pi r s+2 \pi r^{2}$
D) $\pi r s+4 \pi r^{2}$

## Question 15

Sarah has prepared $7000 \mathrm{~cm}^{3}$ of soup in a pot. She uses a ladle, in the shape of a hemisphere, to serve the soup into bowls. The ladle has a diameter of 10 cm .


Determine the number of full ladles of soup that she can serve. Show your work.

Nashida wants to build an outdoor kitchen.
She is going to place patio stones in the layout shown to the right. 9 ft .
She is going to p
The patio stones

- are in the shape of a square with 18 -inch sides
- cost $\$ 3.00$ each

a) Determine the number of patio stones Nashida needs. Show your work.

Nashida must buy a grill, a countertop, and a patio set for the outdoor kitchen. She has the following options:

| Grill |  | Countertop |  | Patio Set |  |
| :--- | :---: | :--- | ---: | :--- | :---: |
| Propane | $\$ 1400.00$ | Granite | $\$ 2700.00$ | Basic | $\$ 600.00$ |
| Pellet | $\$ 3000.00$ | Soapstone | $\$ 3600.00$ | Elegant | $\$ 1000.00$ |

She also plans to buy three of the following items:

| Side Burner | Fridge |  | Cabinet |  | Sink |  | Warming Drawer |  |  |
| :--- | :---: | :--- | :---: | :--- | :---: | :--- | :--- | :--- | :---: |
| Single | $\$ 400.00$ | Small | $\$ 800.00$ | 30 -inch | $\$ 700.00$ | Single | $\$ 350.00$ | Small | $\$ 800.00$ |
| Double | $\$ 650.00$ | Large | $\$ 1100.00$ | 42 -inch | $\$ 1250.00$ | Double | $\$ 500.00$ | Large | $\$ 1300.00$ |

b) Nashida can spend a maximum of $\$ 11000.00$, taxes included. Calculate the total cost, plus GST and PST, of the patio stones and components of the outdoor kitchen. Show your work. (Note: GST $=5 \%, \mathrm{PST}=7 \%$ )
(2 marks)

## FinAncial Mathematics

c) Nashida is financing the outdoor kitchen with a financial institution that gives her an interest rate of $5.00 \%$, compounded monthly. She wants to make $\$ 300.00$ monthly payments on the loan. Calculate how many payments it will take Nashida to pay off the loan. Show your work. (2 marks)

Kazoo is looking for a house. He has the following options:
Option 1: He can buy a house with a monthly mortgage payment of $\$ 1150.00$ amortized over 25 years.

Option 2: He can rent a similar house for $\$ 1150.00$ per month.
State which option Kazoo should choose. Provide one reason for your choice.

Ham and Sylvie each had \$10 000.00 to invest.
a) Ham invested $\$ 10000.00$ in a mutual fund at an interest rate of $6.00 \%$, compounded monthly. Determine the value of the mutual fund at the end of the first year. Show your work.
(2 marks)
b) Sylvie invested $\$ 10000.00$ in a guaranteed investment certificate (GIC) with interest compounded semi-annually. The value of the GIC was $\$ 11261.62$ at the end of the third year. Determine the interest rate for the GIC. Show your work.
(2 marks)
c) Using the Rule of 72, determine approximately how much longer it will take for Sylvie's GIC to reach a value of $\$ 40000.00$ compared to Ham's mutual fund. Show your work.
(2 marks)

Simba wants to purchase a bed for $\$ 2200.00$ (taxes included). The store offers him a promotion of $0 \%$ interest with no payments for one year. If Simba does not pay the amount in full within one year, interest will be charged from the date of purchase at an interest rate of $19.99 \%$, compounded monthly.
a) If Simba does not make any payments during the first year, calculate the amount the store will bill him one year after the date of purchase. Show your work.
(2 marks)
b) If Simba makes monthly payments over the second year to pay off the amount calculated in (a), determine his monthly payment. Show your work.
(2 marks)
c) Using your answer in (b), calculate the interest Simba would pay over the two-year period. Show your work.

## (1.5 marks)

d) Give one reason why Simba would buy his bed using the promotion. (0.5 mark)

The Ramilo family moved to The Pas. They bought a house with a purchase price of $\$ 229000.00$ and made a down payment of $\$ 20000.00$. Their mortgage has an interest rate of $3.15 \%$, compounded semi-annually, and is amortized over 25 years.
a) Calculate their monthly mortgage payment. Show your work.
(2 marks)
b) Calculate the balance owing on the mortgage after 10 years if they have been making regular monthly payments.
(1 mark)

In 2009, the value of a cottage was $\$ 325000.00$. In 2019, the same cottage had a value of \$425 000.00.

Determine the average annual appreciation rate. Show your work.

## LOGICAL REASONING

## Question 22

Total: 1 mark

Select the best answer.
Ari, Boba, and Cora are shopping at a farmer's market. The Venn diagram below represents the items found in their shopping baskets.


Identify which of the following statements is true.
A) $n(A)<n(C)$
B) $n\left(B^{\prime}\right)=n(A)+n(C)$
C) $n(C)>n(B)$
D) $n\left(C^{\prime}\right)=n(B)+n(A)$

An arithmagon is a puzzle in which the number in each box is the sum of the two numbers in the circles adjacent to that box. Here is an example:

Given:


Therefore, given:


Fill in each circle above with an appropriate number.

There are 7 students in Ms. Sanduk's class. She knows that some of her students have part-time jobs and some of her students participate in extra-curricular activities. However, 2 students neither have a part-time job nor participate in extra-curricular activities.

$$
\begin{aligned}
& A=\{\text { students with part-time jobs }\} \\
& B=\{\text { students who participate in extra-curricular activities }\} \\
& n(A \cap B)=1
\end{aligned}
$$

Fill in the blank diagrams below to show two possibilities in this situation.


Complete the truth table, including the missing symbol in the box, based on the following logical statement:

A number is even if and only if a number is a multiple of two.

| $\boldsymbol{p}$ | $\boldsymbol{q}$ | $\boldsymbol{p} \square \boldsymbol{q}$ |
| :---: | :---: | :---: |
| True | True |  |
| True | False |  |
| False | True |  |
| False | False |  |

To form a group, 4 students are randomly chosen from 7 students. Jean writes the following conditional statement:
"If all 7 students have an equal chance of being chosen, then there are 840 different groups that could be formed."
a) Write the contrapositive of the conditional statement.
(1 mark)
b) Is the original conditional statement true? Justify your answer.
(1 mark)

